[논 문] - 비 파괴 검사 학회지 Journal of the Korean Society for Nondestructive Testing Vol. 21, No. 1 (2001, 2)

Sensitive NDE of Small Fatigue Cracks

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Abstract Some techniques developed recently for sizing small fatigue cracks are described. One is an ultrasonic technique which deals with a small closed crack, where both the stress closing the crack and the crack size are determined by analyzing inverse problem. Here, difficulties encountered in NDE of closed cracks by usual ultrasonic techniques are summarized in advance. Secondly, the closely coupled probes potential drop (CCPPD) technique, which is based on d-c potential drop measurement, is explained for sizing small cracks. The CCPPD technique is not affected by crack closure. Finally, a discussion is given on NDE of materials degradation in conjunction with sensitive NDE of small cracks.

Keywords: ultrasonic testing, potential drop technique, small closed crack

1. Introduction

Quantitative NDE of cracks is of prime importance for assessing the integrity of structural components based on fracture mechanics. However, in practice, mechanics had hardly gained that much popularity in its application mainly due to the obvious lack of useful researches in the field of quantitative NDE. In order to assess the reliability as well as serviceability of actual structures, it is imperative to detect cracks sensitively and also evaluate them quantitatively in advance. The usual methods of nondestructive testing were primarily aimed to detect the cracks, but most of them were found quite helpless in evaluating the crack quantitatively after their detection. Although the use of ultrasonics, for example, to detect cracks is reliable and wellestablished, reliable sizing of closed cracks, specially, the smaller ones had not been successful in the past mainly because of the inability of detecting them sensitively as well as taking care of their closure appropriately.

The present paper describes a sensitive ultrasonic technique which is based on the use of an oblique longitudinal wave with small angle of incidence and the closely coupled probes potential drop (CCPPD) technique for NDE of small closed fatigue cracks.

2. Ultrasonic Testing of Small Closed Cracks

The potential of using ultrasonic methods in evaluating a crack quantitatively has been greatly recognized and also verified by fundamental experiments. Ultrasonic interrogation towards smaller cracks has constantly been looked into by the researchers in the field of NDE of cracks with increasing sophistication in their methods of evaluation, as the reliable quantitative evaluation of the cracks has got increased importance by the modern

Received January 5, 2001. The paper is based on the presentation at the KSNT Spring Conference celebrating the 20th Anniversary of the Korean Society for Nondestructive Testing held on May 12, 2000 in Seoul, Korea.

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defect-tolerant design philosophy. Although many noteworthy investigations aimed ultrasonic evaluation of cracks have been done, reliable sizing of closed cracks, specifically, the smaller cracks, has still remained the biggest hurdle in the evaluation methods of ultrasonics. Crack closure [1], a result of individual contact of asperities caused by a mismatch of the fractured surfaces, makes the closed cracks almost transparent to ultrasonic beams, thereby offering significant problems to their reliable characterization under external load-free condition. And this problem becomes more serious in the case of smaller cracks, since the smaller closed cracks suffer tighter closure than larger ones. Therefore, the need for development of a suitable NDE method for sizing a tightly closed crack under no-load conditions has been recognized with great importance.

2.1. Standard Ultrasonic Sizing Techniques

Common ultrasonic sizing techniques for the evaluation of crack size use tip diffraction of bulk shear and longitudinal waves, surface Rayleigh waves, back-scattering resonance and also mode conversion (e.g. Buck and Tittmann [2], Buck and Skillings [3], Buck et al. [4], Buck et al. [5], Clark et al. [6], Date et al. [7], Golan et al. [8], Hudgell et al. [9], Schneider et al. [10], So and Sinclair [11], Tittmann and Buck [12] and Walte et al. [13]). In practice, cracks often undergo nondestructive inspection opening them. However, sizing a tightly closed crack is a difficult proposition. This section describes the difficulties encountered in the use of some typical techniques.

