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# 로봇 매니퓨레이터의 강건한 적응 슬라이딩 모드제어

# (On the Robust Adaptive Sliding Mode Control of Robot Manipulators)

裵 焌 垌 *

(Jun Kyung Bae)

요 약

피드-포워드 보상부분과 불연속 제어 부분으로 구성되는 강건한 적응 슬라이딩 모드 로봇제어 알고리즘을 유도하였다. 미지의 매개변수는 실시간으로 추정되는 매개변수를 포함하는 그룹과 실시간으로 추정하지 않는 매개변수를 포함하는 그룹으로 나누어진다. 그런 다음 외란 및 실시간으로 추정하지 않는 매개변수에서의 불확실성 효과를 보상하기 위해 슬라이딩 제어 항이 토크 입력에 포함된다. 또한 매니퓨레이터 동역학 구조의 효율적인 이용으로 인하여 알고리즘은 계산이 간단하다. 매개변수 불확실성과 외부 외란의 존재에도 불구하고 제어기는 대국적 점근적으로 안정하며 추적오차가 영에 수렴함을 보여준다.

**Abstract**

A robust adaptive sliding mode robot control algorithm is derived, which consists of a feed-forward compensation part and discontinuous control part. The unknown parameters is categorized into two groups, with group containing the parameters estimated on-line, and group containing the parameters not estimated on-line. Then a sliding control term is incorporated into the torque input in order to account for the effects of uncertainties on the parameters not estimated on-line and of disturbances. Moreover, the algorithm is computationally simple, due to an effective exploitation of the structure of manipulator dynamics. It is shown that, despite the existence of the parameter uncertainty and external disturbances, the controller is globally asymptotically stable and guarantees zero tracking errors.

## I. Introduction

Advanced manipulators applications often require effective control design to achieve accurate tracking of fast desired motions. If the parameters of a manipulator's links and its load are known a priori, the well-known computed-torque control design can be used for this purpose, and theoretically guarantees exact tracking. However, for a manipulator handling various loads, the inertial parameters of the load change from time to time without being accurately known by the controller, and the performance of the computed-torque controller degrades substantially or may even go unstable. This parameter sensitivity is particularly severe for direct-drive robots and/or fast manipulator motions. Therefore, there has been active research in sliding mode control of robot manipulators as a robust approach, which intends to

* 正會員, 晉州産業大學校 產業自動化工學科

(Dept. of Industrial Automation Engineering, Chinju National University)

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provide stable and consistent performance in spite of large parameter uncertainties.

Among developed algorithms using the theory of VSS, several approaches have been considered. T.P.leung, C.Y.Su, and Q.J.Zhou[1] proposed a new sliding mode control algorithm which consists of a discontinuous compensator and a discontinuous control part. S.W.Wijesoma and R.J.Richards[2] presented the scheme for robust accurate trajectory tracking of manipulators based on the computed torque technique and variable structure systems theory. H.Yu, L.D.Seneviratne, and S.W.E.Earles[3] derived a control scheme which combines a direct adaptive control law with a variable structure adaptive control law for a nonlinear robot manipulator system. C.Y.Su and Y.Stepanenko[4] proposed for an adaptive sliding mode control of robot manipulators by using a general sliding surface, which can be nonlinear or time-varying. Q.J.Zhou and C.Y.Su[5] developed an adaptive sliding mode control scheme for accurate tracking control of robotic manipulators, with unknown manipulator and payload parameters being estimated on-line.

In this paper, a robust adaptive sliding mode control scheme for accurate trajectory tracking of robot manipulators is presented, which consists of a feed-forward compensation part and discontinuous control part, with the unknown parameters being estimated on-line. However, since the algorithm can be simplified by not explicitly estimating all unknown parameters, we categorized the unknown parameters into two groups, with group containing the parameters estimated on-line, and group containing the parameters not estimated on-line. A sliding control term is then incorporated into the torque input in order to account for the effects of uncertainties on the parameters not estimated on-line and of disturbances. In addition, the algorithm is computationally simple, due to an effective exploitation of the structure of manipulator dynamics.

The layout of paper is as follows: In Section II, the robot dynamics and its structure properties are reviewed. Section III provides our main results for designing a robust adaptive sliding mode controller. Section IV discuss the problem of chattering. To illustrate the performance of the controller, a numerical simulation example is presented in Section V. Finally, in Section VI, we give brief concluding remarks.

