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A Comparison between 3-D Analytical and Finite Difference Method for a Trapezoidal Profile Fin

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Abstract

A comparison is made of the temperature distribution and heat loss from a trapezoidal profile fin using two different 3-dimensional methods. These two methods are analytical and finite difference methods. In the finite difference method 78 nodes are used for a fourth of the fin. A trapezoidal profile fin being the height of the fin tip is half of that of the fin base is chosen arbitrarily as the model. One of the results shows that the relative error in the total convection heat loss obtained by using 78 nodes in the finite difference method as compared to the heat conduction through the fin root obtained by analytic method seems to be good (i.e., -3.5% < relative error < 1.0%) for the following range ; Bi < 0.3, L < 2 and 0.4 < w < 10.

Keywords : *3-D analytical method, 3-D finite difference method, conduction, convection, Biot number*

1. Introduction

Fins as extended surfaces are widely used to enhance the rate of heat transfer to a surrounding fluid in many applications such as the cooling devices of internal combustion engines, electronic equipments and heat exchangers, etc.. In the analysis of heat transfer from the fins, it is traditionally assumed that the heat flow is one-dimensional (Look, 1995; Look, 1997; Aparecido and Cotta, 1990), although convenient, may cause error under certain physical conditions (e.g., when Biot number is comparatively large or height of the fins is high). To make more accurate prediction of the heat flow in the fins for efficient designs, some papers present multidimensional approach (Look and Kang, 1992; Kang and Look, 1999; Manzoor, Ingham and Heggs, 1983; Onur, 1996; Abrate and Newnham, 1995; Oh, Jo and Cho, 1989) taking into account the transversal temperature gradients. Also many kinds of methods (i.e., the analytical method (Look, 1995; Look, 1997; Aparecido and Cotta, 1990; Look and Kang, 1992; Onur, 1996), the heat balance integral method (Oh, Jo and Cho, 1989), the finite difference method (Kang and Look, 1999), the finite element method (Abrate and Newnham, 1995), and the boundary integral equation method (Manzoor, Ingham and Heggs, 1983)) have been used to analyze the heat transfer problem. But no literature seems to be available which presents the comparison of two different three- dimensional analysis on the heat transfer problem.

In this study, by using two different three-dimensional methods (i.e., the analytical method and the finite difference method), the convection heat loss at the each surface, the total heat loss from the fin and the temperature distribution are studied in the specific trapezoidal profile fin (i.e., the height of the fin tip is one half of that of the fin base) and then the comparisons of the results obtained using these two methods are presented with respect to the

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non-dimensional fin length, width and Biot number to establish the range of the validity of both methods in this study. For simplicity, the base temperature, the heat transfer coefficient and the thermal conductivity of the fin material are assumed constant and the condition is assumed to be steady-state.

2. Three-Dimensional Analysis

2.1 Analytical Method

Consider a three-dimensional trapezoidal profile fin geometry being the height of the fin tip is one half of that of the fin base as shown in Fig. 1.

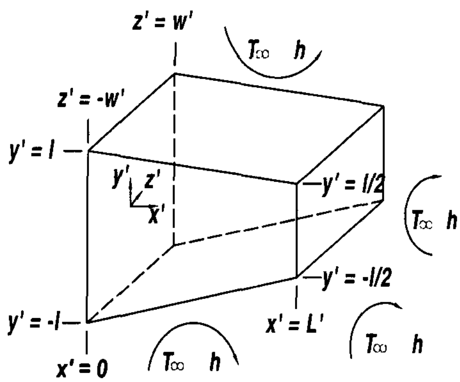


Fig. 1 Geometry of a trapezoidal fin.

Three-dimensional governing differential equation under steady state for this figure is

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2} = 0 \qquad (1)$$

Five boundary conditions and one energy balance equation are required to solve the Eq. (1). These conditions are shown as Eqs. (2)~(7)

$$\theta = 1 \qquad \text{at } x = 0 \qquad (2)$$

$$\frac{\partial\theta}{\partial y} = 0 \qquad \text{at } y = 0 \qquad (3)$$

$$\frac{\partial\theta}{\partial z} = 0 \qquad \text{at } z = 0 \qquad (4)$$

$$\frac{\partial\theta}{\partial x} + Bi\cdot\theta = 0 \qquad \text{at } x = L \qquad (5)$$

$$\frac{\partial\theta}{\partial z} + Bi\cdot\theta = 0 \qquad \text{at } z = w \qquad (6)$$

$$\begin{aligned} &-\int_0^1\int_0^w \frac{\partial\theta}{\partial x}\Big|_{x=0} dz\cdot dy \\ &= -\int_0^{\frac{1}{2}}\int_0^w \frac{\partial\theta}{\partial x}\Big|_{x=L} dz\cdot dy \\ &+\int_0^L Bi\cdot\theta\Big|_{z=w} y\cdot dx \\ &+\int_{\frac{1}{2}}^1\int_0^w Bi\cdot\sqrt{1+4L^2}\cdot\theta\cdot dz\cdot dy \end{aligned} \qquad (7)$$

where

$$\theta = (T-T_\infty)/(T_w-T_\infty),\quad x=\frac{x'}{l},\quad y=\frac{y'}{l},$$

$$z=\frac{z'}{l},\quad w=\frac{w'}{l},\quad L=\frac{L'}{l} \text{ and } Bi=\frac{hl}{k}$$

