

## A NEW CHANNEL TO SEARCH FOR EXTRA-SOLAR SYSTEMS WITH MULTIPLE PLANETS VIA GRAVITATIONAL MICROLENSING

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### ABSTRACT

Gaudi, Naber & Sackett pointed out that if an event is caused by a lens system containing more than two planets, all planets will affect the central region of the magnification pattern, and thus the existence of the multiple planets can be inferred by detecting additionally deformed anomalies from intensive monitoring of high magnification microlensing events. Unfortunately, this method has important limitations in identifying the existence of multiple planets and determining their parameters (the mass ratio and the instantaneous projected separation) due to the degeneracy of the resulting light curve anomalies from those induced by a single planet and the complexity of multiple planet lensing models. In this paper, we propose a new channel to search for multiple planets via microlensing. The method is based on the fact that the lensing light curve anomalies induced by multiple planets are well approximated by the superposition of those of the single planet systems where the individual planet-primary pairs act as independent lens systems. Then, if the source trajectory passes both of the outer deviation regions induced by the individual planets, one can unambiguously identify the existence of the multiple planets. We illustrate that the probability of successively detecting light curve anomalies induced by two Jovian-mass planets located in the lensing zone through this channel will be substantial. Since the individual anomalies can be well described by much simpler single planet lensing models, the proposed method has an important advantage of allowing one to accurately determine the parameters of the individual planets.

*Key words* : gravitational lensing – planets and satellites: general

### I. Introduction

Although most of best candidate extra-solar planets were discovered by the spectroscopic radial velocity technique (e.g. Marcy & Butler 1996, see also <http://exoplanets.org/mult.shtml> for updated planet candidates), they can also be detected by using other techniques (Perryman 2000). Microlensing is one of the promising techniques. While the radial velocity technique has limited applicability only to planets belonging to stars located in the solar neighborhood, the lensing technique enables one to detect planets located at very large distances, and thus it is a complement method to the radial velocity technique.

Planet detection via microlensing is possible because the lensing event caused by a lens system containing a planet can produce noticeable anomalies in the resulting light curve when the source passes close to the lens caustics, which represents the source positions on which the lensing magnification of a point source event becomes infinity (Mao & Paczyński 1991; Gould & Loeb

1992).<sup>1</sup> For a lens system with a planet, there exist two or three disconnected sets of caustics. Among them, one is located close to the primary lens (central caustic) and the other(s) is (are) located away from the primary lens (planetary caustic[s]). Accordingly, there exist two types of planet-induced anomalies: one affected by the planetary caustic (type I anomaly) and the other affected by the central caustic (type II anomaly) (Han & Kim 2001). Compared to the frequency of type I anomalies, type II anomaly occurs with a relatively low frequency due to the smaller size of the central caustic compared to the corresponding planetary caustic. However, the efficiency of detecting type II anomalies can be high because intensive monitoring is possible due to the predictable time of anomalies, i.e. near the peak of magnification, and the known type of candidate events for intensive follow-up monitoring, i.e. very high magnification events (Griest & Safizadeh 1998).

Keeping the high efficiency of type II anomaly detections in mind, Gaudi, Naber, & Sackett (1998) pointed out that if an event is caused by a lens system having multiple planets located in the lensing zone, within

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<sup>1</sup>See § 2 for more details about the caustics.

which the chance for the occurrence of planet-induced anomalies is maximized,<sup>2</sup> all planets will affect the central region of magnification pattern, and thus the existence of multiple planets can be inferred by detecting additionally deformed anomalies from intensive monitoring of high magnification events. This method, however, has important limitations in identifying the existence of multiple planets and determining their parameters (the mass ratio and the instantaneous separation between each planet and the host star). This is because the anomalies induced by multiple planets are in many cases qualitatively degenerate from that induced by a single planet and even if the existence of the multiple planets is known, accurate determination of the individual planet parameters will be difficult due to the complexity of multiple planet lensing models.