## II. Problem formulation

A manipulator is defined as an open kinematic chain of rigid links. Each degree of freedom of the manipulator is powered by independent torques. Using the Lagrangian formulation, the equations of motion of an n-degree-of-freedom manipulator can be written as

$$M(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) + F(q,\dot{q}) = \tau(t) \quad (1)$$

where $q$ is the n×1 vector of joint displacements, $\tau(t)$ is the n×1 vector of applied joint torques, $M(q)$ is the n×n symmetric positive definite manipulator inertia matrix, $B(q,\dot{q})\dot{q}$ is the n×1 vector of centripetal and Coriolis torques, $G(q)$ is the n×1 vector of gravitational torques, $F(q,\dot{q})$ is the n×1 vector of Coulomb and viscous friction.

As remarked by several authors([6-8]), the robot model (1) is characterized by the following structural properties, which are of importance to our stability analysis.

Property 1. There exists a vector $\theta$ with components depending on manipulators parameters (masses, moments of inertia, etc.), such that

$$\begin{aligned} M(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) + F(q,\dot{q}) \\ = Y(q,\dot{q},\ddot{q})\theta = \tau(t) \end{aligned} \quad (2)$$

where $Y(q,\dot{q},\ddot{q})$ is the n×m matrix of functions called the regressor and $\theta$ is the m×1 vector containing the unknown manipulator and payload parameters.

Property 2. The two n×n matrices $M(q)$ and $B(q,\dot{q})$ are not independent. Specially, given a proper definition of $B(q,\dot{q})$, the matrix $(\dot{M}(q)-2B(q,\dot{q}))$ is skew-symmetric.

It is noted that the choice of parameters in the above representation is not unique and that the dimension of the parameters space may depend on the particular choice of the parameters. In practice, some parameters may have relatively minor importance in the dynamics, in which case we may choose to make the controller robust to the uncertainty on these parameters, rather than explicitly estimate them on-line. Similarly, some geometric parameters may already known with reasonable precision, or may have been estimated through sorting devices or visual information. Further, the controller must be robust to residual time-varying disturbances, such as stiction or torque ripple, for instance.

Therefore, the algorithm may be simplified by not explicitly estimating all unknown parameters. We categorize the unknown parameters $\theta$ into two groups, with group $\theta_E$ containing the parameters estimated on-line, and group $\theta_R$ containing the parameters not estimated on-line. Assume, without loss of generality, that only the first $L$ unknown parameters are to be actually estimated :

$$\theta=[\,\theta_E^T\ \ \theta_R^T]^T \tag{3}$$

with

$$\theta_E = [\,\theta_j]^T{}_{j=1,\cdots,L}$$
$$\theta_R = [\,\theta_j]^T{}_{j=L+1,\cdots,m}$$

and let, correspondingly[9,10]

$$Y(q,\dot{q},\ddot{q}) = [\,Y_E(q,\dot{q},\ddot{q})\ \ Y_R(q,\dot{q},\ddot{q})] \tag{4}$$

with $Y_E$ and $Y_R$ are respectively the n×L matrix and n×(m - L) matrix. Using the above result, the left-hand side of equation (2) can be rewritten as

$$\begin{aligned} &M(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) + F(q,\dot{q}) \\ &\quad = Y_E(q,\dot{q},\ddot{q})\theta_E + Y_R(q,\dot{q},\ddot{q})\theta_R \end{aligned} \tag{5}$$

However, although we may already known with reasonable precision for some geometric parameters, it is difficult to obtain completely accurate values for these parameters, because the parameters of (5) depend on the manipulator structure and payload it carries. We can only get estimated values of manipulator payload parameters. Thus if $\theta_E$ and $\theta_R$ are replaced by their estimate $\hat{\theta}_E$ and $\hat{\theta}_R$, then equation (5) can be expressed as

$$\begin{aligned} &\hat{M}(q)\ddot{q} + \hat{B}(q,\dot{q})\dot{q} + \hat{G}(q) + \hat{F}(q,\dot{q}) \\ &\quad = Y_E(q,\dot{q},\ddot{q})\hat{\theta}_E + Y_R(q,\dot{q},\ddot{q})\hat{\theta}_R \end{aligned} \tag{6}$$

where $\hat{M}(q)$, $\hat{B}(q,\dot{q})$, $\hat{G}(q)$ and $\hat{F}(q,\dot{q})$ are estimates of $M(q)$, $B(q,\dot{q})$, $G(q)$ and $F(q,\dot{q})$ respectively.

