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확률적 수요하에서의 자동창고의 필요 저장능력 추정

조 면 식[1†] · Yavuz A. Bozer[2]

[1]경기대학교 산업공학과 / [2]미시간대학교

Storage Capacity Estimation for Automated Storage/Retrieval Systems under Stochastic Demand

[1]Myeonsig Cho · [2]Yavuz A. Bozer

[1]Department of Industrial Engineering, Kyonggi University, Suwon, 442-760
[2]Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, MI 48109, USA

Most of studies on automated storage/retrieval (AS/R) system assumed that storage capacity is given, although it is a very important decision variable in the design phase. We propose a simple algorithm to estimate the required storage capacity, i.e., number of aisles and number of openings in vertical and horizontal directions in each aisle, of an AS/R system under stochastic demand, in which storage requests occur endogenously and exogenously while the retrieval requests occur endogenously from the machines. Two design criteria, maximum permissible overflow probability and maximum allowable storage/retrieval (S/R) machine utilization, are used to compute the storage capacity. This model can be effectively used in the design phase of new AS/R systems.

Keywords : AS/RS, S/R machine, storage capacity estimation, stochastic demand

1. Introduction

Automated storage/retrieval(AS/R) systems continue to play a significant role in manufacturing and warehousing due to higher space utilization and accurate inventory control, among other benefits. The AS/R systems are not only used to store the raw materials and/or finished goods, but also used more often to store the work-in-process (WIP) in automated factories. Although the AS/R system in an automated factory is a supporting facility to store and retrieve WIP, if it is not properly designed, it would be a bottleneck to meet the manufacturing requirement. Furthermore, initial investment cost for AS/R system is high and reconfigurability of the system is limited so that AS/R system should be designed carefully.

One of the common types of AS/R systems is the unit load AS/RS, where pallet loads are stored and subsequently retrieved, one at a time, by a storage / retrieval (S/R) machine. An AS/R system consists of one or several aisles and each aisle consists of an input/output (I/O) point, a S/R machine, and storage racks on both sides. Typically the I/O point is located at the lower left hand corner of the rack and a pair of short conveyors serves as the I/O point; one conveyor for input and one conveyor for output. Loads to be stored wait at the input point until the S/R machine is available and loads retrieved by the S/R machine are deposited at the output point.

The storage and retrieval requests are served by the S/R machine which performs either single command (SC) or dual command (DC). While the S/R machine serves either a storage or a retrieval request in SC, it serves storage and retrieval requests sequentially in

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† Corresponding author : Professor Myeonsig Cho, Deparment of Industrial Engineering, Kyonggi University, Suwon, Kyonggi-Do, 442-760 Korea, Fax : 82-31-244-3534, e-mail : mscho@kuic.kyonggi.ac.kr

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DC. There are two types of the SC: storage SC and retrieval SC.

In most of the studies on AS/R system, the S/R machine dwells at the I/O point if there is no more requests to serve. That is, S/R machine travel always starts from I/O point and ends up at the I/O point. Another S/R machine dwell point strategy is to keep the idle S/R machine at the point of deposit (within the rack) following the storage SC, and at the I/O point following either retrieval SC or DC. If the S/R machine is idle within the rack and the next request is retrieval (storage), then the S/R machine travels directly to the retrieval point (I/O point) to pick up the load. Such dwell point strategy is discussed in Bozer and White (1984) and Egbelu and Wu (1993).

In this paper, using the latter dwell point strategy and assuming that storage and retrieval requests occur randomly and independently, we propose an algorithm to estimate the required storage capacity of an unit load AS/RS under two design constraints; maximum permissible overflow probability and maximum allowable S/R machine utilization. Using this algorithm, AS/R system designer can easily obtain the number of aisles and number of openings in vertical and horizontal directions in each aisle to satisfy the projected throughput capacity of the AS/RS.

