Journal of the Korean Institute of Industrial Engineers
Vol. 27, No. 4, pp. 384-393, December 2001.

등급별 저장방식하에서의 이중 캐로절시스템의 최적구성

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An Optimal Design of Double Carousel System Under Two Class Based Assignment Policy

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The performance of a double carousel system depends on its design and operation strategy. The two-class based assignment policy is expected to be an appropriate storage policy for its efficient operation, since it has two independently-moved carousels, lower and upper sub-carousels. This paper conducts a comparative analysis of the effects of the two-class based assignment policy and those of randomized assignment policy on the throughput of a double carousel system. We also show that, by allowing the sub-carousels not to be equal in size, a further improvement in the throughput can be obtained.

Keywords: double carousel system, design, operation, two-class based assignment policy

1. Introduction

The carousel is a type of automated storage / retrieval systems (AS / RS) which is suitable for storing small and medium parts. It has been widely used in warehousing applications to support order picking, assembly and kitting operations. A typical carousel has a driving mechanism which rotates a set of multi-shelf storage carriers, allowing a S/R machine or picker to select the desired item while remaining in one position. Recent developments in this system include the automation of S/R operations where both the carousel and automated S/R machine are operated under computer control. Compared with unit-load AS/RS, this system has two distinct operating characteristics. One is that while pickup / deposit operations are being performed at the Input / Output (I/O) station by the S/R machine, the carousel may rotate at the same time. The other is that the position of each rack opening relative to the I/O station may change from cycle to cycle.

Two types of automated carousel are currently in use in industries, i.e., standard (single) carousel and rotary rack system. The standard carousel is composed of single carousel body and a storage / retrieval (S/R) machine. The rotary rack system has multi sub-carousels, each of which requires a drive unit for each sub-carousel. In this study, as a special type of rotary rack system, we consider double (or two-level) carousel system introduced in Auguston (1989). <Figure 1> shows typical configurations of standard and double carousels.

Compared with the standard carousel, the double carousel has two subcarousels and is known to have a

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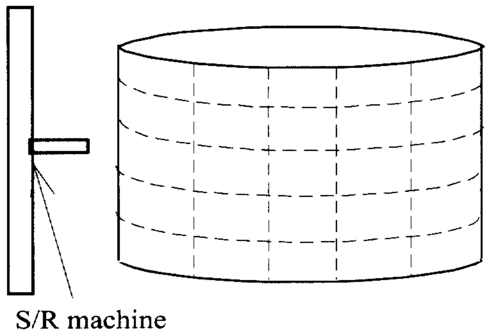
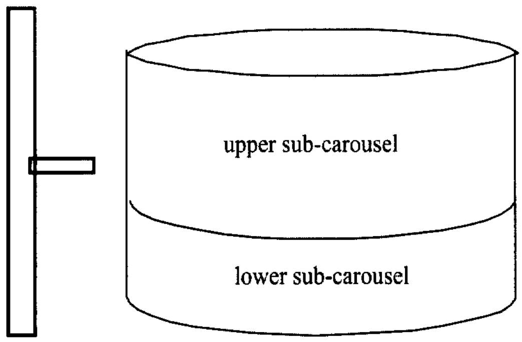
Received January 2001, accepted June 2001 after 1 revision.



(a) A standard carousel system.



(b) A double carousel system.

Figure 1.

higher level of throughput (the number of unit loads handled per unit time) due to its extra driving mechanism and control, which makes storage locations ready to be accessed by the S/R machine. Han and McGinnis (1986) analyzed standard carousel system in which unit loads are stored and retrieved. They described a control algorithm for sequencing retrievals and selecting storage locations, and developed a predictive model for estimating throughput. For a carousel serviced by a single picker, Glgen Beng (1995) introduced a simple and robust storage assignment policy that minimizes the expected turnaround time for accessing an item. Mardix and Sharp (1985) examined a carousel system served by order pickers, in terms of cost and efficiency. Using analytical as well as simulation method, comparisons of long-low carousel vs. short-high ones were analyzed. Han *et al.* (1988) presented an indirect solution method to approximate the expected dual command (DC) cycle time in a rotary rack system. Kobuki (1987) introduced a multi-cross carousel system which is a variation of the rotary rack system. It has two vertically rotating tray conveyors at both sides of the horizontally rotating racks where a tray conveyor functions as a storage machine and the other as a retrieval machine. Hwang and Ha (1991) developed the expected cycle time models for standard and double carousel systems with randomized assignment policy. Su (1998) evaluated the performances of the standard carousel, the rotary rack, and the independently-moved carousel with one motor under various operation strategies. Recently, Hwang *et al.* (1999) analyzed the effects of double shuttle of the S/R machine on the throughput of standard and double carousel systems.

Pallet storage assignment is one of key issues which affect the performance of carousel system. Three most popular assignment rules are random storage assignment, turnover based assignment, and class-based assignment. Under class-based storage assignment rule, storage racks are divided into several areas, and the items are divided into the same number of groups. Hwang and Ha (1994) studied the effects of two class based assignment policy on the throughput performance of standard carousel system. Since the double carousel system already has two separate storage areas, i.e., lower and upper sub-carousels, two-class based assignment policy (C2) seems to be natural choice for the operation of the double carousel system. Therefore, the following questions could be raised.