In this paper, we propose a new channel to search for extra-solar systems composed of multiple planets via microlensing. The method is based on the fact that the lensing light curve anomalies induced by multiple planets are well approximated by the superposition of those of the single planet systems where the individual planet-primary pairs act as independent lens systems (Bozza 1999; Han et al. 2001). Then, if the source trajectories passes both of the outer deviation regions around the planetary caustics of the individual planets, one can identify the existence of the multiple planets. We illustrate that the probability of successively detecting lensing light curve anomalies induced by two Jovian-mass planets located in the lensing zone through this channel will be substantial. We discuss the advantages of the proposed method over the previous method of monitoring high magnification events.

## II. Basics of Multiple Planet Lensing

If a source located at  $\mathbf{r}_s$  on the projected plane of the sky is lensed by a lens system composed of  $N$ -point masses, where the individual components' masses and locations are  $m_i$  and  $\mathbf{l}_i$ , the positions of the resulting images,  $\mathbf{r}$ , are obtained by solving the lens equation of the form

$$\mathbf{r}_s = \mathbf{r} - \theta_E^2 \sum_{i=1}^N \frac{m_i}{m} \frac{\mathbf{r} - \mathbf{l}_i}{|\mathbf{r} - \mathbf{l}_i|^2}, \quad (1)$$

where  $m = \sum_i^N m_i$  is the total mass of the lens system and  $\theta_E$  is the angular Einstein ring radius. The angular Einstein ring radius is related to the physical parameters of the lens system by

$$\theta_E = \sqrt{\frac{4Gm}{c^2}} \left( \frac{1}{D_{ol}} - \frac{1}{D_{os}} \right)^{1/2}, \quad (2)$$

where  $D_{ol}$  and  $D_{os}$  are the distances to the lens and the source from the observer, respectively. Since the

lensing process conserves the surface brightness of the source, the magnification of each image is simply given by the ratio between the surface areas of the image and the unamplified source, and mathematically its value corresponds to the inverse of the Jacobian of the lens equation evaluated at the image position  $\mathbf{r}_j$ ;

$$A_j = \left( \frac{1}{|\det J|} \right)_{\mathbf{r}=\mathbf{r}_j}; \quad \det J = \left| \frac{\partial \mathbf{r}_s}{\partial \mathbf{r}} \right|. \quad (3)$$

The total magnification is then given by the sum of the individual images' magnifications, i.e.  $A_{\text{tot}} = \sum_j^{N_I} A_j$ , where  $N_I$  is the total number of images.

For a single point-mass lens ( $N = 1$ ), there are two solutions for the lens equation ( $N_I = 2$ ) and the resulting total magnification is expressed in a simple analytical form of