The solution for the temperature distribution $\theta(x,y,z)$ within the fin obtained with Eqs. (2)~(5) is

$$\theta(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} N_{nm}\cdot f(x)\cdot\cos(\lambda_m\cdot y)\cdot\cos(\mu_n\cdot z) \qquad (8)$$

where

$$N_{nm} = \frac{4\sin\lambda_m}{2\lambda_m+\sin(2\lambda_m)}\cdot\frac{4\sin(\mu_n\cdot w)}{2\mu_n\cdot w+\sin(2\mu_n\cdot w)} \qquad (9)$$

$$f(x) = \cosh(\rho_{nm}\cdot x) - f_{nm}\cdot\sinh(\rho_{nm}\cdot x) \qquad (10)$$

$$f_{nm} = \frac{\rho_{nm}\cdot\tanh(\rho_{nm}\cdot L)+Bi}{\rho_{nm}+Bi\cdot\tanh(\rho_{nm}\cdot L)} \qquad (11)$$

$$\rho_{nm} = \sqrt{(\lambda_m{}^2 + \mu_n{}^2)} \qquad (12)$$

The eigenvalues μ_n in Eq. (8) can be obtained from Eq. (13) which comes from Eq. (6).

$$\mu_n \cdot \tan(\mu_n \cdot w) = Bi \qquad (13)$$

Substituting μ_n into Eq. (14) transformed from energy balance equation (7) yields another eigenvalues λ_m .

$$\begin{aligned}&(\rho_{nm}^2 + \frac{1}{4L^2} \cdot \lambda_m)^2 \cdot \sin(\mu_n \cdot w) \cdot \rho_{mn} \\ &\cdot \left\{ \sin\lambda_m \cdot AA_{nm} - Bi \cdot \sin(\frac{\lambda_m}{2}) \right\} \\ &+ Bi \cdot \lambda_m \cdot \mu_n \cdot \cos(\mu_n \cdot w) \\ &\cdot \left\{ \frac{1}{2} \rho_{nm} \cdot \cos(\frac{\lambda_m}{2}) \cdot CC_{nm} \right. \\ &+ \frac{1}{2L} \cdot \sin(\frac{\lambda_m}{2}) \cdot \rho_{nm} \cdot \lambda_m \cdot DD_{nm} \\ &- \rho_{nm} \cdot AA_{nm} \cdot EE_{nm} - \frac{1}{2L} \cdot BB_{nm} \\ &\cdot (FF_{nm} + GG_{nm})\} - Bi \cdot \sqrt{1 + \frac{1}{4L^2}} \\ &\cdot (\rho_{nm}^2 + \frac{1}{4L^2} \cdot \lambda_m^2) \cdot \lambda_m \cdot \sin(\mu_n \cdot w) \\ &\cdot \{ AA_{nm} \cdot \rho_{nm} \cdot \cos\lambda_m - HH_{nm} \\ &\left. + \frac{1}{2L} \cdot BB_{nm} \cdot \lambda_m \cdot \sin\lambda_m \right\} = 0 \qquad (14)\end{aligned}$$

where

$$\begin{aligned}AA_{nm} = &\rho_{nm} \cdot \sinh(\rho_{nm} \cdot L) \\ &+ Bi \cdot \cosh(\rho_{nm} \cdot L) \qquad (15)\end{aligned}$$

$$\begin{aligned}BB_{nm} = &\rho_{nm} \cdot \cosh(\rho_{nm} \cdot L) \\ &+ Bi \cdot \sinh(\rho_{nm} \cdot L) \qquad (16)\end{aligned}$$

$$\begin{aligned}CC_{nm} = &Bi \cdot \rho_{nm}^2 \\ &+ \frac{1}{4L^2} \cdot Bi \cdot \lambda_m^2 - \frac{\rho_{nm}^2}{L} + \frac{\lambda_m^2}{4L^3} \qquad (17)\end{aligned}$$

$$DD_{nm} = \frac{1}{2} \rho_{nm}^2 + \frac{1}{8L^2} \cdot \lambda_m^2 - \frac{Bi}{L} \qquad (18)$$

