휨을 받는 철근콘크리트 단순보의 비선형 해석

Nonlinear Analysis of Reinforced Concrete Simple Beam Subjected by Bending Moment

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요 약

본 연구의 목적은 휨을 받는 철근콘크리트 단순보의 비선형 거동을 해석을 통하여 파악하기 위하여 보의 단면을 Layer로 분할하여 각 Layer에 사용된 재료의 특성을 부여하고, 작용하중에 의한 비선형 특성을 응력-변형도 모델을 통하여 반영할 수 있도록 알고리즘을 작성하는 것이다. 본 연구에서 제안한 알고리즘을 사용하면 실험을 위한 보(beam)를 제작하지 않고도 사용재료의 압축 및 인장 실험을 위한 몰드 만을 제작하여 재료에 대한 정확한 응력-변형도 모델을 작성함으로써 합성보의 거동을 수치해석을 통하여 간단하게 파악할 수 있을 것으로 기대된다.

keywords : Nonlinear analysis, Layered method, Stress-strain model

1. INTRODUCTION

The concrete member as a compressive member is economic material. But when it is used as a tensile member, it is very weak material. As it is reinforced by steel, fiber, etc., which have high strength for tensile force. The rupture of the composition concrete members is shown as various patterns according to both quantity and position of composition material[1].

Many researchers have studied the nonlinear behavior of reinforced concrete members through the analytic and the experimental methods so as to know the results of the exact rupture pattern of composition material. Researchers who have been studied by analytic method through finite element method have studied mainly studies of the nonlinear property of composition material. The research by Ngo and Scordelis[2] in 1967 is on starting point, which model is the reinforced concrete beam, and then many researches were completed after[3],[4].

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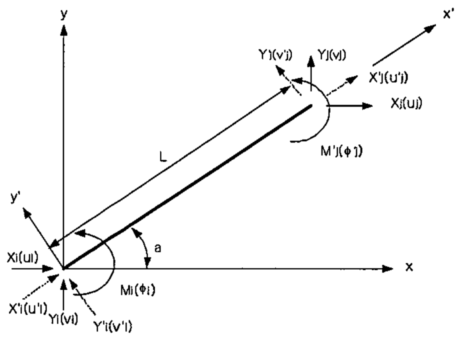
But, in spite of efforts of many researchers, the theory for the nonlinear property of material of composite members is not completed today. Therefor the purpose of this study is to present the algorithm for the nonlinear material property of reinforced concrete beam.

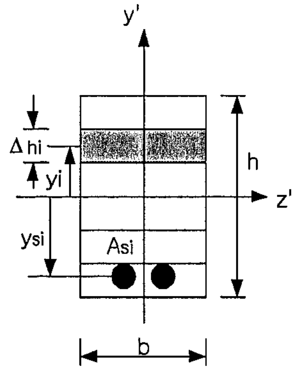
2. THE ANALYSIS OF REINFORCED CONCRETE BEAM

2.1 The theory of analysis

The layer finite element method is used in order to consider the nonlinear behavior of reinforced concrete beam, and the simple beam is used as a numerical analysis model.



(a) The coordinate system of force and displacement for an element



(b) Shape of cross section

Fig. 1 The numerical analysis model

The analytic theory of the model is derived from considering the Fig. 1 with several layers[(5)~(7)]. Displacements of an element of the beam may be written as a compact matrix form as shown in equations (1) and (2).