## Ⅲ. Robust adaptive sliding mode controller

The control of a manipulator will always be challenged by the uncertainty as mentioned before and the disturbances possibly arising from the actual running of the actuator or some other causes. Hence, in this section we will consider a more general class of dynamic models which include input disturbances, i.e.,

$$\begin{aligned} &M(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) \\ &\quad + F(q,\dot{q}) = \tau(t) + d(t,q,\dot{q}) \end{aligned} \tag{7}$$

where $d(t,q,\dot{q})$ is the n×1 vector of disturbances referred to the actuator input.

The adaptive sliding mode design problem is stated as follows : Given the desired trajectory $q_d$ and $\dot{q}_d$, and the sliding surface $S^T=[s_1,\ldots,s_n]=0$, where

$$\begin{aligned} &S=\dot{q}(t)-\dot{q}_d(t) + C(q(t)-q_d(t)) \\ &C=diag(c_1,\ldots,c_n)\quad c_i>0,\quad i=1,\ldots,n \end{aligned} \tag{8}$$

and with some or all the manipulator parameters being not exactly known, derive a control law for the actuator torques and an estimation law for the unknown parameters such that the manipulator tracking errors $\tilde{q}(t)=q(t)-q_d(t)$ are forced to sliding along the sliding surface $S=0$, thus guaranteeing asymptotic convergence of the tracking after an initial adaptation process.

In order to derive the adaptive sliding mode law, the following assumption is required.

Assumption A1 : The desired trajectory $q_d$ is chosen such that $q_d$, $\dot{q}_d$, and $\ddot{q}_d$ are all bounded signals.

Assumption A2 : In many applications, the disturbances introduced into the $i$th actuator will usually be dependent only on the activity associated with the $i$th joint. Therefore, we henceforth consider the disturbances $d=(d_1,\dots,d_n)$ which satisfy the following :

$$|d_i(t,\dot{q}_i,q_i)| \le d_i^1 + d_i^2|q_i| + d_i^3|\dot{q}_i| \tag{9}$$

for some $d_i^j \ge 0$, $i=1,\dots,n$ $j=1,2,3$.

Now the robust adaptive sliding mode control law is chosen as

$$\tau(t)=Y_E(q,\dot{q},\dot{q}_d,\ddot{q}_d)\hat{\theta}_E + Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\hat{\theta}_R + \Delta\tau(t) \tag{10}$$

$$\Delta\tau(t) = -Ksgn(S) \tag{11}$$

$$\dot{\hat{\theta}}_E = -\Gamma Y_E^T(q,\dot{q},\dot{q}_d,\ddot{q}_d)S \tag{12}$$

where $\theta_E$ is the L×1 vector containing the parameters estimated on-line and $\hat{\theta}_E$ is its estimate ; $\theta_R$ is the (m-L)×1 vector containing the parameters not estimated on-line and $\hat{\theta}_R$ is its estimate; $\Gamma$ is a L x L symmetric positive definite matrix, usually diagonal, and $K=diag(k_1,\dots,k_n)$ will be determined in the following.

Theorem :

Consider robotic system defined by (7), with the sliding surface $S=0$ described by (8), then $S$ approaches zero asymptotically provided that the adaptive sliding mode control laws given by (10) - (12) are used. This in turn implies that the tracking error between the desired and actual trajectory converges asymptotically to the zero, i.e,

$$\lim_{t\to\infty}[q(t)-q_d(t)]=0 \quad \text{and} \quad \lim_{t\to\infty}[\dot{q}(t)-\dot{q}_d(t)]=0.$$

Proof :

Define the Lyapunov function candidate as

$$V(t)=\tfrac{1}{2}S^TM(q)S + \tfrac{1}{2}\tilde{\theta}_E^T\Gamma^{-1}\tilde{\theta}_E \tag{13}$$

where $\tilde{\theta}_E=\hat{\theta}_E-\theta_E$ denotes the parameter estimation error vector.

Differentiating $V(t)$ with respect to time yields

$$\dot{V}(t)=\tfrac{1}{2}\dot{S}^TM(q)S + \tfrac{1}{2}S^T\dot{M}(q)S + \tfrac{1}{2}S^TM(q)\dot{S} + \tfrac{1}{2}\dot{\tilde{\theta}}_E^T\Gamma^{-1}\tilde{\theta}_E + \tfrac{1}{2}\tilde{\theta}_E^T\Gamma^{-1}\dot{\tilde{\theta}}_E \tag{14}$$

Using the Property 2, function (14) becomes

$$\dot{V}(t)=S^T(M(q)\dot{S} + B(q,\dot{q})S) + \tilde{\theta}_E^T\Gamma^{-1}\dot{\tilde{\theta}}_E \tag{15}$$