$$\begin{aligned}EE_{nm} = &\rho_{nm}^2 \cdot \cos\lambda_m + \frac{1}{4L^2} \cdot \lambda_m^2 \cdot \cos\lambda_m \\ &- \frac{1}{2L^2} \cdot \lambda_m \cdot \sin\lambda_m \qquad (19)\end{aligned}$$

$$\begin{aligned}FF_{nm} = &\rho_{nm}^2 \cdot \lambda_m \cdot \sin\lambda_m \\ &+ \frac{1}{4L^2} \cdot \lambda_m^3 \cdot \sin\lambda_m \qquad (20)\end{aligned}$$

$$\begin{aligned}GG_{nm} = &\frac{1}{4L^2} \cdot \lambda_m^2 \cdot \cos\lambda_m \\ &- \rho_{nm}^2 \cdot \cos\lambda_m \qquad (21)\end{aligned}$$

$$\begin{aligned}HH_{nm} = &Bi \cdot \rho_{nm} \cdot \cos(\frac{\lambda_m}{2}) \\ &+ \frac{1}{2L} \cdot \rho_{nm} \cdot \lambda_m \cdot \sin(\frac{\lambda_m}{2}) \qquad (22)\end{aligned}$$

By applying Eq. (8) to Fourier's law, the heat loss rate conducted into the fin through the fin base is given by

$$\begin{aligned}Q_k = &4k \cdot l \cdot \theta_0 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} N_{nm} \cdot \rho_{nm} \cdot f_{nm} \\ &\cdot \frac{\sin\lambda_m}{\lambda_m} \cdot \frac{\sin(\mu_n \cdot w)}{\mu_n} \qquad (23)\end{aligned}$$

where

$$\theta_0 = T_w - T_\infty$$

The heat loss conducted into the fin through the fin base should be equal to total convection heat loss from the each surface. The convection heat loss from the each surface (i.e., fin tip, both sides and top & bottom) is obtained using Newton's law of cooling and the equations for the heat loss from each surface are shown as Eq. (24) through Eq. (26).

$$\begin{aligned}\frac{Q_{htip}}{k \cdot l} = &\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} N_{nm} \cdot \rho_{nm} \cdot \frac{Bi}{aa_{nm}} \\ &\cdot \frac{\sin(\frac{\lambda_m}{2})}{\lambda_m} \cdot \frac{\sin(\mu_n \cdot w)}{\mu_n} \qquad (24)\end{aligned}$$

$$\frac{Q_{hbs}}{k \cdot l} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Bi \cdot N_{nm} \cdot \cos(\mu_n \cdot w)$$
$$\cdot \left[\frac{1}{2} \cdot \frac{\rho_{nm}}{bb_{nm}} \cdot \left\{ \cos(\frac{\lambda_m}{2}) - \frac{1}{L^2} \cdot \frac{\lambda_{nm}}{bb_{nm}} \right. \right.$$
$$\left. \cdot \sin(\frac{\lambda_m}{2}) \right\} \cdot cc_{nm} - \frac{1}{2L} \cdot \frac{1}{bb_{nm}}$$
$$\cdot \left\{ \frac{\lambda_m}{2} \cdot \sin(\frac{\lambda_m}{2}) - \frac{dd_{nm}}{bb_{nm}} \cdot \cos(\frac{\lambda_m}{2}) \right\} \cdot ee_{nm}$$
$$- \frac{1}{2} \cdot \frac{1}{bb_{nm}^2} \cdot (\frac{\cos\lambda_m}{L} - f_{nm} \cdot \rho_{nm}$$
$$\left. \cdot \lambda_m \cdot \sin\lambda_m) - \frac{hh_{nm}}{bb_{nm}} \right] \quad (25)$$

$$\frac{Q_{htb}}{k \cdot l} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Bi \cdot \sqrt{1 + \frac{1}{4L^2}}$$
$$\cdot \frac{N_{nm}}{bb_{nm}} \cdot \frac{\sin(\mu_n \cdot w)}{\mu_n} \cdot \left[\frac{\lambda_m}{2L} \cdot \sin\lambda_m \right.$$
$$+ ff_{nm} \cdot \sinh(\rho_{nm} \cdot L) - gg_{nm} \cdot \cosh(\rho_{nm} \cdot L)$$
$$\left. - f_{nm} \cdot \rho_{nm} \cdot \cos\lambda_m \right] \quad (26)$$

where

$$aa_{nm} = \rho_{nm} \cdot \cosh(\rho_{nm} \cdot L) + Bi \cdot \sinh(\rho_{nm} \cdot L) \quad (27)$$