2. Literature Review

A number of papers on AS/R system continue to appear in the literature. In addition to results such as the expected value and/or distribution of SC and DC travel time (see, for example, Bozer and White (1984), Chang *et al.* (1995), and Foley and Frazelle (1991), among others), various papers have investigated operational issues such as S/R machine dwell point strategies, storage-retrieval sequencing, and storage methods (see Chang and Egbelu(1997a and 1997b), Egbelu and Wu (1993), Elsayed and Lee (1996), Hwang and Lim(1993), and Lee and Schaefer(1996), among others), and storage-retrieval matching or AS/RS control / design strategies (see Han *et al.*(1987), Linn and Wysk(1990), Rosenblatt *et al.*(1993), and Wang and Yih(1997), among others).

However, most of these studies tend to consider the AS/RS as a stand alone system, that is, interaction between the unit load AS/RS and manufacturing facility is ignored. Furthermore, until recently, most of the researchers ignore the stochastic nature of an AS/RS, i.e., they assume that all the storage and retrieval requests are known and waiting to be processed at time zero, or depends entirely on simulation to analyze the stochastic nature of the requests, which is costly and time consuming.

Recently, Lee (1998) studies to determine the storage capacity under full turnover based storage policy, which minimizes storage space and shortage costs while satisfying specified service level. However, S/R machine travel time and command types are not explicitly incorporated in the model.

Bozer and Cho (1998a) derive closed form analytical results to evaluate the performance of an AS/R system under stochastic demand and determine whether it meets required throughput or not. The ratio of SC and DC is not predetermined in this study. The S/R machine performs either SC or DC depending on the types of requests to serve. To develop the analytical models, they use the latter dwell point strategy, i.e., the S/R machine idles either at the I/O point or within the rack when it becomes idle. They show empirically that this dwell point strategy is reasonable and performs well compared to the dwell point strategy where the S/R machine always starts and finishes at the I/O point.

Bozer and Cho(1998b) present an approximate analytical model to estimate the expected waiting times for the random storage and retrieval requests, using also the latter dwell point strategy. This model enhances their previous work in the sense that expected waiting times (and the associated mean queue lengths) play an important role in deciding whether the performance of a stable system is actually acceptable or not. Note that, although the system is stable, if we have to provide large space for waiting parts, it would not be desirable. This model can be used to determine the amount of buffer size of the input conveyor and the amount of the rack openings which is required to hold the parts which are requested by machines but waiting in the rack to be retrieved by the S/R machine.

3. Storage Capacity Model

In this section, we present a storage capacity algorithm to determine the number of aisles and openings in horizontal and vertical directions per aisle. <Figure 1> depicts schematic view of an automated manufacturing system consisting of several machines, storage aisles, and conveyors. An incoming part to the system arrives at the incoming conveyor located at the upper left hand side, travels toward the main conveyor, moves to the appropriate storage aisle, travels to the I/O point through storage conveyor, moves to the storage opening by a S/R machine, and waits there until a machine requests it to process. A part retrieved from the storage rack travels to the main conveyor through retrieval conveyor and then moves to the appropriate machine buffer through main conveyor and transporter which connects main conveyor and the machines.

Since machine buffer capacity in the automated manufacturing system is usually limited, we assume that at most one part can reside at each machine buffer. When machine starts to process a part located in buffer, a retrieval request is generated for the next part which is scheduled to be processed by that machine. Machine processing times are exponentially distributed with different means. If the completed part from this machine requires further operation from other machines in the system, it travels to the least utilized storage aisle through conveyor and is stored at the closest open location in the aisle until a request for this part is generated. Otherwise, it leaves the system through outgoing conveyor located at the upper left hand side of <Figure 1>.

