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Strength Evaluation of Slender Steel Reinforced Concrete Beam-Columns

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Abstract

The paper is intended to propose design strength of slender steel reinforced beam-columns by using the modified superposed method. The design of composite members is carried out by a superposed strength method in AIJ (Architectural Institute of Japan) design method. The bearing capacities of the steel part and the concrete part have to be determined separately and then added to a combined capacity. Authors have proposed a new superposed method in a modified form for the slender composite beam-columns and reinforced column. The modified superposed method is adopted for the slender steel reinforced beam-columns. Validation of the modified superposed method is undertaken by comparison with analytical results calculated assuming a sine curve deflected shape of the beam-columns, and with the test results conducted in Japan.

Keywords: SRC beam-columns, superposed method, strength estimation

1. INTRODUCTION

Steel reinforced concrete (SRC) Structures have been used widely for building structures in Japan. AIJ Standards for Structural Calculation of Steel Reinforced Concrete Structures was published first in 1958, and the latest fourth edition was revised in 1987. The superposed strength method has been used for calculating the strength of SRC members since the outset of the Standards.

In the latest revision (1987), the superposed strength of a column section is adopted for slender steel-concrete composite columns considering the effect of additional bending moment ($P\delta$ moment). The strength of SRC beam-columns is obtained by superposing the strength of reinforced concrete (RC) portion and steel portion (Wakabayashi, 1977). The accuracy of the formula, however, was not examined in detail.

A modified superposed method was proposed for the slender composite beam-columns (Tsuda et al., 1997). The difference between the AIJ method and modified superposed method is in the strength of RC portion. In modified method, approximately exact concrete column strength was obtained from the numerical analysis. The accuracy of the modified superposed method for the slender concrete filled steel tubular beam-columns was verified by the analytical results and test results conducted in Japan (Chung and Matsui, 1999). This method was also adopted for Recommendations for Design and Constructions of CFT Structures published in 1997.

In this paper, the modified superposed method is adopted for slender SRC beam-columns. The modified superposed strength and AIJ design strength for the slender SRC beam-columns are examined by the numerical investigation. The numerical analysis is performed by assuming a conventional sine deflected shape of the beam-columns. As the analytical parameters, buckling length-section depth ratio, tensile reinforcement ratio and strength of concrete are selected.

2. PROSED DESIGN FORMULA

The proposed strength of a SRC slender column is calculated by Eq. 1, where conventional equations of simple superposition are modified based on the Wakabayashi's study.

$$\text{When } N_u \leqq {}_{rc}N_{cu} \text{ or } M_u \geqq {}_{s}M_{u0}(1-{}_{rc}N_{cu}/{}_{src}N_0)$$
$$N_u = {}_{rc}N_u$$
$$M_u = {}_{rc}M_u + {}_{s}M_{u0}(1-{}_{rc}N_u/{}_{src}N_0) \qquad (2.1)$$

$$\text{When } N_u > {}_{rc}N_{cu} \text{ or } M_u < {}_{s}M_{u0}(1-{}_{rc}N_{cu}/{}_{src}N_0)$$
$$N_u = {}_{rc}N_{cu} + {}_{s}N_u$$
$$M_u = {}_{s}M_u(1-{}_{rc}N_{cu}/{}_{src}N_0) \qquad (2.2)$$

The fundamental concept of superposed method is shown in Figure 1. The strength of the steel part and the RC part has to be determined separately and then added to the steel portion and RC portion. An example of superposed strength of slender SRC beam-columns on the moment M_u-axial load N_u interaction curve is shown in Figure 2. The ultimate compressive and flexural strength of both steel and RC portion can be obtained by the following calculation.

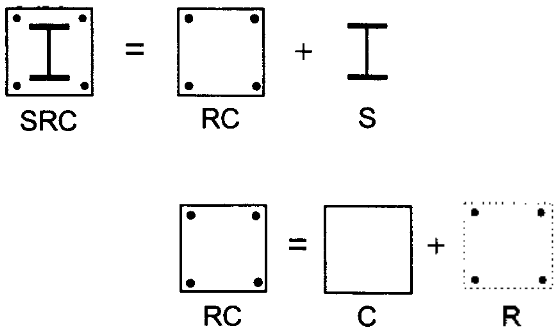


Figure 1. Superposed strength of SRC column

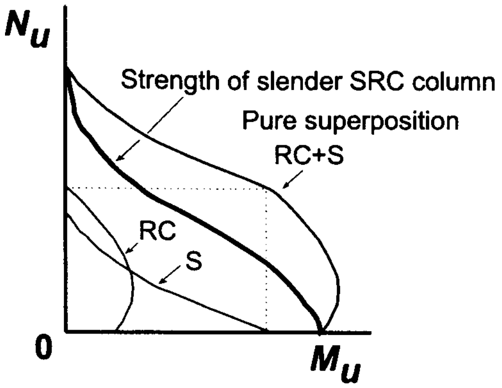


Figure 2. Superposed strength of slender SRC beam-column

(1) Strength of Slender Steel Column

As an interaction between ${}_{s}N_{u}$ and ${}_{s}M_{u}$ appearing in Eq.(2.1) and (2.2), a conventional formula used in the plastic design of steel structures (AIJ, 1975) is adopted in the form of

$$ {}_{s}N_{u}/{}_{s}N_{cr}+{}_{s}M_{u}/\{{}_{s}M_{uo}(1-{}_{s}N_{u}/{}_{s}N_{k})\}=1 \qquad (2.3)$$

in which ${}_{s}N_{u}$ denotes the axial load, ${}_{s}N_{cr}$, critical load, ${}_{s}N_{k}$ Euler buckling load, ${}_{s}M_{u}$, end moment, ${}_{s}M_{uo}$ and full plastic moment.