$$bb_{nm} = \rho_{nm}^2 + \frac{\lambda_m^2}{4L^2} \quad (28)$$

$$cc_{nm} = \sinh(\rho_{nm} \cdot L) + f_{nm} \cdot \cosh(\rho_{nm} \cdot L) \quad (29)$$

$$dd_{nm} = \rho_{nm}^2 - \frac{\lambda_m^2}{4L^2} \quad (30)$$

$$ee_{nm} = \cosh(\rho_{nm} \cdot L) + f_{nm} \cdot \sinh(\rho_{nm} \cdot L) \quad (31)$$

$$ff_{nm} = \rho_{nm} \cdot \cos(\frac{\lambda_m}{2}) - f_{nm} \cdot \frac{\lambda_m}{2L} \cdot \sin(\frac{\lambda_m}{2}) \quad (32)$$

$$gg_{nm} = \frac{\lambda_m}{2L} \cdot \sin(\frac{\lambda_m}{2}) - \rho_{nm} \cdot \cos(\frac{\lambda_m}{2}) \quad (33)$$

$$hh_{nm} = f_{nm} \cdot \rho_{nm} \cdot \cos\lambda_m - \frac{\lambda_m}{2L} \cdot \sin\lambda_m \quad (34)$$

The total convection heat loss from the fin can be calculated using Eq. (35).

$$Q_h = Q_{htip} + Q_{hbs} + Q_{htb} \quad (35)$$

2.2 Finite Difference Method

For this finite difference method, fin length, height and width are divided into respectively 4, 16 and 4 by uniform distances along the x, y, z direction, where these uniform distances are Δx, Δy and Δz. Accordingly 78 nodes in a fourth of the fin is located as illustrated in Fig. 2.

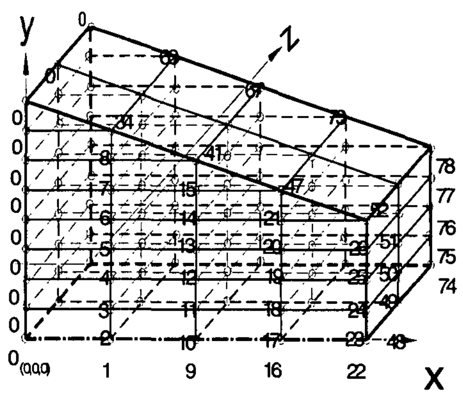


Fig. 2 A fourth trapezoidal profile fin geometry showing the 78 nodes in the finite difference method.

Seventy eight equations are solved simultaneously to obtain the value of the temperature at each node. Examples of the equations used are given by Eqs. (36)~(53).

For node 1 (and a similar form for the points 9, 16)

$$1 - C \cdot \theta_1 + 2A \cdot \theta_2 + \theta_9 + 2B \cdot \theta_{27} = 0 \quad (36)$$

For node 2 (and a similar form for the points 3-7, 10-14, 17-20)

$$1 + A \cdot \theta_1 - C \cdot \theta_2 + A \cdot \theta_3 + \theta_{10} + 2B \cdot \theta_{28} = 0 \quad (37)$$

For node 8 (and a similar form for the points 15, 21)

$$1 + A \cdot \theta_7 - F \cdot \theta_8 + B \cdot \theta_{34} = 0 \quad (38)$$

For node 22

$$\theta_{16} - D \cdot \theta_{22} + A \cdot \theta_{23} + B \cdot \theta_{48} = 0 \quad (39)$$

For node 23 (and a similar form for the points 24-25)

$$\theta_{17} + \frac{1}{2} A \cdot \theta_{22} - D \cdot \theta_{23} + \frac{1}{2} A \cdot \theta_{24} + B \cdot \theta_{49} = 0 \quad (40)$$

For node 26

$$\theta_{20} + \frac{1}{2} A \cdot \theta_{25} - E \cdot \theta_{26} + \frac{3}{4} B \cdot \theta_{52} = 0 \quad (41)$$

For node 27 (and a similar form for the points 35, 42)

$$1 + B \cdot \theta_1 - C \cdot \theta_{27} + 2A \cdot \theta_{28} + \theta_{35} + B \cdot \theta_{53} = 0 \quad (42)$$

For node 28 (and a similar form for the points 29-33, 36-40, 43-46)

$$1 + B \cdot \theta_2 + A \cdot \theta_{27} - C \cdot \theta_{28} + A \cdot \theta_{29} + \theta_{36} + B \cdot \theta_{54} = 0 \quad (43)$$

For node 34 (and a similar form for the points 41, 47)

$$1 + \frac{1}{2} B \cdot \theta_8 + A \cdot \theta_{33} - F \cdot \theta_{34} + \frac{1}{2} B \cdot \theta_{60} = 0 \quad (44)$$

