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# Some Implication of CPMP Developed Curriculum for Korean High School Mathematics Education

-Focused on Algebra and Functions-

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To educate students to adjust and lead their future society oriented on technologies and information communications, most of the countries in the world try to reform their traditional mathematics education. Contemporary Mathematics in Context (CMIC) developed by Core-Plus Mathematics Project in WMU has been developed in order to assist school districts in reforming their high schools mathematics programs in the United States, and 7<sup>th</sup> Korean National Mathematics Curriculum was developed to guide reforming Korean mathematics program in 1997.

In this presentation, by analyzing essential differences between the two curricula according to Korean Mathematics Curriculum, we may find some implications of CMIC for Korean mathematics education.

### I. Introduction

Currently most of countries in the world have been trying to reform their school education to educate students can adjust and lead their future societies oriented on technologies and information communications. In the United States, to guide reform of mathematics education, National Council of Teachers of Mathematics (NCTM) issued Curriculum and Evaluation Standards for School Mathematics in 1989, Professional Standards for Teaching and Learning Mathematics in 1991, Assessment Standards for School Mathematics in 1995, and Principles and Standards for School Mathematics in 2000. To implement those standards, several projects have developed new mathematics curricula and learning and teaching materials. One of the most successful projects is Core-Plus Mathematics Project (CPMP) in Western Michigan University. CPMP has developed Contemporary Mathematics in Context (CMIC) consisted with four courses of materials for high schools students from 1994. These materials are used broadly in the United States.

In Korea, the 7th National Mathematics Curriculum was developed in 1997 to reform traditional

mathematics education, and nowadays they are developing learning and teaching materials. Elementary schools begin to use the 7th curriculum from 2000, junior high schools begin to use it from 2001, and senior high schools begin to use it from 2002. This country is well known as a high achievement of mathematics (2nd / 26 countries) in the Third International Mathematics and Science Study (National Research Council, 1999).

The writer, the director of The Sung Kyun Kwan University Committee of Mathematics Curriculum Research and Revision, which developed the draft of The 7th Elementary and Secondary School Mathematics Curriculum of Republic of Korea, visited Western Michigan University (WMU) for his sabbatical leave for one year. The writer has studied CPMP curriculum and CMIC materials, visited a few classes using CMIC materials, and found some differences between the two curricula. In this paper, the writer analyzes the essential differences according to Korean High School Mathematics Curriculum (KHMC), and finds some implications of CMIC for Korean Mathematics Education. This comparative study of the two curricula will also give some important ideas to understand about the reform of mathematics education of the two countries.

# II. Identifying Differences and Implications

# 1. Fundamental Characteristics

Korean National Mathematics curriculum aims to help students with the followings (MOE, 1997, p.28): Understand basic concepts, principles, and rules of mathematics; mathematically observe and analyze the phenomena of matters; acquire abilities and attitudes needed in thinking and solving problems related to real life in a rational manner. Mathematics in the National Common Basic Education Period (grade 1 grade 10) is organized in a stepwise and level-referenced manner that will allow the teacher to consider the rate of the students cognitive development and to thereby select core contents of the curriculum based on a learning hierarchy and difficulties. And the curriculum separate basic and enriched content to make it possible for each student to maintain his/her own learning paced and to have creative learning experiences.

All of the 10th grade students should take Level 10-A and 10-B Mathematics or lower levels. Mathematics curricula for 11th and 12th grade students are organized in six courses that are optional for all students. Students can take some courses by themselves according to their future directions or needs. The six courses are Mathematics I, Mathematics II, Differentiation and Integration, Mathematics for the Real Life, Probability and Statistics, and Discrete Mathematics.

The essential characteristic of each course is briefly explained in Table 1. All of the 10th grade students may study mathematics for four 50 minutes per week, and the college-bound students may study mathematics for four 50 minutes per week in 11th grade. The students who wish to study natural sciences or engineering may study mathematics for four 50 minutes or more per week in 12th grade.

CPMP curriculum is developed to execute NCTMs STANDARDS to reform school mathematics education of the United States. The curriculum aims to teach all fo the high schools students the same mathematical topics in different methods according to their abilities. Some of the fundamental aims of CPMP are as follows (CPMP, 1998, p.1):

The curriculum builds upon the theme of mathematics as sense-making. Though investigations of real-life contexts, students develop a rich understanding of important mathematics that makes sense to them and which, in turn, enables them to make sense out of new situations and problems.

To achieve these aims, CMIC students discuss social or scientific situations, solve questions from the contexts, find patterns, make tables, draw graphs, transfer the situations into mathematical model, or connect various concepts, and then introduce new mathematical concepts, formalize them, and apply them to various real problems. By this way, students learn meaningful mathematics, doing mathematics and develop mathematical power.