To derive the $M(q)\dot{S}$ term, differentiating $S$ with respect to time yields

$$\dot{S}=\ddot{q}(t)-\ddot{q}_d(t) + C(\dot{q}(t)-\dot{q}_d(t)) \tag{16}$$

Multiplying the matrix $M(q)$ to (16) and inserting (3) gives

$$\begin{aligned} M(q)\dot{S} &= \tau(t)+d(t,q,\dot{q})+((M(q)C - B(q,\dot{q}))\dot{q}(t)-M(q)C\dot{q}_d(t) - G(q)-F(q,\dot{q})-M(q)\ddot{q}_d(t) \\ &= \tau(t)+d(t,q,\dot{q})-Y(q,\dot{q},\dot{q}_d,\ddot{q}_d)\theta \\ &= \tau(t)+d(t,q,\dot{q})-[Y_E(q,\dot{q},\dot{q}_d,\ddot{q}_d)\theta_E + Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\theta_R] \end{aligned} \tag{17}$$

Substituting (10) into (17) yields

$$\begin{aligned} M(q)\dot{S} &= Y_E(q,\dot{q},\dot{q}_d,\ddot{q}_d)\theta_E + Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\hat{\theta}_R - Ksgn(S)+d(t,q,\dot{q}) \\ &\quad -[Y_E(q,\dot{q},\dot{q}_d,\ddot{q}_d)\theta_E + Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\theta_R] \\ &= Y_E(q,\dot{q},\dot{q}_d,\ddot{q}_d)\tilde{\theta}_E + Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\tilde{\theta}_R \end{aligned}$$

$$- Ksgn(S) + d(t,q,\dot{q}) \tag{18}$$

where

$$Y_E(q,\dot{q},\dot{q}_d,\ddot{q}_d)\tilde{\theta}_E = Y_E(q,\dot{q},\dot{q}_d,\ddot{q}_d)\hat{\theta}_E - Y_E(q,\dot{q},\dot{q}_d,\ddot{q}_d)\theta_E$$

$$Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\tilde{\theta}_R = Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\hat{\theta}_R - Y_R(q,q,q_d,q_d)\theta_R$$

Utilizing the above result, function (15) can be represented as

$$\begin{aligned}\dot{V}(t) = S^T(&Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\tilde{\theta}_R \\ &- Ksgn(S) + d(t,q,\dot{q}) + B(q,\dot{q})S) \\ &+ \tilde{\theta}_E^T(\Gamma^{-1}\dot{\tilde{\theta}}_E + Y_E^T(q,\dot{q},\dot{q}_d,\ddot{q}_d)S)\end{aligned} \tag{19}$$

Note that $\dot{\tilde{\theta}}_E = \dot{\hat{\theta}}_E$ , since the unknown parameter estimating on-line $\theta_E$ are constant.

Thus, substituting (12) into (19), the resulting expression of $\dot{V}(t)$ is

$$\begin{aligned}\dot{V}(t) = S^T(&Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\tilde{\theta}_R \\ &+ B(q,\dot{q})S + d(t,q,\dot{q}) - Ksgn(S))\end{aligned} \tag{20}$$

Now, if we can ensure that $\dot{V}$ is negative semidefinite with respect to $S$(i.e. $\dot{V}<0$ for $S\neq 0$, and $\dot{V}=0$ for $S=0$) at all time by choosing $K$ suitably, it may be shown that the switching planes are asymptotically stable from which it may be deduced that the system described by (3) is stable and tracking errors converge to zero. An suitable choice for $K$ to make $\dot{V}$ a negative semidefinite function of $S$ would be

$$\begin{aligned}k_i = &\sum_{j=L+1}^{m} |\ Y_{Rij}\ |\ A_j + \sum_{j=1}^{n} f_{ij}(q,\dot{q})\ |\ s_j\ | \\ &+ d_i^1 + d_i^2\ |\ q_i\ | + d_i^3\ |\ q_i\ | + \delta_i\ , \\ &i=1,\dots,n\end{aligned} \tag{21}$$

where

$$|\ \tilde{\theta}_{Rj}\ | \le A_j,\ j=L+1,\dots,m$$

$$|B_{ij}(q,\dot{q})| < f_{ij}(q,\dot{q})\ ,$$

$$i=1,\dots,n\ ,\qquad j=1,\dots,n$$

$$\delta_i > 0\ ,\quad i=1,\dots,n$$

It is easily verified that the above choice for $K$ ensures the following desired condition for $\dot{V}$.