(2) Strength of Slender RC Column

Strength of slender RC columns is calculated by using the superposed strength of concrete column and reinforced steels that are considered as slender steel column. Therefore strength of reinforced steel can be calculated by Eq. (2.3). Based on the numerical analysis, the strength of slender concrete col-

Table 1. Strength of slender concrete columns

Notation	Equation
${}_{c}M_{u}$-${}_{c}N_{u}$ Interaction	${}_{c}M_{u}/{}_{c}M_{max}=4\,({}_{c}N_{u}/{}_{c}N_{cr})\{\,1-({}_{c}N_{u}/{}_{c}N_{cr})\,\}[\,1+f1(\beta)\,\{\,({}_{c}N_{u}/{}_{c}N_{cr})-0.5\,\}$ $+f2\,(\beta)\,\{\,({}_{c}N_{u}/{}_{c}N_{cr})-0.5\,\}^{2}+f3\,(\beta)\,\{\,(cNu/cNcr)-0.5\,\}^{3}]$
${}_{c}M_{max}$	${}_{c}M_{max}={}_{c}M_{maxo}\,exp\,[\,-3.12\,g1(\phi)\,\beta+2.21\,\beta^{2}-0.731\,g3\,(\phi)\,\beta^{3}]$
	${}_{c}M_{maxo}={}_{c}\sigma_{B}\,D^{3}/8$
$g1$	$g1(\phi)=0.789+0.371\,\phi-0.160\,\phi^{2}$
$g3$	$g3(\phi)=1.17-0.285\,\phi+0.118\,\phi^{2}$
ϕ	$\phi={}_{c}\sigma_{B}/(0.85*960)$
$f1$	$f1\,(\beta)=-\beta/\,(0.248-0.986\,\beta+7.61\,\beta^{2}-7.04\,\beta^{3}+2.11\,\beta^{4})$
$f2$	$f2\,(\beta)=-\beta/\,(0.0257-0.292\,\beta+2.40\,\beta^{2}-15.3\,\beta^{3}+30.0\,\beta^{4})$
$f3$	$f3\,(\beta)=-\beta/\,(0.0036-0.591\,\beta+1.46\,\beta^{2}-1.90\,\beta^{3}+0.702\beta^{4})$
Buckling Load (${}_{c}N_{cr}$)	${}_{c}N_{cr}={}_{c}A\;{}_{c}\sigma_{B}\,\{\,1-(1-\varepsilon_{cr}/\varepsilon_{o})^{a}\}$
$\varepsilon_{cr}/\varepsilon_{o}$	$(1-\varepsilon_{cr}/\varepsilon_{o})^{a}+aK\,(1-\varepsilon_{cr}/\varepsilon_{o})^{(a-1)}-1=0\;,\;\varepsilon_{o}=0.52\,{}_{c}\sigma_{B}^{\,0.25}\,10^{-3}$
Elastic Modulus	${}_{c}E=(0.106*{}_{c}\sigma_{B}^{\,0.5}+0.703)10^{5}$
K	$K=\pi^{2}/(\,24\,\beta\,)$
β	$\beta=0.5\,(Lk/D\,)^{2}\,\varepsilon_{o}$
a	$a={}_{c}E\,\varepsilon_{o}/{}_{c}\sigma_{B}$

Table 2. Proposed formula

Range	Equation
$N_u < {}_cN_{cr}$	$M_u =$ $4(N_u/{}_cN_{cr})\{1-(N_u/{}_cN_{cr})\}\times$ $[1+f1(\beta)((N_u/{}_cN_{cr})-0.5)+f2(\beta)((N_u/{}_cN_{cr})-0.5)^2+$ $f3(\beta)((N_u/{}_cN_{cr})-0.5)^3]\,{}_cM_{max}+{}_mM_{u0}(1-N_u/{}_{rc}N_{km1})+{}_sM_{u0}(1-N_u/{}_{src}N_{km})$
${}_cN_{cr} < N_u < {}_{rc}N_{cr}$	$M_u =$ $\{1-(N_u-{}_cN_{cr})/{}_mN_{cr}\}\{1-(N_u-{}_cN_{cr})/{}_mN_k\}\{1-{}_cN_{cr}/{}_{rc}N_{km1}\}{}_mM_{u0}+{}_sM_{u0}(1-N_u/{}_{src}N_{km})$
$N_u > {}_{rc}N_{cr}$	$M_u =$ $\{1-(N_u-{}_{rc}N_{cr})/{}_sN_{cr}\}\{1-(N_u-{}_{rc}N_{cr})/{}_sN_k\}\{1-{}_{rc}N_{cr}/{}_{src}N_{km}\}{}_sM_{u0}$

umn has been proposed as Table 1.

(3) **Summary of strength of slender SRC column**

Equation for calculating the strength of slender SRC column is summarized in Table 2, by substituting the strength of steel column. In the Table 2, ${}_cN_{cr}$ denotes buckling strength of concrete column.

3. NUMERICAL ANALYSIS

This chapter is concerned with the numerical analysis developed to predict the strength of SRC columns subjected to axial compression and bending moment. In order to obtain the maximum load, an elasto-plastic analysis is performed. In the analysis, a sine curve deflected shape of the beam-column under eccentric axial force with equal end eccentricity is assumed as shown in Figure 3. Corresponding load to a given value of deflections δ can be obtained using an equilibrium condition at the center of column. As to the stress-strain relations of concrete, Sakino-Sun model is adopted for core concrete and cover concrete (Sakino, 1994). For shaped steel and reinforcing bar, Menegotto-Pinto model and elastic-perfectly-plastic model are used, respectively. normal to the longitudinal axis of the members during deformation of members.