1) In comparison with randomized assignment policy, how much the throughput of a double carousel system can be improved by adopting C2 policy?
2) If the size of the sub-carousels is allowed to be unequal in size, how much additional improvement can we expect under C2 policy?

Hereafter, the double carousel system whose sub-carousels are not necessarily equal in size will be called the unbalanced double carousel system (UDCS). To answer the above questions, we first develop the expected cycle time models of single and dual commands. And then an optimal relative size of two sub-carousels is determined. This is equivalent to the problem of Rosenblatt and Eynan (1989) who studied the optimal boundaries of each of the storage area in unit-load AS/RS. Finally, the performance of the UDCS under C2 policy is compared with typical double carousel system under randomized storage assignment policy as well as under C2 policy.

This paper is organized as follows: Section 2 presents the basic assumptions and basis of model development. In section 3 and 4, expected cycle time models are developed for single and dual commands in the UDCS, respectively. In section 5, through numerical evaluation the performance of the UDCS is compared with typical double carousel systems. Finally, conclusions appear in section 6.

2. The assumption and basis of model development

In accordance with Hausman *et al.* (1976), the following assumptions are made:

(1) Each pallet holds only one part number or item type.
(2) All storage locations are the same size, as are the pallets themselves.
(3) The turnover frequency of each item is known and constant through time.
(4) The items are replenished according to the simple EOQ model.
(5) The carousel system is served by a single S/R machine which is located at the bottom front corner and moves only one pallet of items at a time.
(6) The S/R machine can move in vertical direction while the carousel rotates.
(7) The pickup /deposit time of the S/R machine is constant for all cycles.

In two-class based storage assignment policy, the items for storage are partitioned into two classes according to their turnover rates. In general, the items with high turnover rates are assigned randomly within the class I region close to the I/O point and the remaining ones within the class II region. Thus we assume that the lower sub-carousel in <Figure 1(b)> is used as the storage area for class I items and the upper one for class II items.

In the following analysis, the discrete grid of the carousel rack will be approximated by a continuous surface. Suppose we slit the surface of carousel rack vertically at the point farthest from the I/O station, and unwrapped, as illustrated in <Figure 2(a)>, where H denotes its height and L a half of its length. Let t_v be the time for the S/R machine to move from the I/O station to the top level and t_h be the time to rotate a half of the carousel. Assuming that t_h is equal to or greater than t_v without loss of generality, the carousel can be normalized to a rectangle with a length of 2.0 and height of $b = t_v / t_h \leq 1$. The advantage of normalization is to reduce the difficulties of algebraic manipulation. In this normalized carousel, t_h is referred to as the normalization factor and is denoted by T. Let t_k be the height (K) of the lower sub-carousel and k ($= tk / T$) its normalized value. We assume that k is less than or equal to $b/2$ for the convenience of analysis. Let E denote the pickup/deposit time, whose normalized time is represented by e ($= E / T$). We will develop the expected cycle time models in the continuous representation of the normalized time rack as shown in

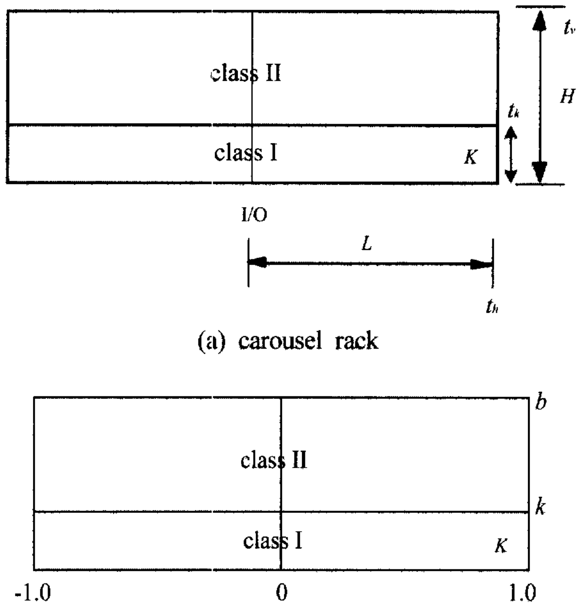


Figure 2. Continuous representation of the normalized time rack.

<Figure 2(b)>. The results of the normalized models must be multiplied by T to obtain actual time values.

To determine the expected cycle time of carousel, the time elements of one cycle needs to be identified. One cycle consists of a sequence of operations beginning with the S/R machine at the I/O station and ending with the S/R machine at the I/O station.