$$\dot{V}(t) = -\sum_{i=1}^{n}\delta_i |\ s_i | < 0 \tag{22}$$

Expression (22) shows that the output tracking error converges to the sliding surface $S=\dot{\tilde{q}}(t)+C\tilde{q}(t)=0$. This in turn implies that $\tilde{q}(t)\rightarrow 0$ as $t\rightarrow\infty$. Thus the robust adaptive sliding mode controller defined by (10) - (12) is globally asymptotically stable and guarantees zero steady state error for joint positions. □

**Remark 1.** From Assumption A1, since the desired trajectory $q_d$ is chosen such that $q_d$, $\dot{q}_d$, and $\ddot{q}_d$ are all bounded signals, $q$ and $\dot{q}$ are also uniformly bounded. Thus, we may bound the entries of $B(q,\dot{q})$, as like[1,5,11].

**Remark 2.** The control law given above is similar to that in[5]. Differences, however, exist in the fact that the unknown parameters are categorized into two groups, and the disturbances arising from the actuator or some other causes are explicitly considered.

## Ⅳ. Elimination of chattering

The control law given above are discontinuous and it is well known that synthesis of such control laws give rise to chattering of trajectories about the surface $S=0$. Chattering is undesirable in practice because it involves high control activity and further may excite high frequency dynamics neglected in the course of modeling(such as unmodelled structural modes, neglected time delays, and the like). To show that the control law proposed here also can be remedy this situation, we utilize the chattering elimination scheme presented in the literature. As suggested by several authors[1],[4],[5],[12], we can eliminate this problem by smoothing out the discontinuous control law in the neighborhood of the sliding surface. To do this, we replace signim nonlinearity by a saturation nonlinearity, which is defined as

$$sat(S)=\begin{cases} 1 & \text{if} \quad S/\Phi\geq 1 \\ S/\Phi & \text{if} \quad -1 < S/\Phi < 1 \\ -1 & \text{if} \quad S/\Phi\leq -1 \end{cases}$$

where $\Phi$ is the boundary layer thickness. With this boundary layer, the robust adaptive sliding mode control law, for example, given by (10) - (12), becomes

$$\tau(t)=Y_E(q,\dot{q},\dot{q}_d,\ddot{q}_d)\hat{\theta}_E + Y_R(q,\dot{q},\dot{q}_d,\ddot{q}_d)\hat{\theta}_R+\Delta\tau(t) \quad (23)$$

$$\Delta\tau_i=-\Big(\sum_{j=L+1}^{m}|Y_{Rij}|A_j+\sum_{j=1}^{n}f_{ij}(q,q)|s_{\Phi j}| + d_i^1+d_i^2|\dot{q}_i|+d_i^3|q_i|+\delta_i\Big)sat(s_i/\Phi_i)$$

$$i=1,\ldots,n \quad (24)$$

$$\dot{\hat{\theta}}_E=-\Gamma Y_E^T(q,\dot{q},\dot{q}_d,\ddot{q}_d)S_\Phi \quad (25)$$

where $S_\Phi=(s_{\Phi 1},\ldots,s_{\Phi n})T$ with $s_{\Phi i}=s_i-\Phi_i sat(s_i/\Phi_i)$ is a measurement of the algebraic distance of the current state to the boundary layer. We can again demonstrate global convergence of the tracking errors to the sliding surface boundary layer by using the Lyapunov function

$$V(t)=\tfrac{1}{2}S_\Phi^T M(q)S_\Phi^T+\tfrac{1}{2}\tilde{\theta}_E^T\Gamma^{-1}\tilde{\theta}_E \quad (26)$$

instead of (13), and noting that $S_\Phi=S$ outside the boundary, while $S_\Phi=0$ inside the boundary layer, which yields

$$\dot{V}(t)=-\sum_{i=1}^{n}\delta_i|s_{\Phi i}|<0 \quad (27)$$

Definition (26) implies that $\dot{V}(t)=0$ inside the boundary layer, which shows that (27) is valid everywhere and thus further guarantees that trajectories eventually converge to the boundary layer.

**Remark 3.** By using boundary layer, similarly to[13], $S$ is then guaranteed to remain in the boundary layers, with corresponding small tracking errors. As like[14], parameter adaptation must then be stopped when the system trajectories are inside the boundary layers. This procedure has also the advantage of avoiding long-term drift of the estimated parameters.