For node 48

$$\frac{1}{2} B \cdot \theta_{22} + \theta_{42} - D \cdot \theta_{48} + A \cdot \theta_{49} + \frac{1}{2} B \cdot \theta_{74} = 0 \quad (45)$$

For node 49 (and a similar form for the points 50, 51)

$$\frac{1}{2} B \cdot \theta_{23} + \theta_{43} + \frac{1}{2} A \cdot \theta_{48} - D \cdot \theta_{49} + \frac{1}{2} A \cdot \theta_{50} + \frac{1}{2} B \cdot \theta_{75} = 0 \quad (46)$$

For node 52

$$\frac{3}{8} B \cdot \theta_{26} + \theta_{46} + \frac{1}{2} A \cdot \theta_{51} - E \cdot \theta_{52} + \frac{3}{8} B \cdot \theta_{78} = 0 \quad (47)$$

For node 53 (and a similar form for the points 61, 68)

$$1 + 2B \cdot \theta_{27} - 2G \cdot \theta_{53} + 2A \cdot \theta_{54} + \theta_{61} = 0 \quad (48)$$

For node 54 (and a similar form for the points 55-59, 62-66, 69-72)

$$1 + 2B \cdot \theta_{28} + A \cdot \theta_{53} - 2G \cdot \theta_{54} + A \cdot \theta_{55} + \theta_{62} = 0 \quad (49)$$

For node 60 (and a similar form for the points 67, 73)

$$1 + B \cdot \theta_{34} + A \cdot \theta_{59} - R \cdot \theta_{60} = 0 \quad (50)$$

For node 74

$$B \cdot \theta_{48} + \theta_{68} - H \cdot \theta_{74} + A \cdot \theta_{75} = 0 \quad (51)$$

For node 75 (and a similar form for the points 76, 77)

$$B \cdot \theta_{49} + \theta_{69} + \frac{1}{2} A \cdot \theta_{74} - H \cdot \theta_{75} + \frac{1}{2} A \cdot \theta_{76} = 0 \quad (52)$$

For node 78

$$\frac{3}{4}B\cdot\theta_{52}+\theta_{72}+\frac{1}{2}A\cdot\theta_{77}-P\cdot\theta_{78}=0 \quad (53)$$

where

$$A=\left(\frac{\Delta x}{\Delta y}\right)^2 \quad (54)$$

$$B=\left(\frac{\Delta x}{\Delta z}\right)^2 \quad (55)$$

$$C=2(1+A+B) \quad (56)$$

$$D=1+Bi\cdot\Delta x+A+B \quad (57)$$

$$E=1+(1+\sqrt{A+1})\cdot Bi\cdot\frac{\Delta x}{2}+\frac{1}{2}A+\frac{3}{4}B \quad (58)$$

$$F=1+Bi\cdot\Delta x\cdot\sqrt{A+1}+A+B \quad (59)$$

$$G=1+Bi\cdot\Delta z\cdot B+A+B \quad (60)$$

$$H=1+Bi\cdot\Delta x+B\cdot(Bi\cdot\Delta z+1)+A \quad (61)$$

$$P=1+\frac{1}{2}Bi\cdot\Delta x\cdot(1+\sqrt{A+1})+\frac{3}{4}B\cdot(1+Bi\cdot\Delta z)+\frac{1}{2}A \quad (62)$$

$$Q=1+Bi\cdot\Delta x\cdot\sqrt{A+1}+B\cdot(Bi\cdot\Delta z+1)+A \quad (63)$$

The convection heat loss from the each surface can be calculated using Eqs. (64)~(66).

$$\frac{Q_{tb}}{k\cdot l}=AA\cdot\left\{1+\sum_i\theta_i+\frac{1}{2}\sum_j\theta_j+\frac{1}{4}\sum_k\theta_k\right\} \quad (64)$$

$$\frac{Q_{bs}}{k\cdot l}=BB\cdot\left\{\frac{31}{8}+\frac{1}{2}\sum_l\theta_l+\sum_m\theta_m+\frac{1}{4}\theta_{74}+\frac{3}{8}\theta_{78}\right\} \quad (65)$$

$$\frac{Q_{tip}}{k\cdot l}=BB\cdot\left\{\frac{1}{4}\sum_p\theta_p+\frac{1}{2}\sum_q\theta_q+\sum_r\theta_r\right\} \quad (66)$$