In a single command cycle, a storage or a retrieval operation is performed, whereas in a dual command cycle, both storage and retrieval operations are performed. For single command cycle, we will analyze it with storage order, without loss of generality. A dual command cycle consists of the following time elements:

(1) time to pick up a load at the I/O station and time to move the S/R machine to the height of storage level while the carousel rotates to the correct open location for storage (Let ST be the maximum of the load picking time plus the S/R machine movement time and the carousel rotating time.)
(2) time to insert the load into the open location $= e$
(3) time to bring the S/R machine and carousel to the retrieval point (Let IT be the maximum of the S/R machine movement time and the carousel rotating time.)
(4) time to extract a load from the retrieval point $= e$
(5) time to return the S/R machine to the I/O station $= RT$
(6) time to discharge at the I/O station $= e$

To facilitate the analysis of the cycle times, the elements (1), (3), and (5) are denoted by ST, IT, and RT, respectively. Once the time values of ST, RT and

IT are identified, the constant terms such as element (2), (4), and (6) are simply added to find dual command cycle time. Let SC and DC be the single command and dual command cycle time, respectively. Then

$$SC = ST + RT + e \tag{1}$$
$$DC = ST + IT + RT + 3e \tag{2}$$

In addition to the assumptions already made, it is further assumed that items are ranked to their contribution to total demand using ABC curve. The ABC curve is a plot of ranked cumulative percentage demand versus percentage of inventoried items, where items are represented on the continuum from 0 to 1. Using the notations of Hausman *et al.* (1976), the ABC curve is represented by:

$$G(i) = i^s,\ 0 < s \leq 1, \tag{3}$$

where $G(i)$ is cumulative percentage of demand in pallet loads, i is the number in percentage of inventoried items and s is the skewness of the ABC curve. Based on the basic EOQ model with the ratio R of ordering cost to holding cost being identical for all items, Hausman *et al.* estimated the turnover rate $\lambda(j)$ of the j^{th} pallet as

$$\lambda(j) = (2s/R)^{1/2} \cdot j^{(s-1)/(s+1)},\ 0 \leq j \leq 1 \tag{4}$$

The j^{th} pallet should be interpreted as the pallet at j^{th} the percentile in the ranked ordering of all pallets. Equation (4) indicates that the turnover rate of each pallet is determined by the demand rate for the items it carries.

Let p_i be the probability that the pallet associated with a storage order is assigned to class i area, $i = 1, 2$. Utilizing the notations in <Figure 2(b)> and equation (4), Hausman *et al.* (1976) derived

$$p_1 = \frac{\int_{j=0}^{\frac{k}{b}} \lambda(j)dj}{\int_{j=0}^{l} \lambda(j)dj} = \left(\frac{k}{b}\right)^{\frac{2s}{s+1}} \tag{5}$$

and

$$p_2 = 1 - p_1 = 1 - \left(\frac{k}{b}\right)^{\frac{2s}{s+1}} \tag{6}$$

3. Development of the single command cycle time model

Let $E(SC)$ be the expected value of single command cycle time. To find $E(SC)$, we only need to analyze ST and RT. In the double carousel system, the S/R machine can pick up a pallet for storage order, while the sub-carousel corresponding to the class of the order rotates to bring a randomly chosen open location to the axis of the S/R machine column. Thus the carousel may save as much rotating time as the time required for the S/R machine picking. If we represent the coordinates of an open location for storage order as $(x,\ y)$ in the normalized carousel rack, where, $0 \leq |x| \leq 1$ and $0 \leq y \leq b$, then

$$ST = \max\{\text{arousel rotating time, S/R machine moving time}\} = \max\{|x|, y + e\} \tag{7}$$

Assuming the x, y coordinates are independently generated,

$$S(z) = \Pr[ST \leq z] = \Pr[|x| \leq z] \cdot \Pr[y + e \leq z] = F(z) \cdot G(z), \tag{8}$$

where $F(z)]\Pr[|x| \leq z]$ and $G(z) = \Pr[y + e \leq z]$

For the UDCS under C2 policy, the location of x coordinate can be assumed to be uniformly distributed. Thus

$$F(z) = \begin{cases} z, & 0 \leq z \leq 1 \\ 1, & z > 1 \end{cases} \tag{9}$$

For $y + e$, we have the following distribution functions;

For $y \in class$ I $region$,

$$G_1(z) = \begin{cases} (z - e)/k, & e \leq z \leq k + e \\ 1, & z > k + e \end{cases} \tag{10}$$

and for $y \in class$ II $region$

$$G_2(z) = \begin{cases} [z - (k + e)]/(b - k), & k + e \leq z \leq b + e \\ 1, & z > k + e \end{cases} \tag{11}$$

Hence, the distribution function of ST can be obtained as

$$S_i(z) = \Pr[ST \leq z] = F(z) \cdot G_i(z),\ \text{for } i = 1, 2, \tag{12}$$

where i indicates the class an order is associated with.