## V. Simulation

A computer simulation is performed to evaluate the performance of control algorithm. Consider the two-link planar manipulator as shown in Fig.1, carrying a load of unknown mass.

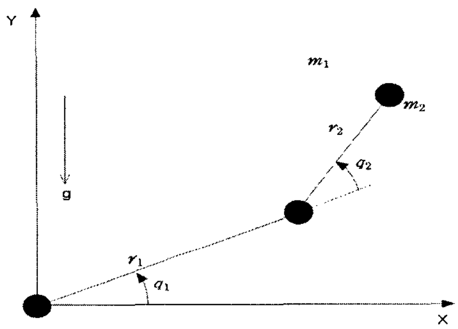



그림 1. 2-링크 매니퓰레이터 모델.
Fig. 1. Two-link manipulator model.

The dynamics of the manipulator with payload can be written as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}+\begin{bmatrix}-B_{12}\dot{q}_2 & -B_{12}(\dot{q}_1+\dot{q}_2)\\ B_{12}\dot{q}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}+\begin{bmatrix}G_1\\ G_2\end{bmatrix}+\begin{bmatrix}F_1\\ F_2\end{bmatrix}=\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$$

where

$$M_{11}=(m_1+m_2)r_1^2+m_2r_2^2+2m_2r_1r_2\cos(q_2),$$
$$M_{12}=m_2r_2^2+m_2r_1r_2\cos(q_2),$$
$$M_{21}=M_{12},\ M_{22}=m_2r_2^2,$$
$$B_{12}=m_2r_1r_2\sin(q_2),$$
$$G_1=(m_1+m_2)r_1^2 g\cos(q_2)+m_2r_1r_2 g\cos(q_1+q_2),$$
$$G_2=m_2r_1r_2 g\cos(q_1+q_2),$$
$$F_1=v_1\dot{q}_1+\xi_1 sgn(\dot{q}_1)$$
$$F_2=v_2\dot{q}_2+\xi_2 sgn(\dot{q}_2)$$

and $g$ is the acceleration of gravity. The unknown parameters were considered to be :

$$\theta=[\alpha \;\; \beta \;\; \gamma \;\; v_1 \;\; v_2 \;\; \xi_1 \;\; \xi_2]$$

where $\alpha=(m_1+m_2)r_1^2$, $\beta=m_2 r_2^2$ and $\gamma=m_2 r_1 r_2$. We categorize the unknown parameters $\theta$ into two groups, with group $\theta_E=[\alpha \; \beta \; \gamma]^T$ containing the parameters estimated on-line, and group $\theta_R=[v_1 \; v_2 \; \xi_1 \; \xi_2]^T$ containing the parameters not estimated on-line.

The parameter values used are selected as $m_1=m_2=0.5kg$, $r_1=1m$, $r_2=0.8m$. Thus the true values of unknown parameters are $\alpha=1$, $\beta=0.32$ and $\gamma=0.4$. The corresponding initial parameter estimates are selected as $\hat{\alpha}=0.72$, $\hat{\beta}=0.25$ and $\hat{\gamma}=0.32$. The constant parameters are chosen as $C=3I$, $\Gamma=0.05I$, $\delta_1=\delta_2=2.0$, $\hat{\theta}_R=[0.5 \; 0.5 \; 1.0 \; 1.0]^T$ and $A=[0.3 \; 0.3 \; 0.5 \; 0.5]^T$. The entries of the matrix $B(q, \dot{q})$ can be upper-bounded

$$|B_{11}| < \eta|\dot{q}_2| < \bar{\eta}|\dot{q}_2| = f_{11}$$
$$|B_{12}| < \eta|\dot{q}_1+\dot{q}_2| < \bar{\eta}|\dot{q}_1+\dot{q}_2| = f_{12}$$
$$|B_{21}| < \eta|\dot{q}_1| < \bar{\eta}|\dot{q}_1| = f_{21}$$
$$|B_{22}| = 0 = f_{22}$$