$$A = \frac{u_s^2 + 2}{u_s \sqrt{u_s^2 + 4}}, \quad (4)$$

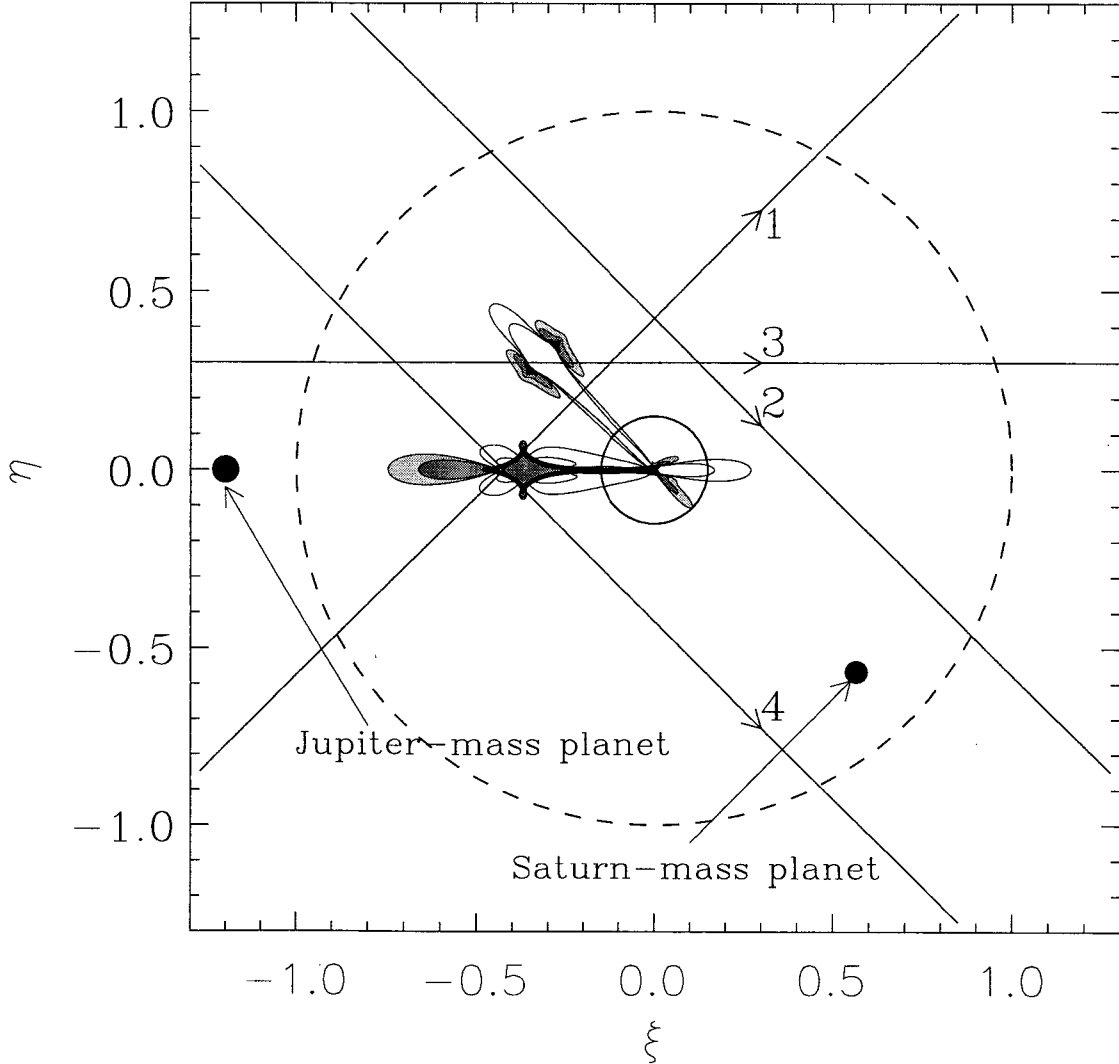
where  $\mathbf{u}_s = (\mathbf{r}_s - \mathbf{l})/\theta_E$  is the dimensionless lens-source separation vector normalized by  $\theta_E$ . For a rectilinear lens-source transverse motion, the separation vector is related to the single lensing parameters by

$$\mathbf{u}_s = \left( \frac{t - t_0}{t_E} \right) \hat{\mathbf{x}} + \beta \hat{\mathbf{y}}, \quad (5)$$

where  $t_E$  represents the time required for the source to transit  $\theta_E$  (Einstein time scale),  $\beta$  is the closest lens-source separation in units of  $\theta_E$  (impact parameter),  $t_0$  is the time at that moment, and the unit vectors  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{y}}$  are parallel with and normal to the direction of the relative lens-source transverse motion, respectively.

The lens system with planets is described by the formalism of multiple lensing with very low mass companions. Since our discussion is focused on multiple lens systems having very low mass companions, we recommend to see Paczyński (1996) and Park & Han (2001) for the lensing behavior of general multiple lens systems. For a binary lens system ( $N = 2$ ), the lens equation becomes a fifth order polynomial in  $r$  and the positions of the individual images are obtained by numerically solving the polynomial (Witt 1990). This yields three or five solutions depending on the source position with respect to the lenses. The main new feature of a multiple lens system is the formation of caustics. For a binary lens, the set of caustics form several disconnected closed curves. The number and locations of the caustic curves are dependent on the separation between the planet and the primary lens. If the separation is larger than  $\theta_E$ , there exist two sets of caustics and one is the central caustic located near the primary lens and the other is the planetary caustic located away from the primary on the planet side with respect to the primary. The lens system having a planet with a separation smaller than  $\theta_E$  also has a single central caustic, but has two planetary caustics located on the opposite

<sup>2</sup>See also § 2 for more details about the lensing zone.