The values of e and k affect the expression of $S_1(z)$ and so we have the following three cases to consider: $0 \leq e \leq 1 - k$, $1 - k < e \leq 1$, and $e > 1$. Let $E_i(ST)$ be the expected single command cycle time for an order in class i. <Table 1> lists the distribution function $S_1(z)$, the probability density function $s_1(z)$, and the expected storage time $E_1(ST)$ in each range of e. For

Table 1. Expected storage times for order in class I

Range of e	Range of z	$S_1(z)$	$s_1(z)$	$E_1(ST)$
$0 \le e \le 1-k$	$e \le z \le e+k$ $e+k < z \le 1$ $z > 1$	$z(z-e)/k$ z 1	$(2z-e)/k$ 1 0	$1/2-3/6k-(e+k)^2/2$ $-(e(e+1)^2/2k+(2(e+k)^3)/3k$
$1-k < e \le 1$	$e \le z \le 1$ $1 < z \le e+k$ $z > e+k$	$z(z-e)/k$ $(z-e)/k$ 1	$(2z-e)/k$ $1/k$ 0	$1/6k-\mathrm{e}/2k-e^3/6k+(e+k)^2/2k$
$e > 1$	$e \le z \le e+k$ $z > e+k$	$(z-e)/k$ 1	$1/k$ 0	$(e+k)^2/2k-e^2/2k$

instance, the expected storage time for the case $0 \le e \le 1-k$ in class 1 order, $E_1(ST)$ can be obtained as,

$$E_1(ST) = \int_e^{e+k} \frac{(2z-e)}{k}\, dz + \int_{e+k}^{1} dz$$
$$= 1/2 - 3/6k - (e+k)^2/2$$
$$- (e(e+k)^2)/2k + (2(e+k)^3)/3k$$

In similar way, <table 2> summarizes the cases for determining $E_2(ST)$.

With the probability P_i, the expected value of ST becomes the weighted average of $E_1(ST)$ and $E_2(ST)$ and

$$E(ST) = \sum_{i=1}^{2} P_i \cdot E_i(ST) \qquad (13)$$

Substituting the results of <table 1> and <table 2> into Equation (13), we have the following results.

1) For $0 \le e \le 1-b$,

$$E(ST) = \left[\begin{array}{l} 3+b^2+3be+e^2+bk+3ek+K^2-b^2\{(k/b)^{(2s/(s+1)}\} \\ -3be\{(k/b)^{(2s/(s+1)}\} - bk\{(k/b)^{(2s/(s+1)}\} \end{array}\right]/6 \qquad (14)$$

2) For $1-b < e \le 1-k$,

$$E(ST) = \{(k/b)^{2s/(s+1)}\}(3+3e^2+3ek+k^2)/6$$
$$-\left[\begin{array}{l} \{k^3-1-3b^2+3e-6be-3e^2+e^3+3k+3e^2k+3ek^2\} \\ \{1-\{(k/b)^{(2s/(s+1)}\}\} \end{array}\right]/6(b-k) \qquad (15)$$

3) For $1-k < e \le 1$,

$$E(ST) = \{(k/b)^{2s/(s+1)}\}(1-3e+3e^2-6ek+3k^2)/6k$$
$$+ [(b+2e+k)\{1-\{(k/b)^{2s/(s+1)}\}\}]/2 \qquad (16)$$

4) For $e > 1$,

$$E(ST) = [b+2e+k-b\{(k/b)^{2s/(s+1)}\}]/2 \qquad (17)$$

The time RT to return the S/R machine to the I/O station depends only on the y-coordinate of storage location. Thus the expected value of RT can be expressed as

$$E(RT) = \sum_{i=1}^{2} P_i \cdot E_i(RT), \qquad (18)$$

where $E_1(RT) = k/2$ for $y \in$ class I region and $E_2(RT) = (k+b)/2$ for $y \in$ class Ⅱ region.

Table 2. Expected storage times for order in class II

Range of e	Range of z	$S_1(z)$	$s_1(z)$	$E_1(ST)$
$0 \le e \le 1-b$	$e+k \le z \le e+b$ $e+b < z \le 1$ $z > 1$	$z(z-e-k)/(b-k)$ z 1	$(2z-e-k)/(b-k)$ 1 0	$1/2-(b+e)^2/2-2(b+e)^3/3(b-k)$ $-(b+e)^2/(e+k)/2(b-k)$ $-(e+k)^3/6(b-k)$
$1-b < e \le 1-k$	$e+k \le z \le 1$ $1 < z \le e+b$ $z > e+b$	$z(z-e-k)/(b-k)$ $(z-e-k)/(b-k)$ 1	$(2z-e-k)/(b-k)$ $1/(b-k)$ 0	$1/6(\mathrm{b}-k)+(b+e)^2/2(b-k)$ $-(e+k)/2(b-k)-(e+k)^3/6(b-k)$
$e > 1-k$	$e+k \le z \le e+b$ $z > e+b$	$(z-e-k)/(b-k)$ 1	$1/(b-k)$ 0	$(b+e)^2/2(b-k)-(e+k)^2/2(b-k)$

4. Development of the dual command cycle time model

Since the expected dual command cycle time $E(DC)$ equals $E(DC)=E(ST)+E(RT)+E(IT)+3e$, we only need to find $E(IT)$. $E(IT)$ in the double carousel system is smaller compared with that in the equivalent standard carousel system. To find $E(DC)$, it is assumed that there are as many retrieval orders as storage orders and orders are processed in a first-come-first-served (FCFS) manner.