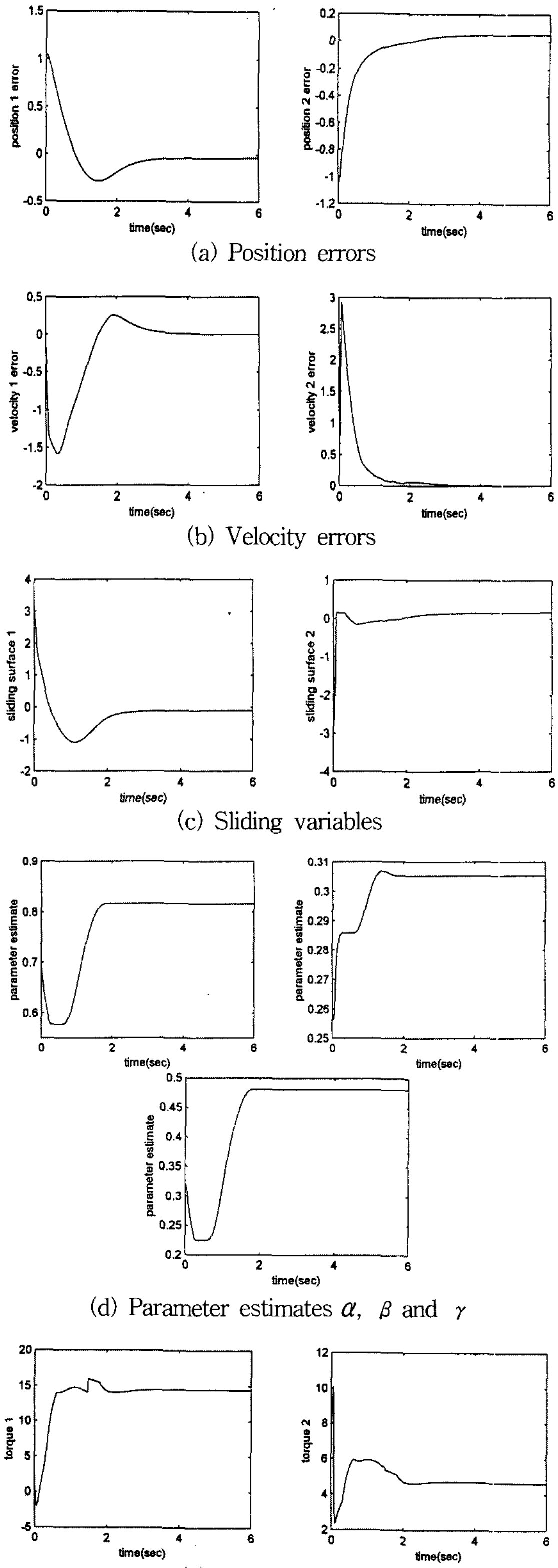
and we select $\bar{\eta}=1$.

**Example 1** : Ideal Case

The desired joint trajectories are chosen to be $q_d(t)=0$, $\dot{q}_d(t)=0$ and the initial position of $q_1(t)$ is $60^0$ and $q_2(t)$ is $-60^0$. The boundary layer thickness is chosen to be $\Phi_1=0.5\rho_1$ and $\Phi_2=0.5\rho_2$. The selection of $\rho_i$ depends on the strength of the discontinuities of control efforts. We choose $\rho_1=1$ and $\rho_2=1$ for this simulation.

**Example 2.** The case which the external disturbances exist

In this case, all the conditions are the same as Example 1. However, we choose external input disturbances as : $d_1(t,q,\dot{q})=d_1(t)+\zeta_1\dot{q}_1$, $d_2(t,q,\dot{q})=d_2(t)+\zeta_2\dot{q}_2$ ; where $d_1^1(t)=d_2^1(t)=0.5\sin(0.25t)$, $\zeta_1=\zeta_2=0, 0\le t<3$ ; $d_1^1(t)=d_2^1(t)=0.5\sin(0.25t)$,



(a) Position errors
(b) Velocity errors
(c) Sliding variables
(d) Parameter estimates $\alpha$, $\beta$ and $\gamma$
(e) Control torques

그림 2. 예제 1에 대한 모의실험 결과
Fig. 2. Simulation results for Example 1.