where,

$AA=4\theta_0\cdot Bi\cdot\Delta z\cdot\sqrt{\Delta x^2+\Delta y^2}$

$BB=4\theta_0\cdot Bi\cdot\Delta y\cdot\Delta z$

i = 34, 41, 47

j = 8, 15, 21, 52, 60, 67, 73

k = 26, 78

l = 53, 60, 61, 67, 68, 73, 75, 76, 77

m = 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 69, 70, 71, 72

p = 22, 26, 74, 78

q = 23, 24, 25, 48, 52, 75, 76, 77

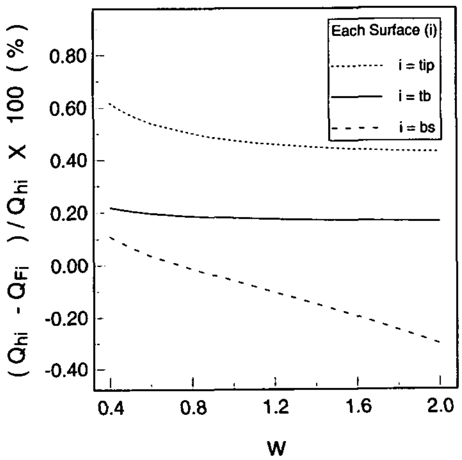
r = 49, 50, 51

The total convection heat loss Q_F using the finite difference method can be obtained by adding convection heat loss from each surface.

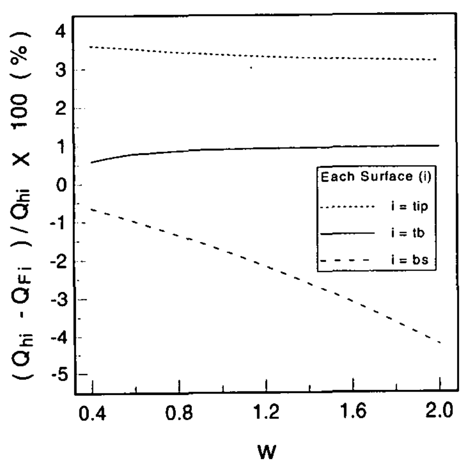
3. Results and Discussions

Fig. 3(a) presents the relative error in the convection heat loss from each surface of the trapezoidal profile fin using the finite difference method as compared to the analytical method as a function of fin width for L=2 and Bi=0.01. It is shown that the relative error decreases as w increases at the tip while it varies from the positive value to the negative value for both sides. And the relative error for top & bottom almost seems to be independent on the fin width. But the relative errors for each surface are all very small and these are within the range from -0.30% to 0.62%. The same description but for Bi=0.1 case is shown in Fig. 3(b). As expected, on the whole, the relative errors in case of Bi=0.1 are larger than those for Bi=0.01. But the variation trends are similar for both Biot numbers. It must be noted that even though the relative error at each surface is large, the total relative error becomes small. For example, in case of w=2, Bi=0.1, the relative error at the tip is 3.2% and that at the both sides is -4.2% but the total relative error becomes 0.6% and it will be shown later in Fig. 6.

Figs 4(a), (b) also show the same sort of comparison as presented in Figs 3(a), (b), but as a function of L, for w=0.4. First the variation



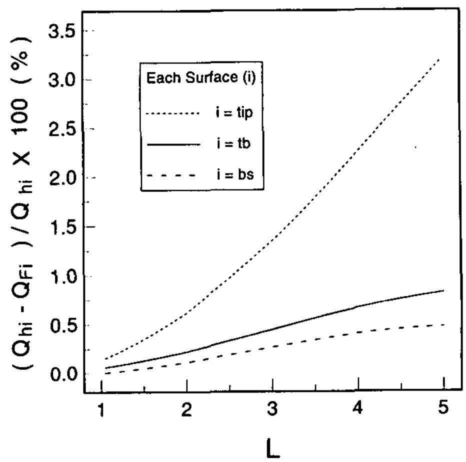
(a) Bi=0.01



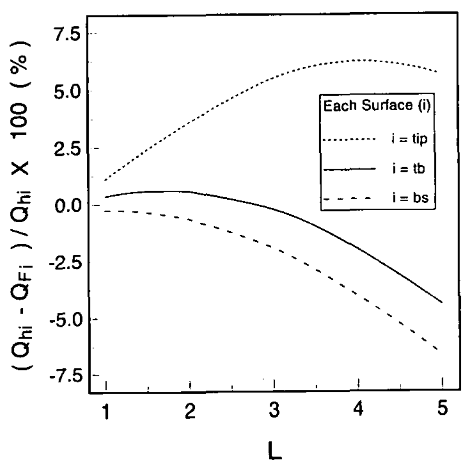
(b) Bi=0.1.

Fig. 3 The relative error in the heat loss from each surface using the finite difference method to the analytical method as a function of w for L=2.

of the relative error for Bi=0.01 is shown in Fig. 4(a). In this case, the relative errors for each surface increase constantly as L increases. Results for the same condition as in Fig. 4(a) except that Biot number is 0.1 are depicted in Fig. 4(b). In contrast the former case, the variation of the relative error is somewhat irregular. In other words the error varies from the positive value to the negative value for top



(a) Bi=0.01.