2.1.1. Surface Rayleigh Wave Technique

The surface Rayleigh wave can effectively be utilized for sizing cracks, provided that the cracks are open. If the closed crack is treated, serious problem happens as indicated in Fig. 1. Tittmann and Buck [12] reported a fundamental study on fatigue lifetime prediction with the aid of surface acoustic wave. The main emphasis

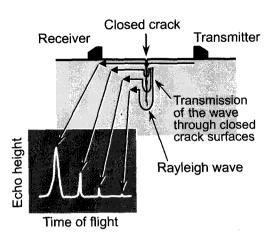


Fig. 1 Evaluation of a closed crack by using the surface Rayleigh wave technique

was placed on evaluating an echo timing technique for determining the crack depth. As reported, to achieve a valid fatigue life prediction using crack propagation data, it was necessary to apply a tensile stress to the cracked specimen such that the crack was fully open. On unloading it was observed that the crack started to close down on itself. Since sound wave travels across the crack surfaces through the contacting asperities, noises showing different travelling passes are observed. Those noise make it difficult to detect the crack-tip signal for the case of a closed crack under no-load conditions.

2.1.2. Tip-Diffracted Wave Technique

The method using the tip-diffracted wave has interested many researchers in the field of ultrasonic NDE. The intensity of the tipdiffracted wave is initially weak. In the case of closed cracks the tip-diffracted wave is further weakened by sound transmission through the closed crack surfaces, see Fig. 2. In this regard, Golan et al. [8] clearly pointed out some of the difficulties faced with ultrasonic major diffraction technique for characterization of fatigue cracks. Because of the high amplification that is needed to enhance the weak signal diffracted from the tip of a crack, oscilloscope trace becomes noisy with signals

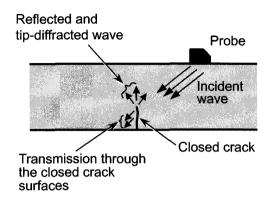


Fig. 2 Evaluation of a closed crack by using the tip-diffracted wave technique

from a variety of sources. Because of the low amplitude of the diffracted wave from closed cracks and the high noise background, signals diffracted from the tip of the crack may level approach the noise and become undetectable. Spurious signals scattered from various points of the crack surface and other sources will form a noisy background, which will interfere with the diffraction measurements and give misleading results. Hence it becomes extremely hard to clearly distinguish the cracktip signal from back-scattering noises.

2.1.3. Decibel Drop Technique

In the specific amount of decibel drop technique, the drop in the intensity of the wave reflected at the crack surface is taken into account. A standard amount of decibel drop is determined by considering the inspection of a fully open crack, as 6 dB for example. However in the case of closed cracks, the reflected wave is weakened by sound transmission through the closed crack surfaces. Hence, the closed crack is evaluated erroneously as being shorter than its actual size. Figure 3 schematically illustrates the technique based on the specific amount of decibel drop in the intensity of the wave reflected at both the open and closed crack surfaces.

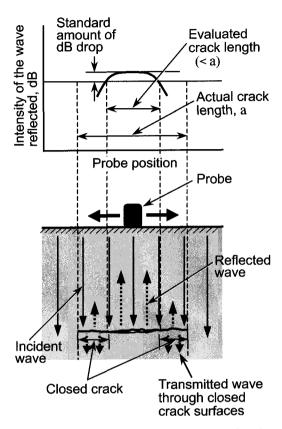


Fig. 3 Evaluation of a closed crack by using the decibel drop technique

2.2. Ultrasonic Angle-Beam Techniques

For effective evaluation of closed cracks, ultrasonic angle-beam techniques with appropriate inverse analyses are noticeable. How tightly a crack is closed is generally unknown. Hence both the crack size and stress closing the crack are the unknown quantities to be determined by solving the inverse problem.