Utilization of the ith machine, ρ_i, can be obtained as λ_i/μ_i, where λ_i is the part arrival rate to machine i and μ_i is the service rate at machine I. Assuming M/M/1 and infinite buffer capacity, mean, ν_i and variance, σ_i^2, of machine i queue length can be computed as follows (Kobayashi, 1978).

$$\nu_i = \frac{\rho_i^2}{1-\rho_i} \tag{1}$$

$$\sigma_i^2 = \frac{\rho_i^2(1+\rho_i-\rho_i^2)}{(1-\rho_i)^2} \tag{2}$$

The probability that there are n parts in queue I, P_{in}, is

$$P_{in} = \begin{cases} 1-\rho_i^2 & n=0 \\ (1-\rho_i)\rho_i^{n+1} & n>0 \end{cases}$$

However, since machine buffer capacity is limited to one, expected number of parts at machine buffer i, γ_i, can be obtained as

$$\gamma_i = 0 \times P_{i0} + 1 \times P_{i1} = \rho_i^2 - \rho_i^3 \tag{3}$$

We present a storage capacity algorithm below and discuss it step by step.

[Storage Capacity Algorithm]

Step 1. Compute mean and variance of each machine queue length using (1) and (2).

Step 2. Let $M = \sum_{i=1}^{N} \nu_i$ and $V = \sum_{i=1}^{N} \sigma_i^2$, where N is the number of machines in the system.

Step 3. Compute the sum of the expected number of parts in each machine queue, $X = \sum_{i=1}^{N} \gamma_i$. Note that buffer capacity of each machine is

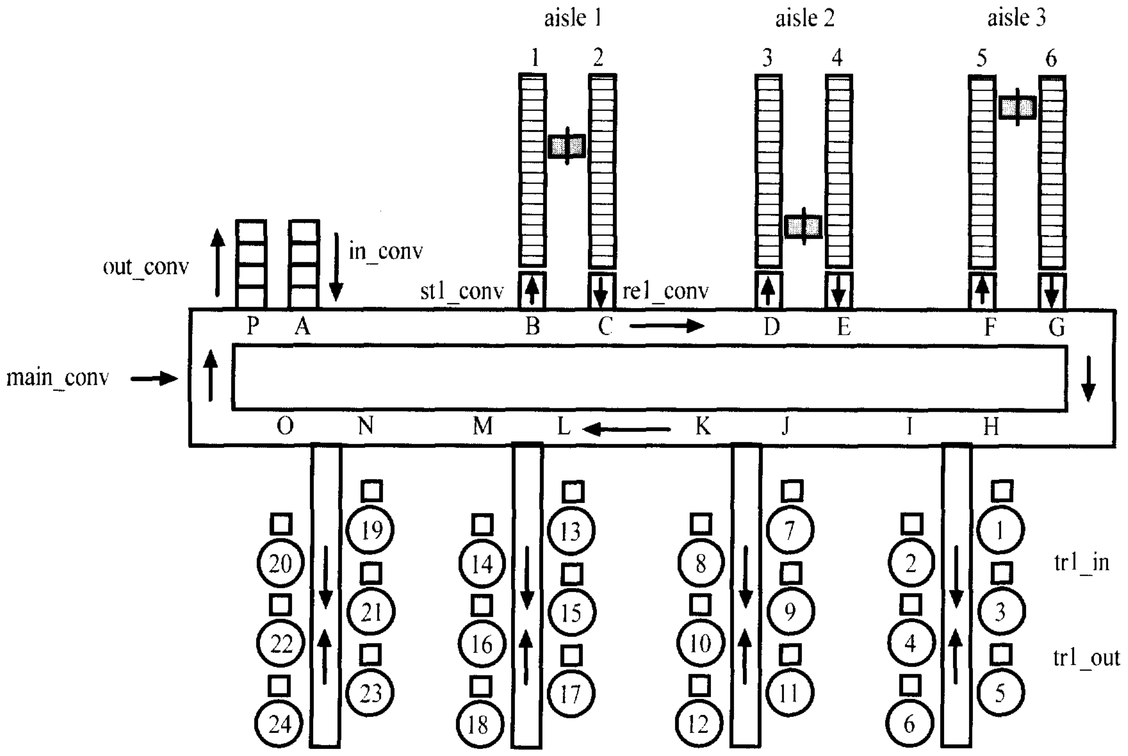

Figure 1. An automated manufacturing system.

limited to 1.