CPMP curriculum provides four courses. All of CMIC students may study the first three courses until grade 11th, and all of the college-bound students may study course 4 in 12th grade. Each course features of algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics (CPMP, 1998). These strands are connected within units by fundamental ideas such as symmetry, functions, matrices, data analysis, and curve-fitting. The strands also are connected across units by mathematical habits of minds such as visual thinking, recursive thinking, searching for and describing patterns, making and checking conjectures, reasoning with multiple representation, inventing mathematics and providing convincing arguments. The strands are linked further by the fundamental themes of data, representation, shape, and change. Important mathematical ideas are continually revisited through these connections so that students can develop a robust of understanding of mathematics. Fundamental characteristics of both curricula are summarized in Table 1.

Table 1. Essential Differences of Characteristics

	KHMC	СРМР
Aims	Understanding basic concepts, principle,	Understanding mathematics that make
	and rules	sense to them
	Problem solving abilities	
	Mathematical disposition	
	Level-preferred learning	
Courses		
Times		

We can find a few differences between KHMC and CPMP. First, KHMC is organized in a level referenced manner that allows students can study selected mathematics according to their cognitive development level. But, CPMP curriculum is organized for all students can study the same mathematics in different methods according to individual abilities. CPMP students study mathematics more long times than KHMC. Most of Koreans have believed that their students study pretty high level mathematics than American students. But, we can find that KHMC students study less mathematics than CMIC students do. If the importance of mathematics is increased in future society oriented to technology, KHMC may consider those characteristics of CPMP.

#### 1. Fundamental Contents

Contents of curriculum are one of the most important elements of mathematics education. We decide the contents by considering of several factors such as nature of mathematics, philosophy of mathematics education, future society, current social environment, learners cognitive abilities and dispositions, and so on.

#### Table 2: Essential Differences of Contents

Fundamental contents of algebra and functions in the two curricula are summarized in Table 2. We can find some important differences as follows. First, for the contents compulsory for all high school students, KHMC emphasizes more on sets, logical statements, expressions (factor theorem, remainder theorem, etc.), inequalities (quadratic inequalities, proof of absolute inequalities), and functions (formal definition, properties of functions, inverse functions). CPMP curriculum emphasizes more on exponential expressions (modeling, solving exponential equations and inequalities), calculus underpinning (intuitive concept of limit), and matrices (transformation

matrices). The contents more emphasized in KHMC are relatively traditional and formal mathematics, and the contents in CMIC are relatively new and to solve real-life problems with aids of calculators. All of the formal contents in 10th grade of KHMC, compulsory for all students, are included in the Course 4 of CMIC for college-bound students. KHMC intends to teach basic formal mathematics to all students and then make students apply the mathematics according to their needs. Conversely, CPMP intends to teach applicable mathematical concepts and experiences to all students and then teach formal mathematics to the college-bound students.

Second, as the fundamental contents of algebra and functions for all college-bound students, KHMC treats exponential and logarithmic functions, matrices, mathematical induction and algorithms and flowcharts. Among these concepts, sequences and limits of sequences are treated more formally in KHMC. CPMP treats rate of change derivative function and definite integral, logarithmic functions and data models, problem solving with spreadsheets, and mathematical induction. Among these concepts, rate of change and problem solving with spreadsheets are treated in CPMP. The concepts of derivative functions and definite integrals are treated intuitively and computed approximately.

KHMC has treated differentiation and integration concepts a little formally in Mathematics I until 2001. But, from 2002, those concepts are treated in Mathematics ¥± and Differentiation and Integration for the students who are intending to study natural sciences and engineering courses in college levels.

Third, as the fundamental contents of algebra and functions for students intending to mathematical natural sciences and engineering, KHMC treats fractional equations and inequalities, irrational equations, natural logarithms, limits and continuity of functions, differentiation and integration. CMIC treats polynomial and rational functions, complex numbers, factor theorem and remainder theorem, natural logarithm and equations. KHMC treats those concepts as formal mathematics. CMIC students intending to social, management, and health sciences study on informatics. This topic is useful to understand applications of technologies to real-life situations.

Through comparisons of contents, we can find that KHMC students begin to study formal mathematical concepts from earlier grades and CMIC students learn more applicable concepts in earlier grades and then learn formal concepts for college-bound students.