**Fig. 1.**— The map of magnification excess of an example lens system composed of two planets with mass ratios of 0.003 and 0.001, which are comparable to those of Jupiter-mass and Saturn-mass planets around a  $\sim 0.3 M_{\odot}$  star, respectively. The coordinates  $(\xi, \eta)$  are set so that the primary is located at the origin, and  $\xi$  axis is parallel with the axis connecting the primary and the Jupiter-mass planet. All lengths are scaled by the angular Einstein ring radius,  $\theta_E$ . The projected separations (normalized by  $\theta_E$ ) of the individual planets from the host star are  $b_J = 1.2$  and  $b_S = 0.8$ , respectively. The angle between the position vectors to the individual planets from the host star is  $\alpha = 135^\circ$ . Contours are drawn at the levels of  $\epsilon = -0.1, -0.05, 0.05, \text{ and } 0.1$  and the regions of positive deviations are distinguished by grey scales. The small figures drawn by thick solid lines are the caustics and the dashed circle is the Einstein ring. The region enclosed by the solid circle around the primary with a radius  $u_s = 0.15$  represents the central deviation region, which is excluded in our estimation of the probability for detecting both planets. The straight lines with arrows represent the source trajectories of several example events for which one can detect both (the trajectory designated by a number ‘1’), one (‘3’ and ‘4’), and none (‘2’) of the planets, respectively.

side of the planet. The planetary caustic(s) is (are) located within the Einstein ring when the projected separation between the planet and the primary (normalized by  $\theta_E$ ) is in the lensing zone of  $0.618 \leq b \leq 1.618$ . Since the sizes of both the central and planetary caustics are maximized at around this separation, planet detection probability is also maximized for systems having planets located in this lensing zone (Gould & Loeb 1992).

For a lens system with triple lenses ( $N = 3$ ), the lens equation becomes a tenth order polynomial and it is still numerically solvable. For these lens systems, there are a maximum of ten images and a minimum of four images, and the number of images changes by a multiple of two as the source crosses a caustic. Unlike the caustics of a binary lens system forming separate sets of closed curves, those of a triple lens system can exhibit self-intersection and nesting.

For lens systems with even larger number of lenses ( $N \geq 4$ ), numerically solving the lens equation becomes very difficult. However, one can still obtain the magnification patterns of these lens systems by using the inverse ray-shooting technique (Schneider & Weiss 1986; Kayser, Refsdal & Stabell 1986; Wambsganss, Paczyński, & Schneider 1990). In this method, a large number of light rays are uniformly shot backwards from the observer plane through the lens plane and then collected (binned) in the source plane. Then, the magnification pattern is obtained by the ratio of the surface brightness (i.e., the number of rays per unit area) on the source plane to that on the observer plane. Once the magnification pattern is constructed, the light curve resulting from a particular source trajectory corresponds to the one-dimensional cut through the constructed magnification pattern. This technique has an advantage of allowing one to construct magnification patterns regardless of the number of lenses, but it has a disadvantage of requiring large computation time to obtain smooth magnification patterns.

### III. A New Channel

It was recently known by Bozza (1999) and Han et al. (2001) that the light curve anomalies induced by multiple planets are well approximated by the superposition of those of the single planet lens systems where the individual planet-primary pairs act as independent lens systems. As pointed out by Gaudi et al. (1998), then, the anomaly pattern in the central region caused by one of the planets can be significantly affected by the existence of other planet(s) because the central deviation regions caused by the individual planets occur in this same region. However, the anomaly pattern in the outer deviation region caused by each planet is hardly affected by other planet(s) because the outer deviation regions of the individual planets occur, in general, at different locations.