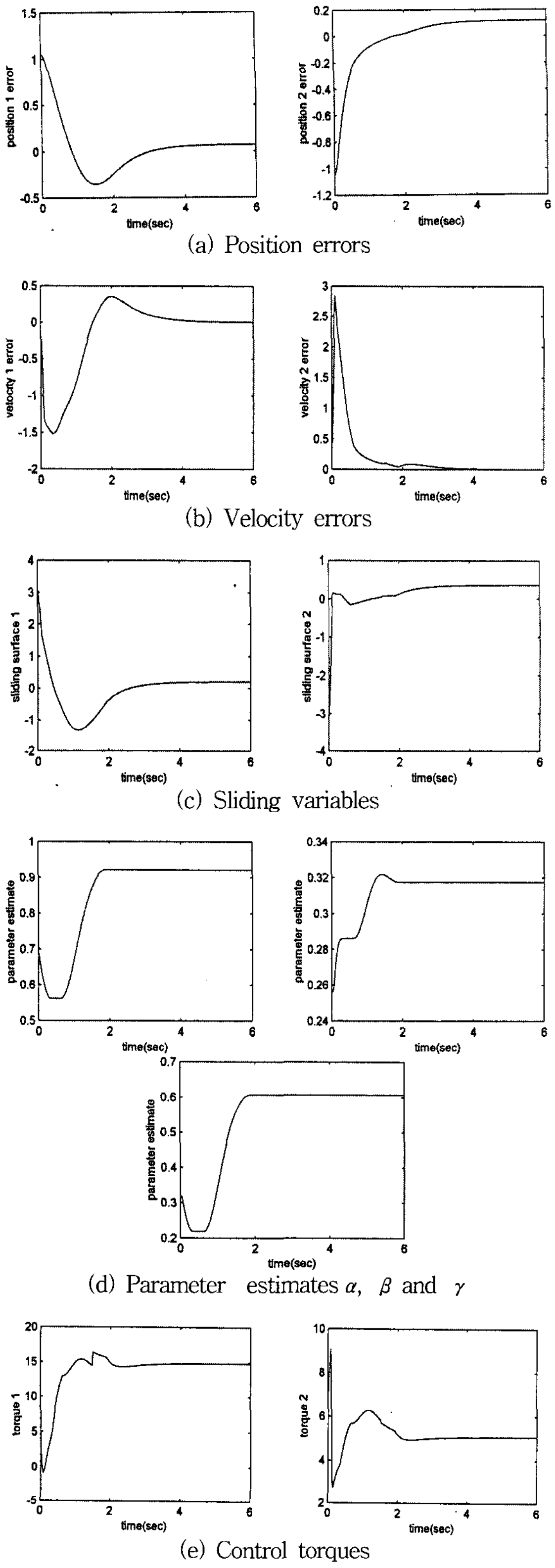



그림 3. 예제 2에 대한 모의실험 결과
Fig. 3. Simulation results for Example 2.

$\zeta_1=\zeta_2=0.5$, $3\le t<6$: and the corresponding values as : $d_1^1=d_2^1=1.0$ , $\zeta_1=\zeta_2=0$, $0\le t<3$ ; $d_1(t,q,\dot{q})=1.0+0.5|\dot{q}_1|$, $d_2(t,q,\dot{q})=1.0+0.5|\dot{q}_2|$, $3\le t<6$.

As a result from computer simulation, Fig. 2 (a)-(e) illustrate the time trajectories of the position error $\tilde{q}(t)$, velocity error $\dot{\tilde{q}}(t)$, values of $s_1$ and $s_2$ , parameter estimates $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ , and input torque $\tau(t)$ under no input disturbances, whereas Fig. 3 (a)-(e) depict the trajectories of the same signal with input disturbances.

From Fig. 2 and Fig. 3, we see that there does not exist a significant difference in both tracking performance. However, Fig. 2(a) and Fig. 3(a) shows tracking precisions of 0.038rad, 0.053rad for $q_1(t)$ and $q_2(t)$ without input disturbances and 0.058rad and 0.12rad with input disturbances. These tracking errors may be reduced by decreasing boundary layer thickness $\Phi_i$.

Although convergence of the trajectory tracking is guaranteed in the simulations, the parameter estimates do not converge to their exact value, since the desired trajectory is not persistently exciting[15]. When the desired trajectory is chosen to be persistently exciting, simulations do yield convergence of the parameter estimation.

## Ⅵ. Conclusion

A robust adaptive sliding mode control scheme has been presented for trajectory tracking of robot manipulators, with unknown parameters being estimated on-line. The controller is designed based on a Lyapunov method, which consists of a feed-forward compensation part and discontinuous control part. In addition, since the controller is exploit the particular structure of manipulator dynamics and does not explicitly estimate all unknown parameters, it is especially structurally simple and computationally fast. The algorithm is able to achieve zero tracking error in the presence of external disturbances and parameter uncertainties. Ongoing research will be on extending the algorithm to the combination of learning control.

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저 자 소 개


裵 焌 垌(正會員)
1982년 : 국민대학교 전자공학과(공학사). 1984년 : 경희대학교 전자공학과(공학석사). 1990년 : 경희대학교 전자공학과(공학박사). 1990년 3월~현재 : 진주산업대학교 산업자동화공학과 부교수. <주관심분야> 로보틱스, 적응제어, 가변구조제어 및 지능제어