#### 3. Fundamental Objects of Study

Even though we study a same mathematical concept there are a lot of different ways in studying it. For an example, to study a concept of linear function, we can think two ways. One

way is the traditional way in which we can study the concept of linear function by defining the function, analyzing some properties of the function and then applying them to problems. Alternative way is constructive way 8in which we find some patterns varying linearly in real situations, represent the pattern in mathematical models such as table, graph, and symbolic expression, an then investigate some properties of the function in various ways with aids of technology. By studying in constructive way, we can learn meaningful mathematics and develop mathematical power.

Fundamental objects of study in KHMC are as follows; understanding basic mathematical concepts, principles, and rules; developing thinking abilities and reasoning abilities; and solving problems reasonably and creatively. KHMC has treated mathematical concepts in formal ways traditionally, but nowadays the trends are being changed to constructive ways. The fundamental objects of study in CPMP are as follows: recognizing, representing, and solving problems involving relations among quantitative variables; using functions as mathematical models; investigating algebraic models in at least three linked representations-graphic, numeric, and symbolic with aids of technology; and connecting various concepts. These objects of study can be seen as coherent to a target to teach meaningful mathematics and mathematical power. KHMC may consider of teaching students more meaningful mathematics and mathematical power.

Table 3. Essential Differences of Concepts of Algebra and Functi

	KHMC	CMIC
Fundamental objects of study	-understanding basic mathematical concepts, principles, and rules	quantitative variables  -Using functions as mathematical models  -Investigating algebraic models in at least three linked representations

	KHMC	CMIC
Typical problems in the Curriculum	-Symbolic expressions, equations, and functions to compute, evaluate, solve, graph, or prove -Verbal problems with requests to find numerical values, explain, or verify so on	requests to find values and describe, make tables or graphs, explain, conjecture and test, or prove so on —Symbolic expressions or equations are algorithms.
Typical solution methods	manipulations in correct order -Write mathematics models such as equation or inequalities, solve them, and interpret the results	-Find patterns by manipulating or conjecturing -Make function models, graphs, or tables with helps of calculators, and
	-Practice the same type problems as given examples to master understanding and applying new concepts learned -Supplement basic concepts	-Apply ideas and methods -Integrate and connect formal mathematics
Role of technology for Representing and calculating		-Supports students' work on almost all learning activities and problem solving
Elements in typical lesson	-Collaborative learning -Teacher centered instruction	-Launch: full-class discussion- Explore: collaborative small group investigate -Share and Summarize: full-class discussion, teacher is moderator -Apply: individual work to reinforce understanding -Teacher is director, moderator, facilitator, and intellectual coach

#### 4. Typical Problems

Typical problems that students are asked to solve have been deeply corresponded to typical objects of study. KHMC include essentially two types of problems in algebra and functions: (1) symbolic expressions that students are directed to compute, evaluate, solve, graph, or prove and (2) verbal problems with requests to find values, explain, verify, or prove. Solving numeric or symbolic expressions skillfully, some times, can be helpful for students, understand and apply mathematical concepts or rules to solve problems, of KHMC request to find numerical values and explain or verify the results by applying the mathematics they have learned. Most of the verbal provblems in textbooks are routine problems and less open-ended problems even though they are related to real life situations.

Typical problems in Courses 1 to 3 of CMIC are verbal problems that include tables, graphs, or symbolic expressions, with requests to find numerical values, describe, make tables or graphs, explain, conjecture and test, or prove. There are few problems asking only computation and manipulation of numerical of symbolical expressions. In CMIC, basic procedural skills are assumed that they can be achieved through various context verbal provblems. For some students who need more practice for computational skill of manipulating skill, supplementary materials named "CMIC-RAP: Contemporary Mathematics in Context Reference and Practice" are developed and used individually in a class or at home. Course 4 supplies PUMP material for the college-bound students who need basic skills to compute or manipulate numeric or symbolic expressions to prepare for undergraduate mathematics placement test. Most of PUMP materials are consisted with basic and fundamental numeric or symbolic expressions.

By comparing typical problems between the two curricula, we can find that KHMC may consider to treat more contextual and open-ended real-life problems.

#### 5. Typical Solution Methods

Because typical problems of the two curricula are different, typical solution methods also differ. More over, same problems can have multiple different solution methods. So, we may find some implications by comparing typical solution methods of the two curricula.

Typical solution methods of KHMC are as follows: complete the correct manipulations in correct order; write mathematics models such as equations or inequalities, solve them, and interpret the results; prove some properties by using logical rules; and solve real life problems by using suitable or creative strategies. In his case, efficiency and streamlined development, and creative thinking are valued attributes of students' work. Once they master the basic procedures,

students are expected to develop shortcuts and recognize special cases.