The fact that the anomaly patterns in the outer deviation regions are scarcely affected by other planet(s) provides a new channel to search for multiple planets

and to determine their parameters. In this channel, multiple planets are detected when the source trajectory passes both of the outer deviation regions around the planetary caustics of the individual planets. The greatest advantage of this method over the method of monitoring high magnification events is that one can unambiguously identify the existence of multiple planets from the unique pattern of consecutive anomalies and accurately determine the planet parameters due to the applicability of much simpler single planet lensing models to the individual anomalies.

### IV. Efficiency of the New Channel

In this section, we illustrate that the efficiency of detecting multiple planets through the new channel will be substantial. To demonstrate this, we estimate the probability of successively detecting lensing light curve anomalies induced by multiple planets. The probability is determined for a lens system having two Jovian-mass planets (i.e. Jupiter and Saturn) orbiting a host star with a mass of  $\sim 0.3 M_\odot$ . Then, the mass ratios of the planets (to that of the host star) are  $q_J = 0.003$  and  $q_S = 0.001$  for the Jupiter-mass and the Saturn-mass planets, respectively. If the lens system is located at  $D_{ol} \sim 6$  kpc (and with a source at  $D_{os} \sim 8$  kpc) and the component planets have intrinsic orbital separations similar to those of Jupiter and Saturn of our solar system, the orbital separations (normalized by  $\theta_E$ ) of the planets correspond to  $a_J \sim 2.7$  and  $a_S \sim 5.0$ , respectively.

To estimate the probability, we first construct maps of magnification excess. The magnification excess is defined by

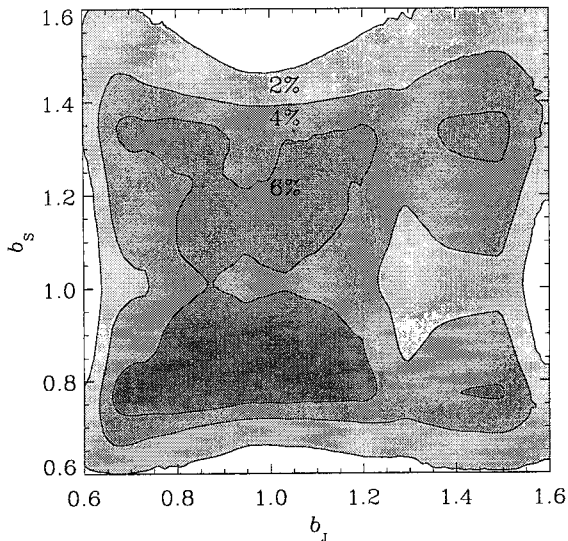
$$\epsilon = \frac{A - A_0}{A_0}, \quad (6)$$

where  $A$  and  $A_0$  represent the magnifications expected with and without the planets, respectively. Figure 1 shows the constructed map when the projected separations of the individual planets are  $b_J = 1.2$  and  $b_S = 0.8$  and the angle between the position vectors to the individual planets from the host star (orientation angle) is  $\alpha = 135^\circ$ . We note that the projected separation,  $b$ , is related to the intrinsic orbital separation,  $a$ , by

$$b = a \sqrt{\cos^2 \phi + \sin^2 \phi \cos^2 i}, \quad (7)$$

where  $\phi$  is the phase angle and  $i$  is the inclination of the orbital plane.

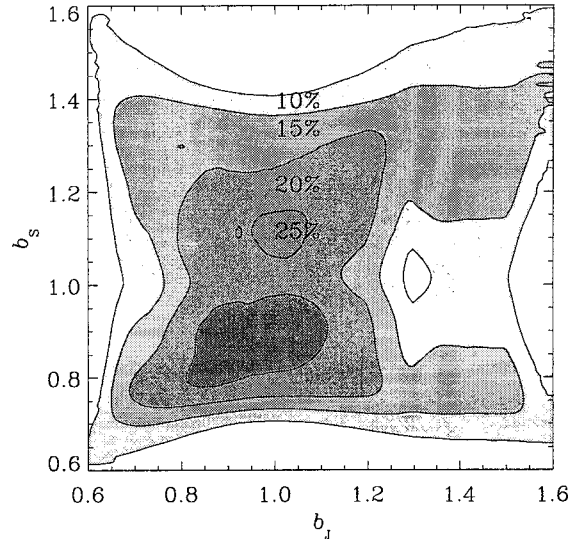
Once the map is constructed, the probability is estimated by computing the ratio of events whose source trajectories pass both of the outer deviation regions induced by the individual planets (for example, the event resulting from the source trajectory designated by '1' in Fig. 1) during the measurements to the total number of trial events. For given lens positions, the source trajectory orientations (with respect to the Jupiter-primary axis) and the impact parameter (with respect to the