Let $p_s=(x_s, y_s)$ and $p_r=(x_r, y_r)$ denote a storage location and a retrieval location in the normalized carousel rack, respectively. The interleave time from P_s to P_r is max{carousel rotating time, S/R machine movement time}. Both the carousel rotating time and S/R machine movement time depend on the classes the orders in dual command cycle are associated with. Thus the following four cases need to be considered.

(case-1): $P_s \in$ class Ⅰ region and $P_r \in$ class Ⅱ region
(case-2): $P_s \in$ class Ⅱ region and $P_r \in$ class Ⅰ region
(case-3): $P_s \in$ class Ⅰ region and $P_r \in$ class Ⅰ region
(case-4): $P_s \in$ class Ⅱ region and $P_r \in$ class Ⅱ region

Let be the expected interleave time for case-i, then $E(IT)$ can be expressed as

$$E(IT)=P_1*P_2*E_1(IT)+P_2*P_1*E_2(IT) + P_1*P_1*E_3(IT)+P_2*P_2*E_4(IT) \quad (19)$$

1) For case-1

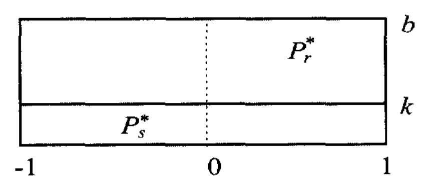


Figure 3. Representation of a pair of orders for case-1.

Suppose a pair of orders as shown in <Figure 3> arrives at the I/O station. Then two sub-carousels start to rotate simultaneously to bring each target location to the axis of the S/R machine column while the S/R machine begins the pickup operation at the I/O station. It takes (y_r-y_s) unit of time for the S/R machine to reach p_r as soon as it completes inserting the storage pallet at p_s. Thus the interleave time IT_1 can be expressed by the following equation:

$$\begin{aligned} IT_1 &= \max[\max\{|x_r|-(ST+e),\ 0\},\ (y_r-y_s)] \\ &= \max\{|x_r|-(ST+e),\ (y_r-y_s)\} \end{aligned} \quad (20)$$

where $ST=\max\{|x_s|,\ y_s+e\}$

Let $L_1(z)$ denote the cumulative distribution function of IT_1. Assuming that the carousel rotating time and the S/R machine movement time are independent, we have

$$\begin{aligned} L_1(z) &= \Pr(IT_1 \le z) \\ &= \Pr[\max[|x_r|-(ST+e),(y_r-y_s)\} \le z] \\ &= \Pr[\{|x_r|-(ST+e)\} \le z] \\ &\quad *\Pr[(y_r-y_s)\le z] \\ &= R_1(z)*H_1(z), \end{aligned} \quad (21)$$

where $R_1(z)=\Pr[\{|x_r|-(ST+e)\}\le z]$ and $H_1(z)=\Pr[(y_r-y_s)\le z]$

The distribution function $H_1(z)$ of S/R machine movement time was determined by Hwang and Ha (1994) and

$$H_1(z)=\begin{cases} z^2/[2k(b-k)], & 0\le z\le k \\ (2z-k)/[2(b-k)], & k\le z\le b-k \\ 1-(b-z)^2/[2k(b-k)], & b-k\le z\le b \\ 1, & z>b \end{cases} \quad (22)$$

With the help of Mathematica (Wolfram, 1991), the distribution function $R_1(z)$ of carousel rotating time is obtained from the joint probability density function of $|x_r|$and $(ST+e)$. $R_1(z)$ has three types of expressions as follows:

i) For $0\le e\le(1-k)/2$,

$$R_1^1(z)=\begin{cases} 1/2+e+ek/2+k^2/6+z-ez-z^2/2, & 0\le z\le(1-2e-k) \\ (9e-2-12e^2+4e^3+6k+6z-18ez+12e^2z-6z^2+9ez^2+2z^3)/6k, & (1-2e-k)\le z\le(1-2e) \\ 1, & z>1-2e \end{cases} \quad (23)$$

ii) For $(1-k)/2<e\le 1/2$,

$$R_1^2(z)=\begin{cases} (9e-2-12e^2+4e^3+6k+6z-18ez+12e^2z-6z^2+9ez^2+2z^3)/6k, & 0\le z\le(1-2e) \\ 1, & z>1-2e \end{cases} \quad (24)$$

iii) For $e>1/2$,

$$R_1^3(z)=1. \quad (25)$$

Since $L_1(z)=R_1(z)*H_1(z)$, we can obtain $L_1(z)$ comparing the magnitude of lower and upper interval values of e and z for $R_1(z)$ and $H_1(z)$ respectively.

$L_1(z)$ has a unique distribution function in each of the following eight intervals.