Step 4. Let $\hat{M} = M - X$. That is, $\hat{M}$ is the expected number of parts to be stored in the storage rack. Recall that we assume there is no time delay due to material handling device such as conveyor.

Step 5. Initialize NAISLE = OLD NAISLE = 0, where NAISLE is the number of storage aisles.

Step 6. Set NAISLE = NAISLE + 1 and NSIDE = 2 × NAISLE, where NSIDE is the number of storage rack sides. Note that we have two rack sides per storage aisle.

Step 7. Compute real number of openings required, T, using truncated normal distribution with mean $\hat{M}$ and variance V, to account for the maximum allowable overflow probability. Then, the real number of openings per rack side can be obtained as $T/NSIDE$.

Since we do not know the distribution of parts to be stored, one may want to use Chebyshev's one sided inequality to compute the required number of openings (Ross, 1984) (Note that number of openings should be positive). However, we observed empirically that the rack size tends to be overestimated with Chebyshev's one sided inequality.

Step 8. Find the discrete number of columns and rows per storage rack, since we cannot build a storage rack with real number of openings. Note that the rack size, A, should be smallest but greater than required area per side ($T \times (area\ per\ opening) / NSIDE$) and as close to the square in time (SIT) rack as possible to minimize S/R machine travel time. Let K denote the number of discrete openings per side. Note that $K \geq \frac{T}{NSIDE}$. Also compute the shape factor of the discrete rack (Bozer and White, 1984).

Step 9. Find closest open location (COL) area per side, C, as follows.

$$C = \frac{\hat{M} \times (area\ per\ opening)}{NSIDE}.$$

We also compute rack utilization as $R = \frac{\hat{M}}{K \times NSIDE}$. Recall that there are $\hat{M}$ parts, in average, in the rack.

Step 10. Compute the horizontal and vertical lengths of the COL area.

Step 11. Using the throughput capacity model presented in Bozer and Cho(1998a), compute the stability condition and the expected S/R machine utilization based on COL area (Although the required number of openings should be greater than $\hat{M}$, there are in average $\hat{M}$ parts in the storage). If the stability value is less than 1 and the computed S/R machine utilization is less than the allowable S/R machine utilization, then go to Step 12. Otherwise, go to step 6.

Step 12. If *OLD NAISLE = NAISLE*, stop this algorithm. Otherwise, set *OLD NAISLE = NAISLE* and go to Step 13.

Step 13. Using the waiting time model shown in Bozer and Cho(1998b), compute the retrieval queue length per aisle, L, and add L to $\hat{M}$ ($\hat{M} = \hat{M} + L \times NAISLE$) to account for the parts which are requested but waiting for the S/R machine. Then, go to Step 7. Here, we assume that V remains unchanged.

4. Experiment and Results

Recall that S/R machine becomes idle either at the I/O point or within the rack and the S/R machine performs either SC or DC depending on the types of service requests. However, ratio of SC and DC and their sequences are not known in advance. Storage and retrieval requests are served FCFS by a S/R machine. A part to be stored will be assigned to the least utilized storage rack side. If tie occurs, nearest storage rack side from the incoming conveyor or machines will be assigned for this part. Once the part arrives at the storage rack, it is stored in the closest open location in that storage rack side.

Horizontal and vertical lengths of a storage opening are 9 feet, respectively. Horizontal and vertical speeds of the S/R machine are 400 ft/min and 100 ft/min, respectively. Pick up and deposit times are 0.05 minutes. If the number of parts stored in the storage rack exceeds the storage capacity, then storage rack is overflowed and parts to be stored cannot enter the main conveyor until a part is retrieved from the storage (In this study, we assume that time delay due to material handling device such as conveyor is negligible).