**Fig. 2.**— The distribution of the absolute probabilities to detect both planets of the example lens system with two Jovian-mass planets as functions of the projected separations of the individual planets from the host star,  $P_{abs}(b_J, b_S)$ . The observational conditions and the detection criteria are described in § 4.

primary) of the trial events are randomly selected in the ranges of  $0 \leq \theta \leq 2\pi$  and  $0 \leq \beta \leq 1.0$ , respectively. Measurements for each event are assumed to be carried out during  $-t_E \leq t_{obs} \leq t_E$  with a frequency of 10 times/night, which corresponds to that of the current microlensing followup observations (Rhie et al 1999; Albrow et al. 2001; Bond et al. 2001). We consider each planet is detected if excesses greater than a threshold value of  $\epsilon_{th} = 5\%$  are consecutively detected more than 5 times during the measurements. Since we are interested in anomalies occurred only in the outer deviation regions, we do not count the detections of anomalies occurred in the central deviations region (the region enclosed by the solid circle around the primary with a radius  $u_s = 0.15$ ). In addition, since identifying the existence of both planets and determining their parameters will be difficult if both of the planets' outer deviation regions are located at a similar place due to the resulting complexity of the interfered anomaly pattern, we also do not count detections if the time interval between the anomalies induced by the individual planets is shorter than  $0.1t_E$ .

In Figure 2, we present the distribution of the *absolute* probabilities of detecting both planets as functions of their projected separations,  $P_{abs}(b_J, b_S)$ . The presented probabilities are the values averaged over the random orientation angles in the range of  $0 \leq \alpha \leq 2\pi$ . In Figure 3, we also present the distribution of *conditional* probabilities to successively detect the second planet under the condition that the first planet is detected,  $P_{cond}(b_J, b_S)$ . When  $b \sim 1.0$ , the planetary and the central caustics merge together and the merged



**Fig. 3.**— The distribution of the conditional probabilities to successively detect the second planet under the condition that the first planet is detected,  $P_{abs}(b_J, b_S)$ .

caustic is located at the central region. Since we do not count planet detections identified from the anomaly occurred in the central region, the probability patterns in Fig. 2 and 3 do not peak at  $(b_J, b_S) = (1.0, 1.0)$  although the caustic size becomes maximum at this separation. One finds that if the two planets are located in the lensing zone, detecting both of them is possible with absolute probabilities of  $P_{abs} \gtrsim 2\%$ . One also finds that once a planet is detected, the probabilities to successively detect the second one are  $P_{cond} \gtrsim 10\%$ .

## V. Conclusion

We propose a new channel of detecting extra-solar systems composed of having multiple planets by using microlensing. In this method, multiple planets are detected when the source trajectory passes both of the outer regions of deviations induced by the individual planets. From the estimation of the probabilities to detect both planets of an example Galactic lens system composed of two Jovian-mass planets around a star with  $\sim 0.3 M_\odot$ , we find that if they are located within the lensing zone, both planets can be detected with a non-negligible absolute probabilities ( $P_{abs} \gtrsim 2\%$ ) and a substantial conditional probabilities ( $P_{cond} \gtrsim 10\%$ ) of successively detecting the second planet under the condition that the first planet is detected. The proposed method has an important advantage of allowing one to accurately determine the planet parameters because the light curve anomalies induced by the individual planets can be well described by simple single planet lensing models.

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