Typical solution methods in CMIC are much different. They involve finding patterns by manipulating or conjecturing; making function models, graphs, or tables with helps of calculators, and analyzing them; writing equations or inequalities, solving them, and interpreting the results. The clarity, logic, and thoroughness of students' explanations are nalued attributes of the solutions to this problem and ones like this.

KHMC may have to consider those typical solution methods used in CPMP before using formal and logical methods so that students can solve problems with varous available methods for them.

#### 6. Role of Practice

Sufficient practice plays a very important role to deeper understand and apply the new concepts learned. Role of practice is changed according to the objectives of learning. In KHMC, there are two fundamental practice tasks. One is the exercise task followed just after an example showing how to apply the new concepts they learned. By practicing this exercise students can understand more clearly the new concepts. The other one is the exercise tasks given at the end of each unit. This practice task involves two types of exercises: one is to practice fundamental skills and the other one is to develop problem solving ability. By practicing these tasks students can understand more clearly what they learned and can solve more efficiently problems related to the concepts.

CMIC supplies two types of tasks to be practiced individually – "On Your Own" and "Modeling, Organizing, Reflecting, and Extending(MORE)". By completing individually the tasks of "On Your Own" and selected tasks from MORE, students can reinforce their initial understanding of a concept or method achieved by collaborative learning. Selection of MORE tasks should be based on student performance and the availability of time and technology. The characteristics of tasks in a typical MORE set can be described as follows:

Modeling: Modeling tasks are related to, or provide new contexts to which students can apply, the ideas and methods that they have developed in the lesson.

Organizing: Organizing tasks offer opportunities for integrating the formal mathematics underlying the mathematical models developed in the lesson and for making connections with other strands.

Reflecting: Reflecting tasks encourage thinking about thinking itself, about mathematical meanings, about processes, and promote self-monitoring and evaluation of understanding.

Extending: Extending tasks permit further, deeper, or more formal study of the topics under investigation.

Most of the practice tasks in KHMC are isolated. To make students' holistic understanding of mathematical concepts, KHMC may supply some practice tasks consisted sequentially with various questions to applications of the new concepts, as CMIC does.

# 7. The Role of Technology

Technology is used as one of the most useful and necessary tools in current and future society. We can not avoid using technology for mathematics education since it is found as a useful for it. But the role of technology is changed according to the notions students learn.

KHMC encourages each level of schools to use technology in any area of mathematics if it can be used in pedagogically effective way. But, most of teachers do not use efficiently it yet. They think using calculators only for computations is not much necessary since students can compute simple numeric or algebraic expressions used in to understand basic mathematical concepts. But, they agree that calculators should be used to compute complex numerical expressions such as irrational, exponential, logarithmic, trigonometric, and statistics to solve problems. Some teachers are much concerned on how we can use computers for mathematics education such as Geometer Sketchpad that can not be replaced by any traditional materials.

CMIC materials treat technology as an integrated part of learning and teaching materials. The curriculum makes quite strong commitments to technology (Preface of students' materials, Curriculum Overview in teacher's of CMIC, and Implementing the Core-Plus Mathematics Curriculum, 1998):

Numerical, graphics, and programming/link capabilities of graphics calculators are assumed and capitalized on. This technology permits the curriculum and instruction to emphasize multiple representations (numerical, graphical, and symbolic) and to focus on goals in which mathematical thinking and problem solving are central.

Technology is embedded and used throughout CPMP curriculum. Calculators and computers and used both in computation and in creating and manipulating representations. In many problems, students are asked to use their calculators to make tables or construct graphs, then are asked to explain, interpret, predict, and compare these representations. Moreover, CPMP developed Calculator Software can be used in computers and calculators of TI-82 through TI-92. They are as follows:

For Course 1:

STEM - Produces a stem-and-leaf plot or a back-to-back stem-and-leaf plot to help analyze the shape of a distribution or compare two related distributions.

COLLECTIONS - Run Monte Carlo simulations to study waiting-time distributions and their corresponding probability distributions.

TABLEPLOT - Graphs discrete functions from tables.

PERT - Applies the Program Evaluation and Review Technique to examine scheduling of projects.

For Course 2:

GeoXplor - Supports exploration and analysis of plane shapes; explore transformations and their effects on figures on figures in the plane; supports geometric exploration of linear combinations of two equations of the form Ax + By = C; and demonstrates computer animation.

Shortcut-Locates shortest paths and their lengths in weighted graphs.

These types of software supplement the limited functions of calculators of TI 80 series, and make those calculators can be effectively in any area of secondary school mathematics.