(b) Bi=0.1.

Fig. 4 The relative error in the heat loss from each surface using the finite difference method to the analytical method as a function of L for w=0.4.

& bottom and that for both sides increases negatively while the error for the fin tip increases first and then decreases slightly.

Fig. 5 presents the relative error in the total convection heat loss from all the surfaces as compared to the heat loss conducted into the fin through the fin base as a function of Biot number for w=0.4 and 1.0 in case of L=2. This figure shows that the errors for i=h increase while those for i=F vary from positive value to negative value as Biot number increases. In case of i=h, the relative error for w=1.0 is larger than that for w=0.4. On the other hand, for i=F, the relative error with respect to width varies somewhat irregularly.

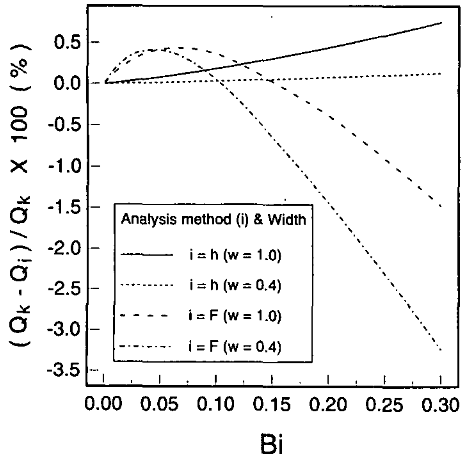


Fig. 5 The relative error in the total heat loss by convection as compared to by conduction as a function of Biot number for L=2.

Fig. 6 illustrates the relative errors in the total convection heat loss using a finite difference method as compared to the heat conducted into the fin through the fin base using the analytical method as a function of w for Bi=0.01, 0.1 and 0.3 in case of L=2. The relative error is near about 0.14% for Bi=0.01 regardless of the increment of w. This figure also shows that the relative error increases from approximately 0.05% to 1.1% as w increases from 0.4 to 10 for Bi=0.1. The relative error for Bi=0.3 has negative value at w=0.4 and it is rapidly reduced to zero and then increases slowly as w increases. Most notable in this figure is that the relative errors are within about ±1% for over w=2. This fact explains the effect of the number of division along the width on the relative error is very small. It seems probably due to the fact that the base temperature is constant.

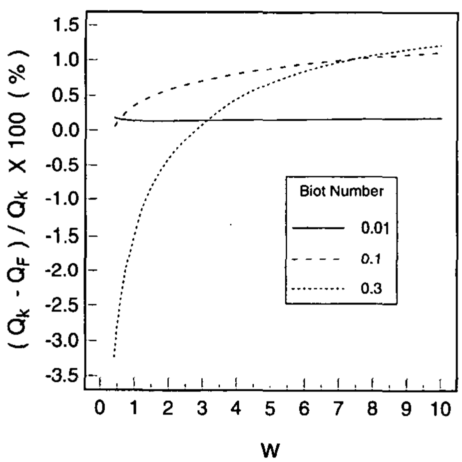


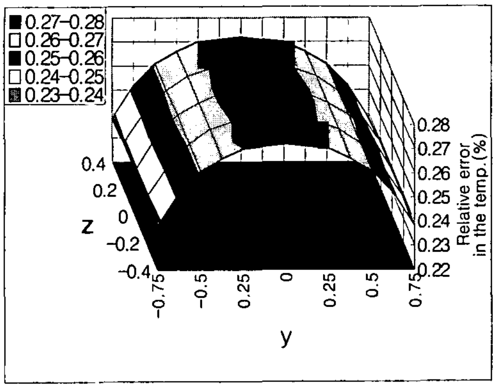
Fig. 6 The relative error in the total convection heat loss using a finite difference method as compared to the heat conduction using the analytical method as a function of w for L=2.

Table 1 Relative error in the temperature along the center line of the fin top surface for L=2, w=0.4.