(1) Correlation with test results

Examples of comparison between the test results and the analytical ones are shown in Figure 4 and Figure 5. Two parameters in the test were the buckling length-section depth ratio (Lk/D), and the eccentricity of the applied compressive load (e). In addition, the mechanical properties, such as the compressive concrete strength, tensile steel strength and reinforced bar, are shown in each figure. Solid lines show experimental results, and results of elasto-plastic analysis are shown by dotted line. The buckling-section depth ratio (Lk/

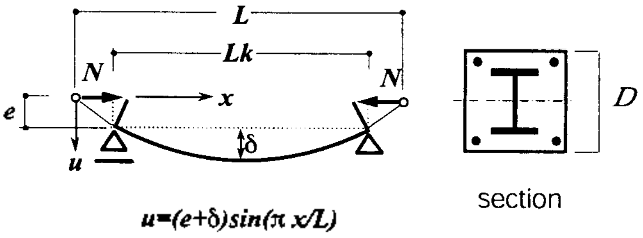


Figure 3. Analytical model

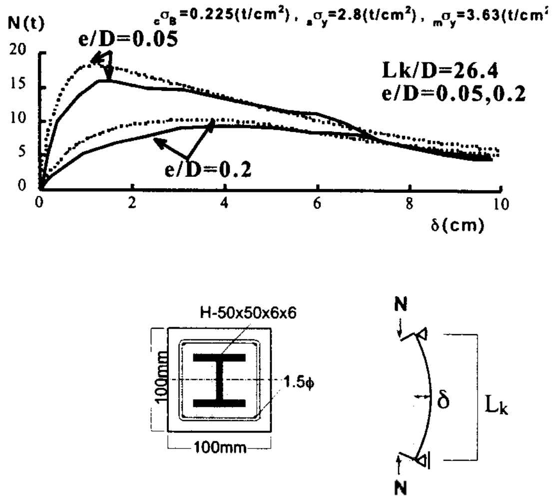


Figure 4. Comparison between test and theoretical results
Wakabayashi's test (Wakabayashi, 1979)

D) ranges from 14.7 to 26.4. Good agreement between test and theoretical results is observed.

(2) Parametric Study

In order to examine the accuracy of proposed strength formula, parametric study is performed using the analysis above mentioned. The extensive investigation embraced a range of parameters that could influence SRC columns, including:

1. buckling length-section depth ratio (L_k/D=4, 8, 12, 18, 24 and 30)
2. tensile reinforcement ratio of reinforcing bar (0.2% and 1.0%)
3. steel ratio of shaped steel (1.33% and 2.90%)
4. strength of concrete (${}_c\sigma_B$=300 and 600kg/cm^2)

By changing the magnitude of the end eccentricity e, end moment (M_u) -axial force (N_u) relations are calculated. Yield stress of shaped steel ${}_s\sigma_y$ and reinforcing steel ${}_m\sigma_y$ are both 3t/cm^2. Assumed stress-strain relations are shown in Figure 6 and Table 3. The column section for analysis is shown in Figure 7.

(3) Examples of analytical results

In Figure 8, examples of axial load-lateral deflection δ relations are shown with the eccentricity e=0.2D, tensile reinforcement ratio of reinforcing bar, ${}_r p_t$=1.0% and strength of concrete, ${}_c\sigma_B$=300 kg/cm^2. In Figure 9, moment (M_{ctr}) - axial compressive force (N_{ctr}) relations at the center of the column are shown by solid lines, and end moment (M_u) and axial compressive force (N_u) relations at the ultimate state are shown by dots.

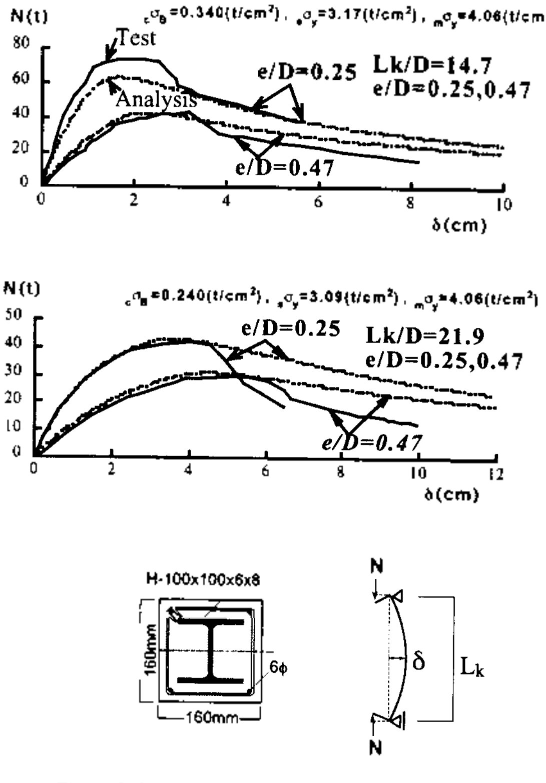
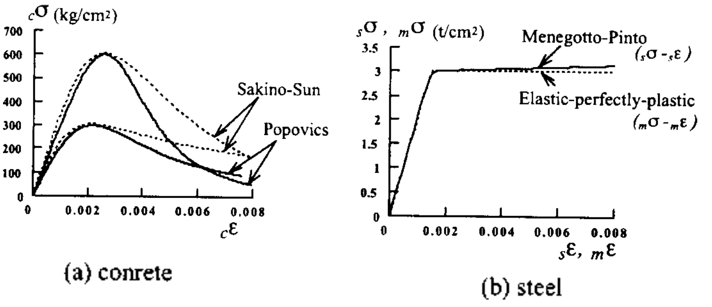


Figure 5. Comparison between test and theoretical results
Matsui's test (Matsui, 1982)