To verify the performance of the algorithm, we test two problems, P1 and P2. In both problems, three types of parts enter the system through the incoming conveyor. 50 % of the incoming parts is part type 1 and its routing is machines 1, 3, 8, 10, 6, 15, 18, 23, 21, and 14. 30 % of the incoming parts is part type 2

Table 1. Machine processing times (min) and expected machine utilizations in P1

Machine number	Processing time	Expected utilization	Machine number	Processing time	Expected utilization	Machine number	Processing time	Expected utilization
1	7	0.8167	9	8	0.6667	17	15	0.75
2	12	0.6	10	6	0.8	18	10	0.8333
3	6	0.8	11	25	0.8333	19	14	0.7
4	16	0.5333	12	15	0.75	20	15	0.5
5	12	0.6	13	22	0.7333	21	8	0.6667
6	5	0.8333	14	10	0.8333	22	17	0.5667
7	20	0.6667	15	10	0.8333	23	10	0.8333
8	6	0.8	16	18	0.9	24	12	0.6

and its routing is 2, 5, 9, 6, 8, 10, 12, 3, 16, 17, 24, and 19. Rest of them is part type 3 and its routing is 11, 9, 6, 7, 4, 1, 13, 22, and 20. The average interarrival time of parts to the system in P1(P2) is exponentially distributed with mean 6(4.5).

There are 24 machines in both problems and their processing times are exponentially distributed with means shown in <Table 1, (2)> for P1 (P2). Expected machine utilizations, which can be easily computed using arrival rate and processing rate, are also summarized in these tables. Note that the main difference in these problems is the level of the expected machine utilizations. Range of machine utilizations in P1(P2) is 0.5~0.9 (0.8~0.89) and arithmetic average utilization over all the machines in P1 (P2) is 0.727 (0.8583). That is, P2 would require more storage space than P1.

In the tested problems, we assume that maximum permissible storage rack overflow probability is 0.05 and maximum allowable S/R machine utilization is 0.8. If overflow is not allowed at all, the required AS/RS capacity should be very large to store maximum number of parts, which is very costly. Furthermore, if we set maximum allowable S/R machine utilization very high, then the S/R machine will be the bottleneck and the required storage capacity will be increased unnecessarily.

We use SIMAN V(1995) to simulate the system. In simulation, we warm up the system to eliminate initial bias until 5,000 parts arrive at the system. After warm up period, we make a single long run and divide the collected observations into 10 replications. Each replication contains 200,000 storage and retrieval requests. We found that there is no significant statistical correlation among replications. That is, to analyze the simulation output statistically, we use the batch means methodology.

Applying the algorithm to P1, we obtain heuristically that 93.35 opening (T) are required, although there are 60.34 parts ($\hat{M}$) in the storage in average. However, in order to have an integer number openings per rack side, we need 96 openings. That is, algorithm computes that we need 2 aisles with 3(8) openings in horizontal (vertical) direction. The shape factor value of this discrete rack is 0.667, which means that this is not a SIT rack. With 96 openings, we obtain following

Table 2. Machine processing times (min) and expected machine utilizations in P2

Machine number	Processing time	Expected utilization	Machine number	Processing time	Expected utilization	Machine number	Processing time	Expected utilization
1	5.5	0.8556	9	7.5	0.8333	17	13	0.8667
2	13	0.8667	10	5	0.8889	18	8	0.8889
3	5	0.8889	11	18	0.8	19	13	0.8667
4	18	0.8	12	13	0.8667	20	18	0.8
5	13	0.8667	13	19	0.8444	21	8	0.8889
6	4	0.8889	14	8	0.8889	22	18	0.8
7	18	0.8	15	8	0.8889	23	8	0.8889
8	5	0.8889	16	13	0.8667	24	13	0.8667

Table 3. Results from the algorithm and simulation in P1

Performance measures	Algorithm	Simulation (95% CI)
Overflow probability	–	[0.0, 0.0401]
S/R machine utilization	0.5058	[0.447, 0.470] 1) [0.427, 0.450] 2)

1) S/R machine utilization in the first aisle.
2) S/R machine utilization in the second aisle.

analytical results from the algorithm and the empirical results from simulation experiment, which are shown in <Table 3>.