$$\{\delta_{e1}\} = [S_1][C_1]\{\delta_1\} \tag{1}$$

$$\{\delta_{e2}\} = [S_2][C_2]\{\delta_2\} \tag{2}$$

where, $\{\delta_{e1}\} = \{u(x',0)\}$,

$\{\delta_{e2}\} = \{\ v(x',0)\ \phi(x',0)\}^T$

$\{\delta_1\} = \{u'_i\ u'_j\}^T$, $\{\delta_2\} = \{v'_i\ \phi_i\ v'_j\ \phi'_j\}^T$

$$[S_1] = [\,1\ \ x\,],\ [S_2] = \begin{bmatrix} 1 & x' & x'^2 & x'^3 \\ 0 & 1 & 2x' & 3x'^2 \end{bmatrix}$$

$$[C_1] = \begin{bmatrix} 1 & 0 \\ -\frac{1}{L} & \frac{1}{L} \end{bmatrix},$$

$$[C_2] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{L^2} & -\frac{2}{L} & \frac{3}{L^2} & -\frac{1}{L} \\ \frac{2}{L^3} & \frac{1}{L^2} & -\frac{2}{L^3} & \frac{1}{L^2} \end{bmatrix},$$

and L is the length of element, x' is the length of element from starting point to arbitrary point, u, v, ϕ are horizontal, vertical, rotational displacements, respectively, the nodes i and j represent starting point and ending point of element, respectively.

Axial displacement of arbitrary point (x', y') on element may be written as equation (3) by using beam theory.

$$u(x',y')=u(x',0)-y'\cdot\frac{\partial v(x',0)}{\partial x'} \quad (3)$$

Then the axial strain and stress of arbitrary point (x', y') are:

$$\varepsilon(x',y')=\frac{u(x',0)}{\partial x'}-y'\cdot\frac{\partial^2 v(x',0)}{\partial^2 x'}$$
$$=\varepsilon(x',0)+\triangle\varepsilon(x'y') \quad (4)$$

$$\sigma(x',y')=\sigma(x',0)+\sigma(x',y')$$
$$=E_y\varepsilon(x',0)+E_y\triangle\varepsilon(x'y') \quad (5)$$

The variables $\varepsilon(x',0)$, $\sigma(x',0)$ represent strain and stress by axial force, and $\triangle\varepsilon(x'y')$, $\triangle\sigma(x'y')$ represent strain and stress caused by bending moment on element.

Also, using equation (1) and equation (2), the strain is rewritten in the form.

$$\varepsilon(x',0)=[0\ \ 1][C_1]\{\delta_1\} \quad (6)$$

$$\triangle\varepsilon(x',y')=-y'\cdot[0\ \ 0\ \ 2\ \ 6x'][C_2]\{\delta_2\} \quad (7)$$

Now, considering the principle of virtual work, the external work and internal work have following relationship.

$$\{\delta_1\}^T\{F_1\}=\int_V\varepsilon(x',0)\sigma(x',0)dV \quad (8)$$

$$\{\delta_2\}^T\{F_2\}=\int_V\triangle\varepsilon(x',y')\triangle\sigma(x',y')dV \quad (9)$$

Substituting equations (4) to (7) into equations (8) and (9), forces $\{F_1\}$ and $\{F_2\}$ were given by

$$\{F_1\}=\int_V[C_1]^T[0\ \ 1]^T E_i[0\ \ 1][C_1]\{\delta_1\}dV$$
$$=[C_1]^T\sum_{i=1}^{n}\frac{E_iA_i}{L}[0\ \ L]^T[0\ \ L][C_1]\{\delta_1\}$$
$$=[K_1]\{\delta_1\} \quad (10)$$

$$\{F_2\}=[C_2]^T\int_0^L\left(\int\int E_y y'^2 dy'dz'\right)$$
$$[0\ 0\ 2\ 6x']^T[0\ 0\ 2\ 6x']dx'[C_2]\{\delta_2\}$$
$$=[C_2]^T\int_0^L\left(\sum_{i=1}^{n}E_iI_i\right)[0\ 0\ 2\ 6x']^T$$
$$[0\ 0\ 2\ 6x']dx[C_2]\{\delta_2\}$$
$$=[K_2]\{\delta_2\} \quad (11)$$

where, A_i and E_i are sectional area and secant modulus of material on ith layer, respectively. Equations (10) and (11) are written as a matrix form as shown in equation (12).

$$\{F\}=[K]\{\delta\} \quad (12)$$

where,

$\{F\}=\{X'_i\ X'_j\ Y'_i\ M'_i\ Y'_j\ M'_j\}^T$,

$\{\delta\}=\{u'_i\ u'_j\ v'_i\ \phi'_i\ v'_j\ \phi'_j\}^T$,

$[K]=\begin{bmatrix}[K_1] & 0\\ 0 & [K_2]\end{bmatrix}$.