Table 3. Stress-strain relations of concrete

	Equation	Notation
I Popovics	$\eta = n\cdot\xi/(n-1+\xi^n)$ $n = (0.57\times10^{-2}\times{}_c\sigma_B)+1$ $\eta=\frac{{}_c\sigma}{{}_c\sigma_B},\ \xi=\frac{{}_c\varepsilon}{\varepsilon_0}$	ε_0 : strain at ${}_c\sigma_B$ $\varepsilon_0 = 0.52\,{}_c\sigma_B^{1/4}\,10^{-3}$ ${}_cE=(0.106\,{}_c\sigma_B^{0.5}+0.703)10^5$ ${}_c\sigma_B$: compressive strength (unit : kg/cm^2)
II Sakino-Sun	$Y=\frac{AX+(D-1)X^2}{1+(A-2)X+DX^2}$ $X=\frac{{}_c\varepsilon}{\varepsilon_{c0}},\ Y=\frac{{}_c\sigma}{{}_c\sigma_{cB}}$ $A={}_cE\varepsilon_{c0}/{}_c\sigma_{cB}$ $D=\alpha+\beta\sigma_B+\gamma[\sigma_{re}]^{1/2}$ ${}_c\sigma_{cB}={}_c\sigma_B+k_e\sigma_{re}$ $\sigma_{re}=\frac{1}{2}\rho_h\sigma_{hs}(\frac{d''}{C})(1-\frac{s}{2D_c})$ $\frac{\varepsilon_{c0}}{\varepsilon_0}=\begin{cases}1+4.7(K-1) & K\le1.5\\ 3.35+20(K-1.5) & K>1.5\end{cases}$ $K={}_c\sigma_{cB}/{}_c\sigma_B$	α =1.50 β =-1.68x10^{-3} γ =0.5 k_e : confinement factork_e =23 ρ_h : volume ratio of longitudinal reinforcement σ_{hs} : stress of longitudinal reinforcement at peak d'' : diameter of longitudinal reinforcement C : Length of longitudinal support s : distance of longitudinal reinforcement D_c : distance between center of reinforcing steel and longitudinal reinforcement



(a) conrete

(b) steel

Figure 6. Stress-strain relations of materials

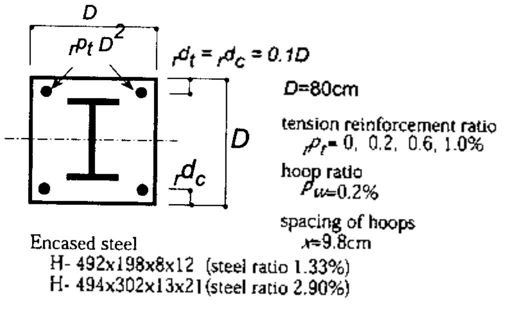


Figure 7. Column section for analysis

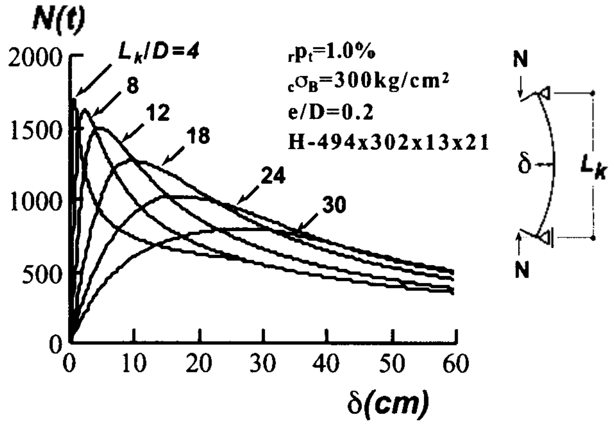


Figure 8. Axial load-lateral deflection relation

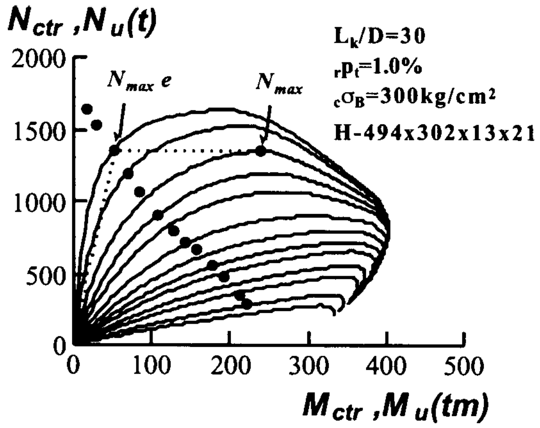


Figure 9. Moment -axial load relation

4. COMPARISON OF STRENGTH

Comparisons of normalized axial force (n) and end moment (m) are shown in Figure 10 and Figure 11. Moment M_u-axial load N_u are normalized by the superposed maximum moment of section, M_{max}(= ${}_{c}\sigma_{B}D^3/8+{}_{r}p_{t}(1\text{-}2\alpha)D^3\,{}_{m}\sigma_{y}+{}_{s}M_{uo}$, $\alpha={}_{r}d_{t}/D$=0.1) and the superposed maximum strength, N_{max} (= ${}_{c}\sigma_{B}D^2+2{}_{r}p_{t}D^2\,{}_{m}\sigma_{y}+{}_{s}A\,{}_{s}\sigma_{y}$), respectively.

Exact maximum loads calculated by elasto-plastic analysis are shown by dots, and solid lines show proposed strength. In addition to these, to examine the current AIJ method (given in Table 4) and moment magnifier method (given in Table 5), strengths obtained by the current AIJ SRC Standard (AIJ, 1990) and moment magnifier method are shown by broken lines and chain lines, respectively. Dotted lines indicate the strength of the section.