(1) $0 \le e \le (1-b-k)/2$

(2) $(1-b-k)/2 \le e \le (1-b)/2$

(3) $(1-b)/2 \le e \le \min\{(1-2k)/2,\ (1-b-k)/2\}$

(4) $(1-b+k)/2 \le e \le (1-2k)/2$

(5) $(1-2k)/2 \le e \le (1-b+k)/2$

(6) $\max\{(1-2k)/2, (1-b+k)/2\} \le e \le (1-k)/2$

(7) $(1-k)/2 \le e \le 1/2$

(8) $e > 1/2$

Let $l_1^k(z)$ be the probability density function in the k^{th} interval of e. $E_1^k(IT)$ is expressed as,

$$E_1^k(IT) = \int_0^{\infty} z \cdot l_1^k(z)\,dz,\ k=1, 2, \cdots, 8 \tag{26}$$

2) For case-2

The procedures of finding $E_2(IT)$ are similar to $E_1(IT)$. We have to consider nine intervals of e and in each interval, $E_2^j(IT)$ is found. Most of E_1^k and E_2^j turn out to have massive mathematical expressions, and so they will not be presented in this paper except the followings:

$$E_1^1(z) = (4+6b+4b^2-b^3-12e+12be-4b^2e+16e^3 -2bk+b^2k-12ek+8bek+24e^2k-2k^2 +bk^2+10ek^2+2k^3)/24$$

$$E_1^4(z) = (15+180b^2-60e+240e^3-240e^4-30k- 180bk+360e^2k-480e^3+210ek^2-420e^2k^2 +45k^3-180ek^3-31k^4)/\{360(b-k)\}$$

$$E_1^7(z) = (1-9e+30e^2-40e^3+48e^5-32e^6+180b^2k^2 -180bk^3)/\{360(b-k)k^2\}$$

$$E_1^8(z) = b/2$$

$$E_2^1(z) = (4+6b+3b^3-12e+14b^2e+24be^2+16e^3 -6bk+5b^2k-12ek+20bek+24e^2k-2k^2 +3bk^2+10ek^2+2k^3)/24$$

$$E_2^9(z) = b/2$$

3) For case-3

$E_3(IT)$ is the same as the expected interleave time in the standard carousel system whose shape factor is k. Utilizing the results of Han and McGinnis (1986), we find that

$$E_3(IT) = 1/2 + k^2/12 \tag{27}$$

4) For case-4

Substitution of k with $(b-k)$ in $E_3(IT)$ gives $E_4(IT)$ and

$$E_4(IT) = 1/2 + (b-k)^2/12 \tag{28}$$

Note that the expected interleave time for dual command cycle is obtained as a weighted average of all the possible cases as in Equation (19). For a given set of values of (s, e, b), an optimum value k^* is found which minimizes $E(DC)$ or $E(SC)$ by a grid search method.

5. Evaluation

Considering the following three double carousel systems, we compared the performance of the C2 policy with randomized assignment policy. Also, the effectiveness of optimum partition of the carousel surface (k^*) over $k=b/2$ (two sub-carousels are equal in size) was studied.

1) **system-1**: double carousel system with $k=b/2$ under randomized storage assignment policy.
2) **system-2**: double carousel system with $k=b/2$ under C2 policy.
3) **system-3**: double carousel system with k^* under C2 policy.

For the evaluation, three inventory turnover distributions; i.e., 20/60, 20/70 , 20/80 were tested, where X/Y means that X percent of the highest demand items (measured in pallet loads) accounts for Y percent of total demand. The corresponding "s" value can be found by solving $Y/100=(X/100)$ from ABC curve function as mentioned in section 2. We tested four values of shape factor b, i.e. 0.4, 0.6, 0.8 and 1 and three values of normalized pickup/deposit time e, i.e. 0.1, 0.3 and 0.5.

Let $E(SC|RAN)$, $E(SC|C2^{\circ})$ and $E(SC|C2^{*})$ be the expected single command cycle time in system-1, 2 and 3, respectively. Similarly, $E(SC|RAN)$, $E(SC|C2^{\circ})$ and $E(SC|C2^{*})$ represent the expected dual command cycle time in system-1, 2 and 3, respectively.

For $3\times3\times4$ number of various combinations of s, e, and b, $E(SC)$ and $E(DC)$ were determined for each system. The results are listed in <Table 3> for single command and <Table 4> for dual command. Also, shown are the relative throughput improvements of system-2 and 3 over system-1, and system-3 over system-2.