Also, as shown in Fig. 1, the local coordinate x' is rotated with angle α as compared with reference coordinate x. Therefore, local coordinate must be transformed into reference coordinate. Using the transformation matrix $[T]$, we rewrite equation (12) as

$$\{F\}=[T]^T[K][T]\{\delta\}=[K_e]\{\delta\} \quad (13)$$

The secant modulus $(E_{c1})_i$ involved in a stiffness

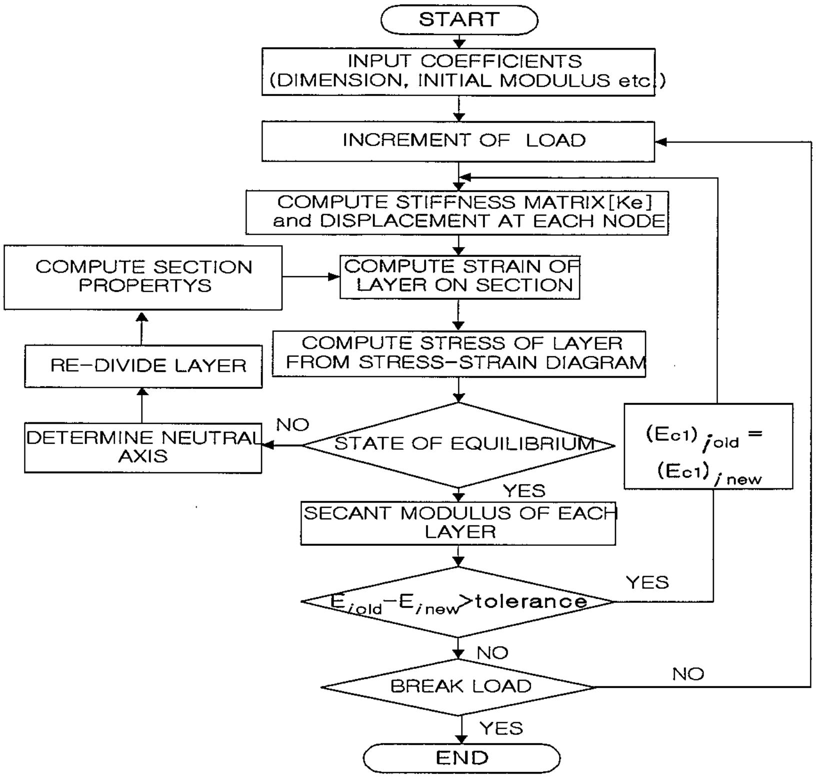
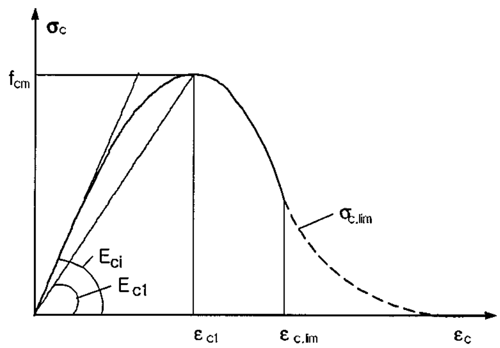


Fig. 2 The flowchart for numerical analysis

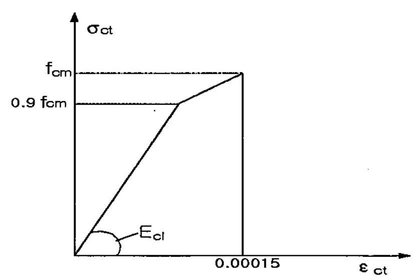
matrix $[K_e]$ in equation (13) is determined by iteration computation as shown in Fig. 2.