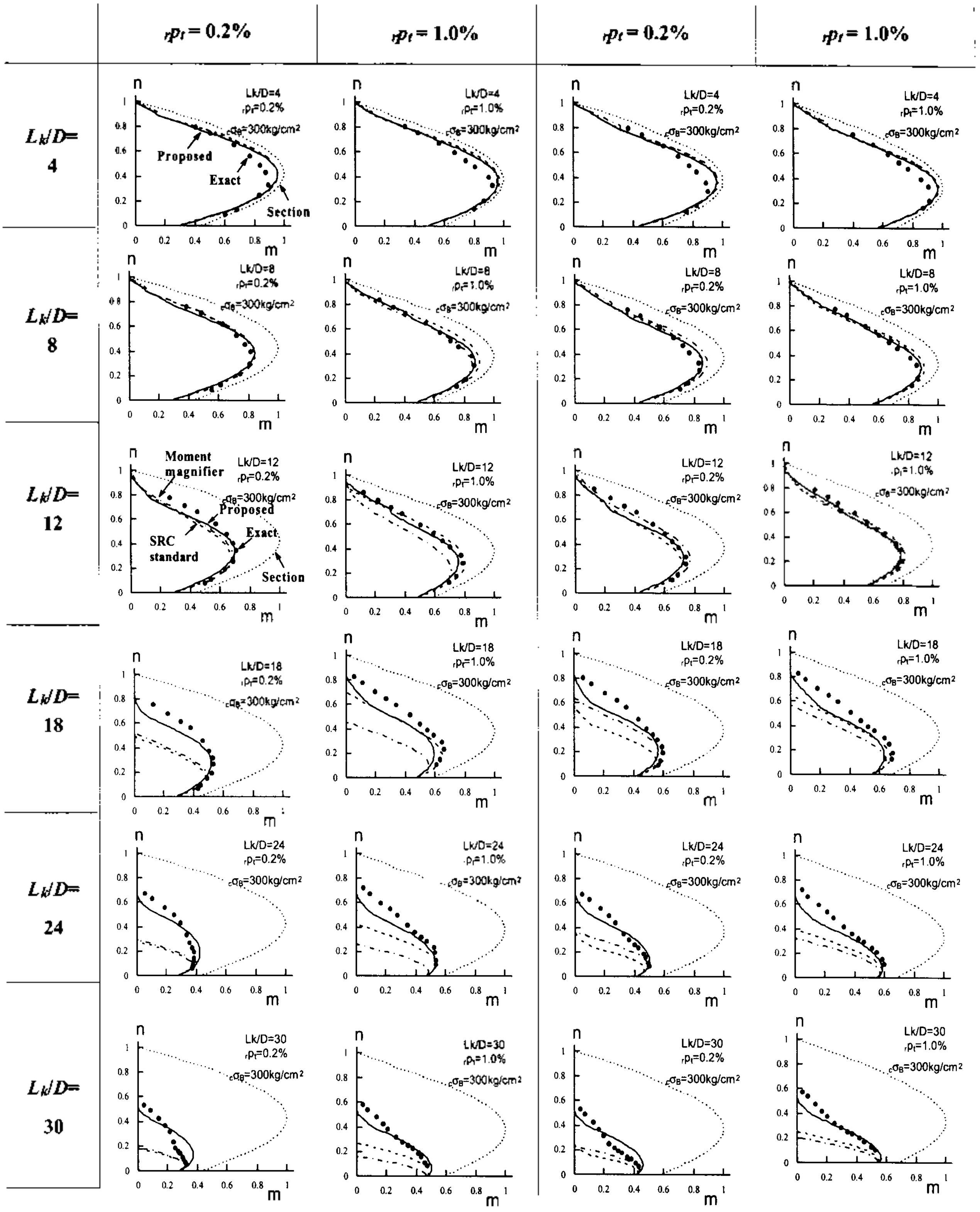
The current AIJ SRC Standard is mainly based on the allowable stress design method, and design formulas to estimate the ultimate strength for slender SRC columns are not provided. The allowable stress design formula is modified

Table 4. AIJ-SRC strength formula

Range	Equation
$N_u < {}_{c}N_{y}$	M_u = $[\,0.5N_u D\{1 - N_u/(D^2 c\sigma B)\} + {}_{m}M_{u0}\,]\ (1 - N_u/{}_{rc}N_{k}) + {}_{s}M_{u0}(1 - N_u/{}_{src}N_{k}')$
${}_{c}N_{y} < N_u < {}_{rc}N_{km2}$	M_u = ${}_{m}M_{u0}\{1 - (N_u - {}_{c}N_{y})/{}_{m}N_{y}\}\ \{1 - N_u/{}_{rc}N_{k}\} + {}_{s}M_{u0}(1 - N_u/{}_{src}N_{k}')$
$N_u > {}_{rc}N_{km2}$	M_u = $\{1 - (N_u - {}_{rc}N_{km2})/{}_{s}N_{cr}\}\ \{1 - (N_u - {}_{rc}N_{km2})/{}_{s}N_{k}\}\ \{1 - {}_{rc}N_{km2}/{}_{src}N_{k}'\}\,{}_{s}M_{u0}$