From the tables, we can make the following observations:

(1) The percentage reductions in cycle time of system-2 (system-3) over system-1 ranges from 5.0% to 20.8% (6.0% to 27.0%) for single command cycle and from 1.6% to 13.3% (3.1% to 17.6%) for dual command cycle. Thus it is apparent that, over a wide range of the system

Table 3. Computational results for single command cycle

s	e	b	$E(SC/RAN)$	$E(SC/C2^{\circ})$	$E(SC/C2^{*})$	k^{*}	I_{SC}^{12} (%)	I_{SC}^{13}(%)	I_{SC}^{23}(%)
0.318 (20%/60%)	0.1	0.4	0.8517	0.7956	0.7852	0.10	6.6	7.8	1.3
		0.6	0.9950	0.9044	0.8887	0.16	9.1	10.7	1.7
		0.8	1.1517	1.0222	1.0011	0.22	11.2	13.1	2.1
		1.0	1.3215	1.1490	1.1223	0.27	13.1	15.1	2.3
	0.3	0.4	1.1317	1.0670	1.0547	0.10	5.7	6.8	1.1
		0.6	1.2950	1.1915	1.1730	0.16	8.0	9.4	1.5
		0.8	1.4715	1.3249	1.3000	0.21	10.0	11.6	1.9
		1.0	1.6572	1.4650	1.4333	0.27	11.6	13.5	2.2
	0.5	0.4	1.4517	1.3783	1.3642	0.10	5.1	6.0	1.0
		0.6	1.6347	1.5184	1.4971	0.16	7.1	8.4	1.4
		0.8	1.8260	1.6645	1.6356	0.21	8.8	10.4	1.7
		1.0	2.0208	1.8141	1.7767	0.26	10.2	12.1	2.1
0.222 (20%/70%)	0.1	0.4	0.8517	0.7796	0.7615	0.09	8.5	10.6	2.3
		0.6	0.9950	0.8785	0.8511	0.13	11.7	14.5	3.1
		0.8	1.1517	0.9852	0.9482	0.18	14.5	17.7	3.8
		1.0	1.3215	1.0997	1.0526	0.23	16.8	20.3	4.3
	0.3	0.4	1.1317	1.0485	1.0273	0.09	7.4	9.2	2.0
		0.6	1.2950	1.1619	1.1297	0.13	10.3	12.8	2.8
		0.8	1.4715	1.2830	1.2395	0.18	12.8	15.8	3.4
		1.0	1.6572	1.4100	1.3546	0.23	14.9	18.3	3.9
	0.5	0.4	1.4517	1.3574	1.3330	0.09	6.5	8.2	1.8
		0.6	1.6347	1.4851	1.4481	0.13	9.2	11.4	2.5
		0.8	1.8260	1.6184	1.5681	0.18	11.4	14.1	3.1
		1.0	2.0208	1.7550	1.6901	0.22	13.2	16.4	3.7
0.139 (20%/80%)	0.1	0.4	0.8517	0.7621	0.7314	0.07	10.5	14.1	4.0
		0.6	0.9950	0.8504	0.8033	0.10	14.5	19.3	5.5
		0.8	1.1517	0.9451	0.8812	0.14	17.9	23.5	6.8
		1.0	1.3215	1.0461	0.9647	0.18	20.8	27.0	7.8
	0.3	0.4	1.1317	1.0284	0.9922	0.07	9.1	12.3	3.5
		0.6	1.2950	1.1297	1.0746	0.10	12.8	17.0	4.9
		0.8	1.4715	1.2374	1.1628	0.14	15.9	21.0	6.0
		1.0	1.6572	1.3503	1.2550	0.18	18.5	24.3	7.1
	0.5	0.4	1.4517	1.3346	1.2930	0.07	8.1	10.9	3.1
		0.6	1.6347	1.4490	1.3858	0.10	11.4	15.2	4.4
		0.8	1.8260	1.5682	1.4822	0.14	14.1	18.8	5.5
		1.0	2.0208	1.6908	1.5803	0.17	16.3	21.8	6.5

$$I_{SC}^{12}(\%) = \frac{E(SC\mid RAN) - E(SC\mid C2^{\circ})}{E(SC\mid RAN)} \times 100,\quad I_{SC}^{13}(\%) = \frac{E(SC\mid RAN) - E(SC\mid C2^{*})}{E(SC\mid RAN)} \times 100$$

$$I_{SC}^{23}(\%) = \frac{E(SC\mid C2^{\circ}) - E(SC\mid C2^{*})}{E(SC\mid C2^{\circ})} \times 100$$

parameters, the two-class storage assignment policy has a better performance than the randomized storage assignment policy.

(2) The values of indicate that a proper choice of the class boundary can further increase the throughput of the double carousel system.

(3) $I^{12}(\%)$, $I^{13}(\%)$ and $I^{23}(\%)$ increase considerably as the skewness of the inventory distribution increases.

(4) As either b becomes larger or e becomes smaller, $I^{12}(\%)$, $I^{13}(\%)$ and $I^{23}(\%)$ tend to increase.