(a) For uniaxial compression

2.2 The stress-strain model of concrete

In this study, stress-strain models of concrete which are required to apply the algorithm for nonlinear analysis are as shown in Fig. 3.



(b) For uniaxial tension.

Fig. 3. Concrete stress-strain diagram for uniaxial compression and tension.

which have been drawn with reference to CEB-FIP MODEL CODE[8].

The relationship of compressive stress-strain for concrete ($\sigma_c - \varepsilon_c$) is written as

$$\sigma_c = -\frac{\frac{E_{ci}}{E_{c1}}\frac{\varepsilon_c}{\varepsilon_{c1}} - \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^2}{1+\left(\frac{E_{ci}}{E_{c1}}-2\right)\frac{\varepsilon_c}{\varepsilon_{c1}}} f_{cm}$$

for $|\varepsilon_c| \leq |\varepsilon_{c,\lim}|$ (14)

$$\sigma_c = -\left[\left\{\frac{1}{\varepsilon_{c,\lim}/\varepsilon_{c1}}\xi - \frac{2}{(\varepsilon_{c,\lim}/\varepsilon_{c1})^2}\right)\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^2 + \left(\frac{4}{(\varepsilon_{c,\lim}/\varepsilon_{c1})} - \xi\right)\frac{\varepsilon_c}{\varepsilon_{c1}}\right]^{-1} f_{cm}$$

for $|\varepsilon_c| > |\varepsilon_{c,\lim}|$ (15)

where, σ_c, ε_c denote the compressive stress and strain of concrete, respectively. E_{c1} is a secant modulus of concrete which is calculated by f_{cm}/ε_{c1} as shown in Fig. 3(a), and

$E_{c0} = 2.15\times10^4 MPa$, $f_{cm0} = 10MPa$,

$f_{cm} = f_{ck} + \Delta f$, $\Delta f = 8MPa$, $\varepsilon_{c1} = -0.002$,

$E_{ci} = E_{c0}[f_{cm}/f_{cm0}]^{1/3}$,

$$\frac{\varepsilon_{c,\lim}}{\varepsilon_{c1}} = \frac{1}{2}\left(\frac{1}{2}\frac{E_{ci}}{E_{c1}}+1\right) + \left[\frac{1}{4}\left(\frac{1}{2}\frac{E_{ci}}{E_{c1}}+1\right)^2 - \frac{1}{2}\right]^{1/2},$$

$$\xi = \frac{4\left[\left(\frac{\varepsilon_{c,\lim}}{\varepsilon_{c1}}\right)^2\left(\frac{E_{ci}}{E_{c1}}-2\right)+2\frac{\varepsilon_{c,\lim}}{\varepsilon_{c1}} - \frac{E_{ci}}{E_{c1}}\right]}{\left[\frac{\varepsilon_{c,\lim}}{\varepsilon_{c1}}\left(\frac{E_{ci}}{E_{c1}}-2\right)+1\right]^2}$$

Also, the relationship of the tensile stress-strain ($\sigma_{ct} - \varepsilon_{ct}$) for concrete is

$$\sigma_{ct} = E_{ci}\cdot\varepsilon_{ct} \text{ for } \sigma_{ct} \leq 0.9 f_{ctm} \quad (16)$$

$$\sigma_{ct} = f_{ctm} - \frac{0.1 f_{ctm}}{0.00015 - 0.9 f_{ctm}/E_{ci}} \times (0.00015 - \varepsilon_{ct}) \text{ for } 0.9 f_{ctm} < \sigma_{ct} \leq f_{ctm} \quad (17)$$

$$\sigma_{ct} = 0 \text{ for } \sigma_{ct} > f_{ctm} \quad (18)$$

where, $f_{ctm} = f_{ctk0,m}\left(\frac{f_{ck}}{f_{ck0}}\right)^{2/3}$.