Table 5. Moment magnifier strength formula

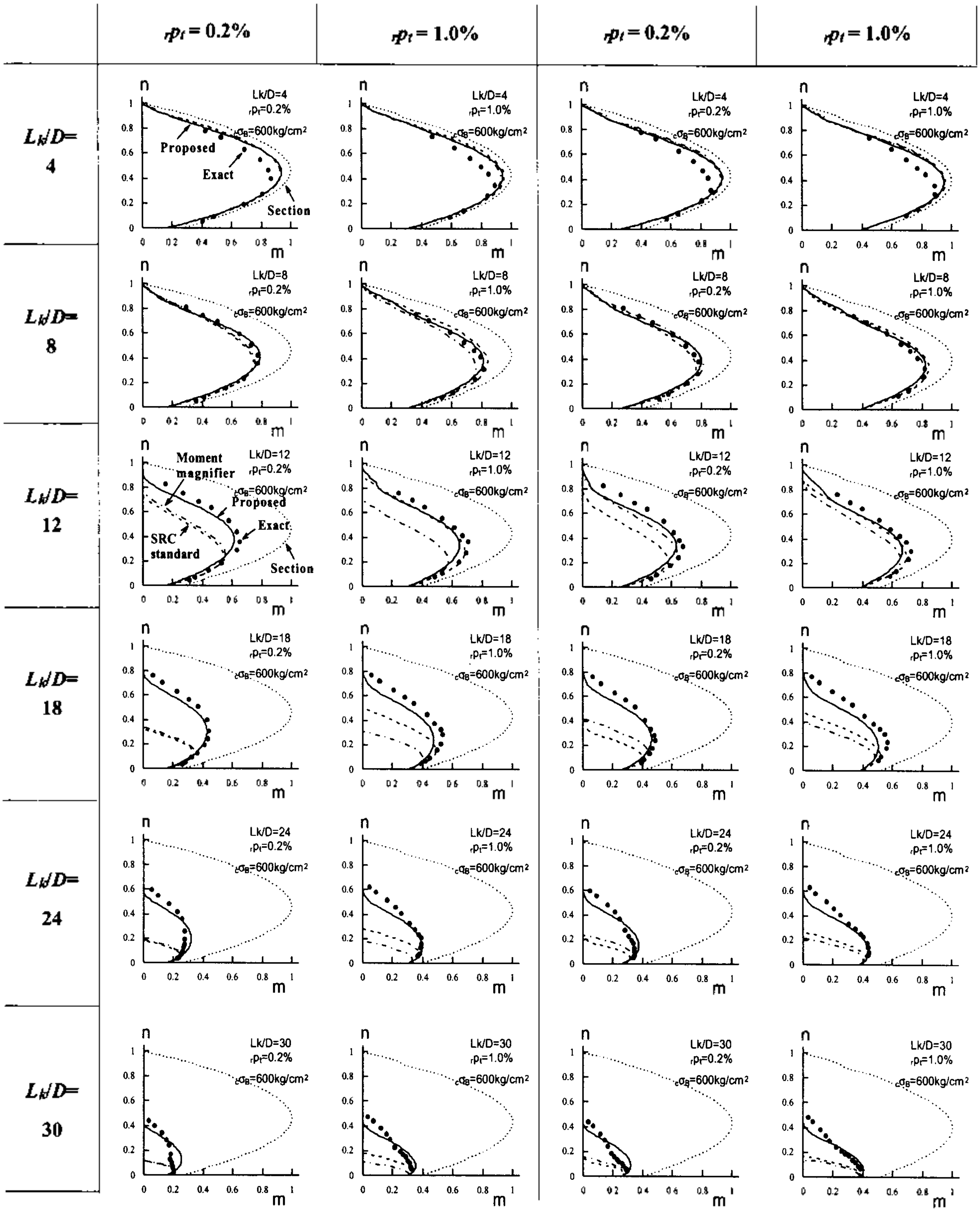
Range	Equation
$N_u < {}_{c}N_{y}$	M_u = $[\,\{0.5N_u\ D(1 - N_u/{}_{c}N_{y}) + {}_{m}M_{u0}\} + {}_{s}M_{u0}\,]\ (1 - N_u/{}_{src}N_{k}')$
${}_{c}N_{y} < N_u < {}_{rc}N_{y}$	M_u = $[\,{}_{m}M_{u0}\{1 - (N_u - {}_{c}N_{y})/{}_{m}N_{y}\} + {}_{s}M_{u0}\,]\ (1 - N_u/{}_{src}N_{k}')$
${}_{rc}N_{y} < N_u <$ ${}_{rc}N_{y}+(A_w/2{}_{s}A){}_{s}N_{y}$	M_u = ${}_{s}M_{u0}\ (1 - N_u/{}_{src}N_{k}')$
$N_u >$ ${}_{rc}N_{y}+(A_w/2{}_{s}A){}_{s}N_{y}$	M_u = ${}_{s}M_{u0}[\,1 - \{N_u - {}_{rc}N_{y} - (A_w/2{}_{s}A)\,{}_{s}N_{y}\}\ /\{{}_{s}N_{y} - (A_w/2{}_{s}A)\,{}_{s}N_{y}\}\,]\ (1 - N_u/{}_{src}N_{k}')$



(a) Steel ratio 1.33%

(b) Steel ratio 2.90%

Figure 10. $M_u \cdot N_u$ relations ($_c\sigma_B$=300kg/cm^2)



(a) Steel ratio 1.33%

(b) Steel ratio 2.90%

Figure 11. $M_u \cdot N_u$ relations ($_c\sigma_B$=600kg/cm^2)

into a formula to estimate the ultimate strength. The strength of slender SRC columns is calculated by adding the strengths of slender steel column and slender RC column. The strength of slender RC columns is obtained using the moment magnification method for RC column section. The difference between AIJ current method and the proposed method is in estimating the strength of RC column. As for the proposed method, approximate exact RC column strength is used. Furthermore, strengths by using the moment magnifier method for SRC cross section are obtained.

Figure 10 and 11 show that both strengths by current AIJ design formula and by moment magnifier method for slender SRC columns are too conservative, while good agreement with exact strengths are observed in case of L_k/D=8. Strengths by proposed design method predict the numerically calculated maximum strength well in a wide range of buckling-section depth ratio.