Above results should be interpreted as follows. For P1, we first found heuristically that we need two storage aisles and 48 openings per aisle for a given set of data. (We need two S/R machines.) That is, with such a configuration of AS/RS, the maximum overflow probability should be less than 0.05 and the maximum S/R machine utilization should be less than 0.8. The heuristic algorithm computes the average S/R machine utilization as 0.5058, which is less than 0.8. Simulation results shows that overflow probability is less than 0.05 and the S/R machine utilization is also less than 0.8. In other words, with 96 openings, we can satisfy the design constraints of maximum permissible storage overflow probability and maximum allowable S/R machine utilization.

From P2, we obtain that 187.40 openings are required, although there are 131.46 parts in the storage in average. However, to have an integer number openings in the horizontal and vertical directions, algorithm computes that we need 3 aisles with 3 (11) openings in horizontal (vertical) direction. The shape factor value of this rack is 0.917, which is very close to SIT rack. With 198 openings, we obtain following results from the algorithm and simulation, which are shown in <Table 4>.

<Table 4> indicates that, if we design an AS/RS with three aisles, the actual overflow probability is less than 0.05 and the expected S/R machine utilization is also less than the maximum allowable S/R utilization, which is 0.8. In other words, the AS/RS designed using the capacity algorithm satisfies the two design constraints; maximum permissible overflow probability and maximum S/R machine utilization.

Table 4. Results from the algorithm and simulation in P2

Performance measures	Algorithm	Simulation (95% CI)
Overflow probability	–	[0.0, 0.0252]
S/R machine utilization	0.5101	[0.451, 0.481]1) [0.440, 0.470]2) [0.428, 0.457]3)

1) S/R machine utilization in the first aisle.
2) S/R machine utilization in the second aisle.
3) S/R machine utilization in the third aisle.

From <Tables 3 and 4>, we observe that S/R machine utilizations obtained from the algorithm are not covered by the 95 % confidence intervals. Note that our objective is not to estimate the S/R machine utilization, but to use the S/R machine utilization as a design criterion. Furthermore, estimating the S/R machine utilization accurately is a very difficult task due to the complexity of our system : storage and retrieval requests occur randomly, ratio and sequence of single and dual commands are not known in advance, and S/R machine travel time and its utilization depend on storage capacity and its configuration.

The phenomenon that simulated S/R machine utilization tends to decrease as the aisle number increases can be explained as follows. Recall that, in simulation, a part to be stored will be assigned to the least utilized storage rack side. If tie occurs, nearest storage rack side is assigned for the part. Therefore, aisle 1 which is nearest both from the incoming conveyor and the machines will be mostly utilized so that the S/R machine utilization in aisle 1 is the highest.

5. Conclusion

A significant majority of published papers on AS/R systems emphasize cycle times and throughput capacity, assuming that storage capacity and its configuration are given. In this paper, we present an algorithm to estimate storage capacity and configuration of AS/R systems, which are important decision variables in the design phase, assuming that storage and retrieval requests arrive randomly. In this study, a S/R machine can be idle either at the I/O point or within the rack. The S/R machine can perform either single command or dual command, depending on the availability of the requests.

Simulation results indicate that the storage capacity obtained from the algorithm can satisfy two design constraints; maximum allowable overflow probability and S/R machine utilization. Our results are approximate in nature but they provide an effective means for estimating the appropriate storage capacity in the design phase. In the future research, we will extend the algorithm to include time delay due to material handling device such as conveyor.

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