$f_{ctk0} = 1.40MPa$, $f_{ck0} = 10MPa$, f_{ck} is the rupture strength of concrete.

2.3 The stress-strain model of steel

In this study, the stress-strain models of steel which are required to apply the algorithm for nonlinear analysis are as shown in Fig. 4, which have been drawn with reference to specification [8],[9].

The relationship of stress-strain for steel ($\sigma_s - \varepsilon_s$) is written as

$$\sigma_s = E_s\cdot\varepsilon_s \text{ for } 0 \leq |\varepsilon_s| \leq |\varepsilon_y| \quad (19)$$

$$\sigma_s = +f_y \text{ for } |\varepsilon_s| > |\varepsilon_y| \quad (20)$$

where, $E_s = 200GPa$, σ_y is the yield strength of steel.

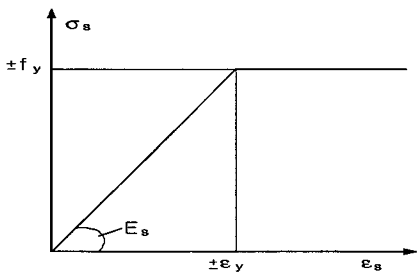


Fig. 4 Steel stress-strain diagram.

3. NUMERICAL ANALYSIS

3.1 Numerical analysis model

The numerical analysis model is used to verify the algorithm of nonlinear analysis model for the reinforced concrete beam subjecting the transverse point load at two points. The analysis model which is simple beam with ten elements and eleven nodes as shown in Fig. 5 has the rectangular cross section divided by several layers as shown in Fig. 6.

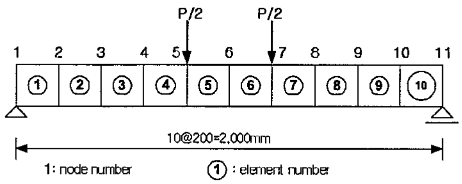


Fig. 5 The simple beam for numerical analysis

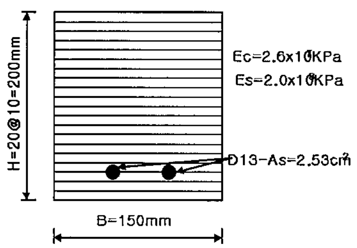


Fig. 6 Initial cross section

3.2 Results of numerical analysis

The relationship between load and deflection which is calculated by nonlinear analysis of reinforced concrete rectangular section divided by several layers is depicted as shown in Fig. 7.

Fig. 7 has shown the deflection of midpoint on element 5 of beam with different steel ratio. When crack is occurred in the tensile part of concrete beam, the deflection is increased in spite of decreasing load.

In addition, we can see that the aspect of the rupture is different according to the steel ratio. In Fig. 7, the curves (1) and (2) have shown that the rupture of beam is occurred by the yield of concrete in the tensile part at once, and curves (3) and (4) have shown that the rupture of beam is occurred after meeting yielding points of the concrete in twice and three times, respectively.