5. CONCLUSIONS

Strength of slender steel reinforced beam-columns is proposed by using the modified superposed method. Validation of the modified superposed method was undertaken by comparison with analytical results calculated assuming a sine curve deflected shape of the beam-columns, the current AIJ design formula and moment magnifier method. As the analytical parameters, buckling length-section depth ratio (L_k/D=4, 8, 12, 18, 24 and 30), tensile reinforcement ratio of reinforcing bar (0.2% and 1.0%), steel ratio of shaped steel (1.33% and 2.90%) and strength of concrete (${}_c\sigma_B$=300 and 600kg/cm^2) were selected.

On the basis of the work carried out in this study, the following conclusions were drawn:

1. Strengths by proposed design method predict the numerically calculated maximum strength well in a wide range of buckling-section depth ratio.

2. Both strengths by current AIJ design formula and moment magnifier method for slender SRC columns are too conservative, with larger buckling-section depth ratio.

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NOTATION

The following symbols are used in this paper:

${}_cA$	: sectional area of concrete
D	: depth
ε_{cr}	: strain at buckling
ε_o	: strain at ${}_c\sigma_B$
N_u	: ultimate compressive strength of member
${}_cN_{cr}$	: buckling load of concrete column (see Table 1.)
${}_cN_y$	: compressive strength of concrete portion
${}_mN_{cr}$	: buckling load replaced steel portion with reinforcing bar
${}_mN_k$	: Euler buckling load replaced steel portion with reinforcing bar
${}_mN_y$	: compressive strength of reinforcement portion
${}_{rc}N_{cr}$	: ${}_{rc}N_{cr} = {}_cN_{cr} + {}_mN_{cr}$
${}_{rc}N_{cu}$	: ultimate compressive strength of RC portion

subjected to compression alone

${}_{rc}N_k$: Euler buckling load of RC column

$=\pi^2({}_sE_sI+{}_cE_cI/5)/Lk^2$

${}_{rc}N_{km1}$: ${}_{rc}N_{km1}=max({}_{rc}N_k,\ {}_cN_{cr}+{}_mN_{cr})$

${}_{rc}N_{km2}$: ${}_{rc}N_{km2}=min({}_{rc}N_k,\ {}_{rc}N_y)$

${}_{rc}N_u$: ultimate compressive strength of RC portion

${}_{rc}N_y$: compressive strength of RC portion

${}_sN_{cr}$: buckling load of steel column

${}_sN_k$: Euler buckling load of steel column

${}_sN_u$: ultimate compressive strength of steel portion

${}_sN_y$: compressive strength of steel portion

${}_{src}N_k$: buckling strength of SRC column

${}_{src}N_k=\pi^2({}_sE_sI+{}_mE_mI+{}_cE_cI/5)/Lk^2$

${}_{src}N_{km}$: ${}_{src}N_{km}=max({}_{src}N_k,\ {}_cN_{cr}+{}_mN_{cr}+{}_sN_{cr})$

${}_{src}N_{k'}$: ${}_{src}N_{k'}=\pi^2({}_sE_sI+{}_cE_cI/5)/Lk^2$

M_u : ultimate flexural strength of member

${}_{rc}M_c$: ultimate flexural strength of RC portion

${}_sM_u$: ultimate flexural strength of steel portion

${}_sM_{uo}$: ultimate flexural strength of steel portion subjected to bending alone

${}_c\sigma_B$: compressive strength of concrete

${}_m\sigma_y$: yield strengthof reinforcing bar

${}_s\sigma_y$: yield strength of steel

APPENDIX (Example)

For an example the force and moment of SRC column with L_k/D=20, shown in Figure A1, are calculated through the following procedure:

Column Depth (D) : 80 cm

Steel Column : H-488 × 300 × 11 × 8 (${}_s\sigma_y$=3.3t/cm²)

Reinforcing steel : 12-D25 (${}_s\sigma_y$=3.0t/cm²)

Concrete Strength : 300 kg/cm²

STEP 1 : Calculate the strength of steel column.

Strength of steel column may be developed by Table A1.

Sectional area of steel, ${}_sA=163.5\text{cm}^2$

Moment inertia of steel, ${}_sI=71000\text{cm}^4$

${}_sN_y=163.5\cdot3.3=540\text{t}$

$\lambda=80\cdot20/(71000/163.5)^{0.5}=76.8$

$\lambda_l=76.8(3.3/2100)^{0.5}/\pi=0.969$

$0.3<\lambda_l<1.3$, accordingly,

${}_sN_{cr}=\{1-0.545(0.969-0.3)\}540=343$

${}_sN_k=\pi^2\cdot2100\cdot71000/(20\cdot80)^2=575\text{t}$

${}_sM_{u0}=30\cdot1.8\cdot3.3\cdot(48.8-1.8)+(48.8/2-1.8)\cdot1.1\cdot3.3\cdot(48.8-2\cdot1.8)/2=10229\text{ tcm}$

Table A1. Strength of slender steel column

	Notation	Formula	
Sectional area	$_sA$	—	
Moment of inertia	$_sI$	—	
Radius of gyration	$_si$	$(_sI/_sA)^{0.5}$	
Slenderness	λ	$Lk/(_sI/_sA)^{0.5}$	
Normalized slenderness	$\lambda1$	$\lambda(_s\sigma y/\ _sE)^{0.5}/\pi$	
Yield strength	$_s\sigma y$	—	
Yield load	$_sNy={_sA}\ _s\sigma y$	—	
Buckling Load	$_sNcr$	$_sNy$	$(\lambda1<0.3)$
		$\{1-0.545(\lambda1-0.3)\}_sNy$	$(0.3<\lambda1<1.3)$
		$_sNy/(1.3\lambda1^2)$	$(1.3<\lambda1)$
Euler Critical Load	$_sNk$	$\pi^2{_sE}\ _sI/Lk^2$	
Full Plastic Moment	$_sMu0$	—	