(5) The relative size of the area for class I items bec

Table 4. Computational results for dual command cycle

s	e	b	$E(SC/RAN)$	$E(SC/C2^{\circ})$	$E(SC/C2^{*})$	k^*	I_{SC}^{12} (%)	I_{SC}^{13}(%)	I_{SC}^{23}(%)
0.318 (20%/60%)	0.1	0.4	1.4215	1.3903	1.3584	0.10	2.2	4.4	2.3
		0.6	1.6067	1.5339	1.5042	0.16	4.5	6.4	1.9
		0.8	1.8121	1.6925	1.6644	0.23	6.6	8.1	1.7
		1.0	2.0339	1.8630	1.8363	0.31	8.4	9.7	1.4
	0.3	0.4	2.0844	2.0477	2.0087	0.10	1.8	3.6	1.9
		0.6	2.2991	2.2148	2.1782	0.16	3.7	5.3	1.7
		0.8	2.5283	2.3922	2.3578	0.22	5.4	6.7	1.4
		1.0	2.7677	2.5774	2.5444	0.29	6.9	8.1	1.3
	0.5	0.4	2.8033	2.7582	2.7163	0.10	1.6	3.1	1.5
		0.6	3.0385	2.9414	2.9016	0.16	3.2	4.5	1.4
		0.8	3.2827	36.1317	3.0932	0.22	4.6	5.8	1.2
		1.0	3.5313	3.3265	3.2879	0.29	5.8	6.9	1.2
0.222 (20%/70%)	0.1	0.4	1.4215	1.3905	1.3368	0.08	2.2	6.0	3.9
		0.6	1.6067	1.5197	1.4686	0.13	5.4	8.6	3.4
		0.8	1.8121	1.6620	1.6125	0.19	8.3	11.0	3.0
		1.0	2.0339	1.8147	1.7659	0.26	10.8	13.2	2.7
	0.3	0.4	2.0844	2.0476	1.9829	0.07	1.8	4.9	3.2
		0.6	2.2991	2.1978	2.1363	0.12	4.4	7.1	2.8
		0.8	2.5283	2.3572	2.2978	0.18	6.8	9.1	2.5
		1.0	2.7677	2.5237	2.4646	0.24	8.8	11.0	2.3
	0.5	0.4	2.8033	2.7557	2.6864	0.07	1.7	4.2	2.5
		0.6	3.0385	2.9208	2.8538	0.12	3.9	6.1	2.3
		0.8	3.2827	3.0925	3.0259	0.18	5.8	7.8	2.1
		1.0	3.5313	3.2686	3.2001	0.24	7.4	9.4	2.1
0.139 (20%/80%)	0.1	0.4	1.4215	1.3953	1.3119	0.04	1.8	7.7	6.0
		0.6	1.6067	1.5075	1.4257	0.09	6.2	11.3	5.4
		0.8	1.8121	1.6306	1.5478	0.14	10.0	14.6	5.1
		1.0	2.0339	1.7625	1.6764	0.20	13.3	17.6	4.9
	0.3	0.4	2.0844	2.0525	1.9532	0.04	1.5	6.3	4.8
		0.6	2.2991	2.1829	2.0857	0.08	5.1	9.3	4.4
		0.8	2.5283	2.3211	2.2229	0.13	8.2	12.1	4.2
		1.0	2.7677	2.4657	2.3630	0.19	10.9	14.6	4.2
	0.5	0.4	2.8033	2.7582	2.6517	0.04	1.6	5.4	3.9
		0.6	3.0385	2.9019	2.7957	0.08	4.5	8.0	3.7
		0.8	3.2827	3.0517	2.9417	0.13	7.0	10.4	3.6
		1.0	3.5313	3.2062	3.0882	0.18	9.2	12.5	3.7

$$I_{SC}^{12}(\%) = \frac{E(SC\,|\,RAN) - E(SC\,|\,C2^{\circ})}{E(SC\,|\,RAN)} \times 100,\quad I_{SC}^{13}(\%) = \frac{E(SC\,|\,RAN) - E(SC\,|\,C2^{*})}{E(SC\,|\,RAN)} \times 100,$$

$$I_{SC}^{23}(\%) = \frac{E(SC\,|\,C2^{\circ}) - E(SC\,|\,C2^{*})}{E(SC\,|\,C2^{\circ})} \times 100$$

omes smaller as s becomes smaller.

(6) Stepwise multiple regression analysis were performed on the data of k^*, with the significance level of 0.0001. We obtained two regression lines of k^*, $k^*(SC)$ from the data of <Table 3> and $k^*(DC)$ from <Table 4>. They are listed in <Table 5>. An approximate optimal value of k^* obtained easily from the regression results can provide a useful reference at the design stage of UDCS.

Table 5. Parameter Estimates

Varialbe	Parameter Estimates	
	$k^*(SC)$	$k^*(DC)$
Intercept	−0.039897	−0.086114
s	0.299856	0.531317
e	0.020833	−0.040833
b	0.119477	0.095554
s^2	−0.706971	−0.798118
eb	−0.041667	−0.066667
sb	0.538639	0.427104
e^2	−	0.104167
b^2	−	0.083333
R-square	0.9976	0.9990

6. Conclusions

In this paper, we examined the effectiveness of two-class based storage assignment policy in double carousel system through the development of the expected cycle time models for single and dual commands. Utilizing the expected cycle time models, we determined an optimum size of each subcarousel based on a grid search method. We confirmed that as in the unit load AS/RS, the storage assignment policy has substantial influence on the performance of the double carousel system. Also, the results of the sensitivity analysis indicated that a proper choice of class boundary can further increase the carousel efficiency.

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