KHMC encourages teachers to use technology effectively, but it does not suggest any lists of technology to avoid illegal recommendation of specific companies' productions. By the results, teachers have difficulties to access available technologies. KHMC may supply teachers some useful information to guide them to use efficient technologies.

#### 8. Elements in Typical Lessons

KHMC, which will be used from 2002, suggests teachers to take into account the following facts to implement the Level Based Differentiated Curriculum effectively (Ministry of Education, 1997, p.104):

- (1) Organize homogenous classes according to the students' individual achievement levels.
- (2) Execute individualized instructions to improve the efficiency of learning.
- (3) Make students learn collaboratively in small groups.

Through various channels such as in-service training or professional meetings about mathematics education, teachers understand that they have to execute various methods of instructions such as small group learning, individualized learning, or students centered learning. But, most of teachers execute teacher-centered instruction. There are a few reasons for teachers are biased on teacher centered instruction. First, most schools in urban area have too big size classes (about 50) to execute student-centered instruction. Second, textbooks are so simple and

abstracted that they are suitable for teacher-center instruction. Third, teachers, parents, and students believe that intensive teacher-centered learning is the most effective way to achieve high scores in the national tests for students college-bound.

CPMP suggests an instructional model of four-phase cycle of classroom activities-launch, explore, share and summarize, and apply. In launch phase, lessons begin with a full-class discussion of a problem situation and of related questions to think about. This discussion sets the context for the student work to follow and helps to generate student interest. It also provides an opportunity for the teacher to assess student knowledge and to clarify directions for the group activities. In this phase teacher is director and moderator. In explore phase, classroom activity is focused on problems and questions related to the launching situation. Students work collaboratively in small groups and the teacher circulates from group to group providing guidance and support, clarifying or asking questions, giving hints, providing encouragement, and drawing group members into the discussion to help groups work more collaboratively. In share and summarize phase, a full-class discussion of concepts and methods developed by different small group is held and students share progress and thinking. This discussion leads to a class summary of important ideas or to further exploration of a topic if competing perspectives remain. In this phase, teacher is moderator. In apply phase, students are given a task to complete on their own. Through this phase individual students reinforce their initial understanding of a concept or method. The teacher circulates in the room assessing levels of understanding.

Through this four-phase cycle of classroom activities, students can learn meaningful mathematics and develop mathematical power. KHMC may consider some methods to encourage students' efficient collaborative learning in small groups.

### III. Conclusions

To educate students to adjust and lead their future society oriented on technologies and information communications, most of the countries in the world try to reform their traditional mathematics education. Contemporary Mathematics in Context (CMIC) developed by Core-Plus mathematics Project(CPMP) in Western Michigan University has been developed in order to assist school districts in reforming their high schools mathematics programs in the United States. 7 Korean Mathematics Curriculum was developed to guide reforming Korean mathematics program in 1997.

In this paper, by analyzing essential differences of algebra and functions between CMIC and

Korean High Schools Mathematics Curriculum (KHMC) according to KHMC, the writer finds some implications of CMIC for Korean mathematics education.

The main differences of the two curricula and some implications of CMIC for Korean mathematics education are as follows.

In characteristics of the two curricula, KHMC organized in a step-wise and level-referenced manner that will allow the teacher to consider the rate of the students' cognitive development and to thereby select core contents of the curriculum based on a learning hierarchy and difficulties. For 11 and 12 grade students, there are 6 courses which individual students can select to study according to their future directions and needs. CPMP curriculum aims to teach all of the high school students from 9 grade to 11 grade the same mathematical topics but in different ways according to their abilities. But, 12 grade college-bound students study differently more advanced and formalized mathematics education are to teach fundamental mathematics necessary for all students, KHMC may consider of teaching all of the high schools students the same fundamental mathematical topics in different ways according to individual abilities.

In algebra and functions, KHMC students study basic formal mathematical concepts in earlier grades and then learn mathematics for real-life if they do not intend to college level education. KHMC students intend to college level education, study more formal mathematics and they have relatively few chances to study mathematics or real-life or discrete mathematics with application of technology. KHMC may consider of teaching algebra and functions by connecting real-life situations or problems.

Nowadays, from the 7 mathematics curriculum, Korean students turn their learning methods from understanding mathematical concepts and principles with teacher's explanations and practicing skillful manipulations of symbolic expressions to investigating the problem situations and modeling them mathematical models. CMIC has been developed for students' learning of meaningful mathematics developing mathematical power. In this study, the writer finds some very important fundamental implications of CMIC for KHMC, even though this studyu is limited on algebra and functions strand.

Further study on other strands of statistics and probability, geometry and trigonometry, and discrete mathematics are needed to find some more implications focused on the specific areas.

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