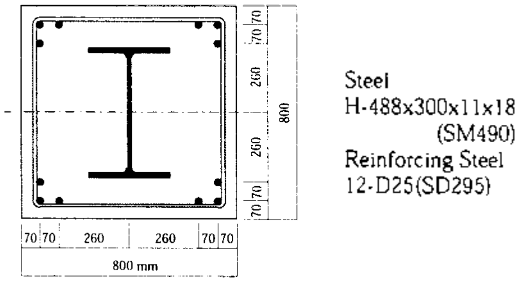

Figure A1. Column section

STEP 2 :Calculate the strength of reinforced concrete column.

a) Strength of concrete portion

Strength of steel column may be developed by Table 1.

${}_c\sigma_B={}_cr_u\cdot F_c=0.85\cdot300=255\text{kg/cm}^2$

$\varepsilon_o=0.52\cdot255^{1/4}\cdot10^{-3}=0.00208$

${}_cE=(0.106\cdot255^{0.5}+0.703)\cdot10^5=239568\text{kg/cm}^2$

$a=239568\cdot0.00208/255=1.95$

$\beta=0.5\cdot20^2\cdot0.00208=0.416$

$f1(\beta=0.416)=-0.585$

$f2(\beta=0.416)=-1.16$

$f3(\beta=0.416)=-3.69$

${}_cM_{maxo}=255\cdot80^3/8=16320000\text{kg cm}=16320\text{tcm}$

$\phi=255/816=0.313$

$g1(\phi)=0.889$

b) Strength of reinforcing steel

Strength of reinforcing steel column may be also developed by Table A1.

Substituting, ${}_sN_y={}_mN_y$

${}_mN_y=2\cdot0.00475\cdot6400\cdot3.0=183\text{t}$

$\lambda=2\cdot20/(1-2\cdot0.117)=52.2$

$\lambda_1 = 52.2(3.0/2100)^{0.5}/\pi = 0.628$

$0.3 < \lambda_l < 1.3$, accordingly,

${}_{m}N_{cr} = \{1-0.545(0.628-0.3)\}183 = 149\text{t}$

${}_{m}N_{k} = 0.5(1-2 \cdot 0.117)^2\pi^2 \cdot 2100 \cdot 0.00475 \cdot 80^2/20^2 = 463\text{t}$

${}_{m}M_{uo} = (1-2 \cdot 0.117) \cdot 0.00475 \cdot 80^3 \cdot 3.0 = 5597\text{tcm}$

c) Strength of reinforced concrete column

Strength of RC column may be developed by Table 2.

${}_{rc}N_k = \pi^2({}_{m}E_m I + {}_{c}E_c I/5)/L_k{}^2 = 1094\text{t}$

${}_{rc}N_{km} = max({}_{rc}Nk, {}_{c}N_{cr} + {}_{m}N_{cr})$
$= max(1094, 1347+149) = 1497\text{t}$

Accrondingly,

when ${}_{rc}N_u < 1347\text{t}$

${}_{rc}M_u = 4 \cdot ({}_{rc}N_u/1347)\{1-({}_{rc}N_u/1347)\}[1-0.585 \cdot (({}_{rc}N_u/1347)-0.5)-1.16 \cdot (({}_{rc}N_u/1347)-0.5)^2-3.69 \cdot (({}_{rc}N_u/1347)-0.5)^3] \cdot 7130+5597 \cdot (1-{}_{rc}N_u/1497)$

when ${}_{rc}N_u > 1347\text{t}$

${}_{rc}M_u = \{1-({}_{rc}N_u-1347)/149\}\{1-({}_{rc}N_u-1347)/463\}\{1-1347/1497\} \cdot 5597$

STEP 3 :Calculate the strength of SRC column.

Strength of RC column may be also developed by Table 2.

${}_{src}N_k = \pi^2({}_{s}E_s I + {}_{m}E_m I + {}_{c}E_c I/5)/L_k{}^2 = 1669\text{t}$

${}_{src}N_{km} = max(1669, 1347+149+343) = 1839\text{t}$

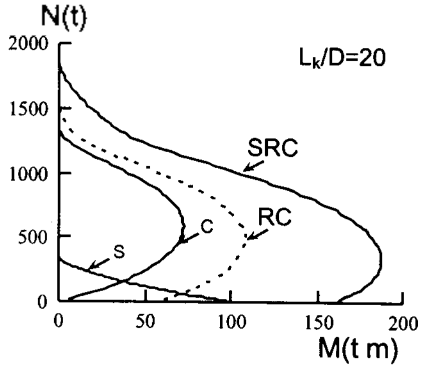

Figure A2. Interaction diagram for a SRC column

Accrondingly,

when $N_u < 1347\text{t}$

$M_u = 4 \cdot (N_u/1347)\{1-(N_u/1347)\}[1-0.585 \cdot ((N_u/1347)-0.5)-1.16 \cdot ((N_u/1347)-0.5)^2-3.69 \cdot ((N_u/1347)-0.5)^3] \cdot 7130+5597 \cdot (1-N_u/1497)+10229 \cdot (1-N_u/1839)$

when $1347\text{t} < N_u < 1497\text{t}$

$M_u = \{1-(N_u-1347)/149\}\{1-(N_u-1347)/463\}\{1-1347/1497\} \cdot 5597+10229(1-N_u/1839)$

when $N_u > 1497\text{t}$

$M_u = \{1-(N_u-1497)/343\}\{1-(N_u-1497)/575\}\{1-343/1839\} \cdot 10229$

The moment capacities calculated is shown Figure A2.