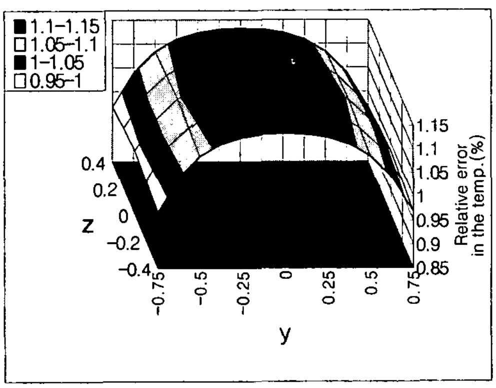
Biot No.	Y	X	θ AM	θ FDM	ε (%)
0.01	0.875	0.5	0.973468	0.973583	-0.012
	0.750	1.0	0.953201	0.950919	0.239
	0.625	1.5	0.939295	0.934263	0.536
	0.500	2.0	0.931636	0.925253	0.685
0.1	0.875	0.5	0.811932	0.814593	-0.328
	0.750	1.0	0.679388	0.672696	0.985
	0.625	1.5	0.593437	0.575548	3.014
	0.500	2.0	0.547968	0.525102	4.173

* *Note* : $\varepsilon = \{(\theta_A - \theta_F)/\theta_A\} \times 100$ (%)

Table 1 lists the relative error in the temperature along the center line of the fin top surface using a finite difference method as compared to the analytical method for L=2 and w=0.4 with Bi=0.1 and Bi=0.01. This table shows the relative error for Bi=0.01 is smaller than that for Bi=0.1. It also shows that the relative error increases as x increases.



(a) Bi=0.01



(b) Bi=0.1

Fig. 7 The relative error in the temperature distribution on the fin cross section at the half length x=1 using a finite difference method as compared to the analytical method for w=0.4, L=2.

The relative errors in the temperature distribution for w=0.4, L=2 on the fin cross section at the half length (i.e., x=1) using the finite difference method as compared to the analytical method are illustrated in Fig. 7. First, Fig. 7(a) presents the relative error for Bi=0.01. This figure shows that the relative error in the temperature is somewhat small, being the highest (0.273%) at the center (y=0, z=0), and the lowest (0.239%) at the edge lines (y=±0.75). It also shows the gradient of the error becomes more steep towards the top and bottom from the center of the fin while the gradient of the error along the z direction is almost negligible. The same condition but for Bi=0.1 is presented in Fig. 7(b). The particularly different characteristics in Fig. 7(b) as compared with Fig. 7(a) can be observed in view of two aspects. One is that the error range is a little greater and the other is that the slope of the variation of the error is considerably steep at the smaller area in the both end lines.

4. Conclusions

The following conclusions can be made from the results.

1) The relative error in the heat convection from each surface using the finite difference method to the analytical method varies somewhat irregularly and in the overall the largest relative error occurs at the tip.

2) For given nodes in the finite difference method the relative error in the total convection heat loss as compared to heat conduction seems to be good (i.e., $-3.5\% < RE < 1.0\%$) for the following region ; $Bi < 0.3$, $L < 2$ and $0.4 < w < 10$.

3) In the view of the non-dimensional temperature distribution, the relative error increases toward fin tip while decreases toward both sides and top & bottom surfaces.

4) The effect of the number of division along the width on the relative error is very small because it probably is due to the assumption that the base temperature is constant.

Nomenclature

Bi	: Biot number, $h\,l\,/\,k$
h	: heat transfer coefficient [W/m^2 °C]
k	: thermal conductivity [W/m ℃]
l	: one half fin height at the base [m]
L'	: fin length (base to tip) [m]
L	: non-dimensional fin length, $L'/\,l$
Q_F	: total convection heat loss from the fin obtained using a finite difference method [W]
Q_{Fbs}	: convection heat loss from both sides obtained using a finite difference method [W]

Q_{Ftb} : convection heat loss from top & bottom obtained using a finite difference method [W]

Q_{Ftip} : convection heat loss from tip obtained using a finite difference method [W]

Q_h : total convection heat loss from the fin obtained using the analytical method [W]

Q_{hbs} : convection heat loss from both sides obtained using the analytical method [W]

Q_{htb} : convection heat loss from top & bottom obtained using the analytical method [W]

Q_{htip} : convection heat loss from tip obtained using the analytical method [W]

Q_k : heat loss conducted into the fin through the base obtained using the analytical method [W]

T : fin temperature [℃]

T_w : fin base temperature [℃]

T_∞ : ambient temperature [℃]

w' : one half fin width [m]

w : non-dimensional a half fin width, w'/l

x' : length directional variable [m]

x : non-dimensional length directional variable, x'/l

y' : height directional variable [m]

y : non-dimensional height directional variable, y'/l

z' : width directional variable [m]

z : non-dimensional width directional variable, z'/l

Δx : increment of x along the length direction

Δy : increment of y along the height direction

Δz : increment of z along the width direction

Greek characters

θ_0 : adjusted temperature, $(T_w - T_\infty)$

θ : non-dimensional temperature, $(T - T_\infty)/(T_w - T_\infty)$

λ_m : eigenvalues (m= 1, 2, 3, ⋯)

μ_n : eigenvalues (n= 1, 2, 3, ⋯)

ρ_{nm} : eigenvalues ($\sqrt{\lambda_m^2 + \mu_n^2}$)

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