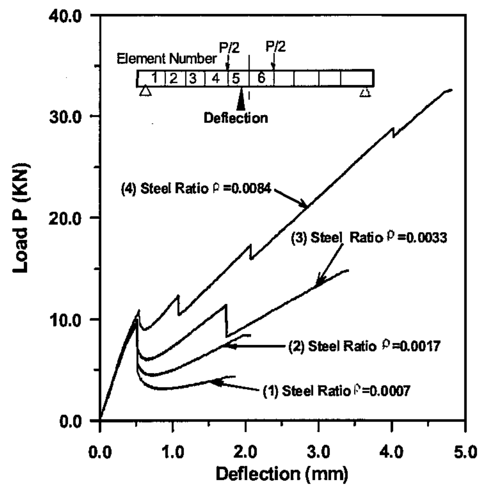
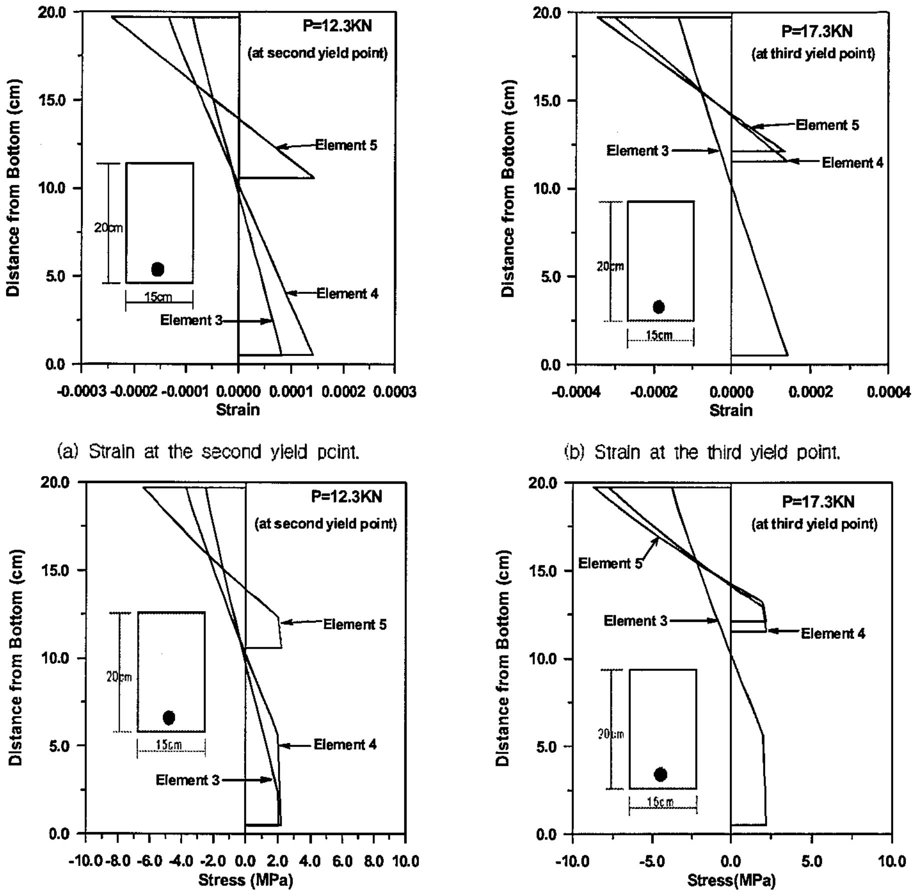


Fig. 7 Load-deflection curve of reinforced concrete beam.

Also, strain and stress of each element of the beam which are applied yielding point load shown in curve (4) of Fig. 7 are presented in Fig. 8, respectively. In Fig. 8, the crack is occurred at the bottom of cross section according to the increasing load. And then at the first yielding point, the depth of crack approached to the position of steel. At the third yielding point, the depth of crack reached to the layer above the neutral axis in the cross section.



(a) Strain at the second yield point.

(b) Strain at the third yield point.

(c) Stress at the second yield point

(d) Stress at the third yield point.

Fig. 8 Strain and stress at the yield point in cross section

4. CONCLUSIONS

This study has presented the algorithm for the nonlinear analysis of beam which is composed of concrete and steel or FRP(Fiber-Reinforced-Plastic) bar. According to the proposal algorithm in this study for the analysis of reinforced concrete simple beam, the element of the beam cross sections have been divided into several layers horizontally, and the material property of the beam have been assigned to each layers, and the structures of beam have been analyzed by the finite element method, and the results have been compared with the values of stress-strain model which are drawn by the tensile and the compressive experiment of specimens.

As a result, this study has exactly shown the behavior of beam which is subjected by the bending moment through both the algorithm which we have proposed and the stress-strain model of specimens without the expensive and vexatious experiment of beam.

Thus we can expect that the nonlinear layered method proposed in this study have the better effects on the estimating nonlinear behavior of reinforced concrete beam.

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