In the conventional angle-beam techniques back-wall cracks in structural components are inspected by obliquely incident sound wave beam. A schematic view of testing configuration is illustrated in Fig. 4. In these techniques the cracks are analyzed by the normalized crackecho-signal corresponding to a particular angle

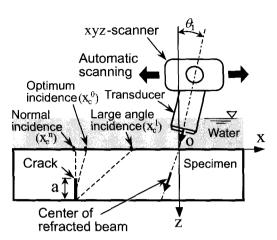


Fig. 4 Schematic of the ultrasonic testing configuration based on the use of an oblique longitudinal wave

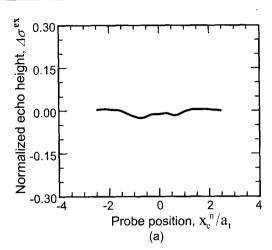
in incidence θ_i . This normalized echo-signal $\Delta \sigma^{\rm ex}$ is defined as

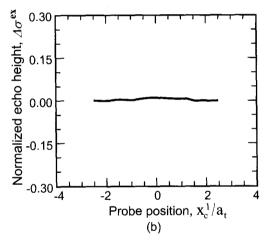
$$\Delta \sigma^{\text{ex}} = \frac{\left| \overline{\sigma} \right|_{\theta_{\perp}}^{\text{ex}} - \left| \overline{\sigma}_{0} \right|_{\theta_{\perp}}^{\text{ex}}}{\left| \overline{\sigma}_{0} \right|_{\theta_{\parallel}=0}^{\text{ex}}}$$
(1)

where $\left|\overline{\sigma}\right|_{\theta_{\dot{1}}}^{\mathbf{ex}}$ and $\left|\overline{\sigma}_{0}\right|_{\theta_{\dot{1}}}^{ex}$ are the amplitudes of

the first back-wall echo of cracked and uncracked specimens.

For a closed crack of 4mm in depth, Fig. 5 shows the variation of normalized amplitude difference with respect to probe position for three different angular incidences, where at is the probe radius. It is well-known that the ultrasonic waves are attenuated as they propagate past the tip of a crack due to the reflection of the energy at the crack surface and diffraction at crack-tip, and, eventually, a W-shaped variation of backwall echo height is encountered around the crack-tip position (Fig. 5(a)), when the specimen is probed using a conventional normal incidence method ($\theta_i = 0$) with longitudinal wave (Saka and Abé [14]; Saka and Uchikawa [15]). When the crack is probed by sound beam with large angle of incidence ($\theta_i = 10^\circ$), very little rise in amplitude of the reflected pulse is observed (Fig. 5(b)). For large size of cracks these two techniques give good characteristic





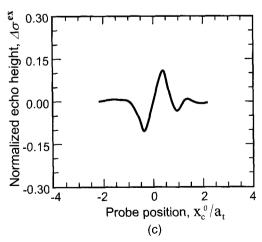


Fig. 5 Normalized echo height versus probe position obtained by using the methods of (a) normal incidence, (b) large-angle incidence, and (c) optimum incidence

response. The recent work (Ahmed and Saka [16]), carried out by using a small angle ($\theta_i = 2^{\circ}$) to the incident longitudinal wave, shows different phenomena of the back-wall echo signal. Here, for this tiny angle of incidence, both the rise in the amplitude and the attenuation phenomenon of the reflected pulse are observed, which are the corresponding effects contributed primarily by the crack-corner and crack-tip. Here the same cracks are identified with much better ultrasonic responses compared to normal and large angle-incidence methods (Fig. 5(c)).

2.3. Theoretical Modeling of Crack Closure

The surfaces of closed crack are contacting and are exerting stress, σ_p , each other, which is namely the stress closing the crack. Consider a coefficient γ of wave reflection at the closed crack surfaces as a ratio of the stresses corresponding to reflected and transmitted waves. Then the received averaged stress, $\overline{\sigma}$, the first back-wall echo obtained for the plate containing a closed crack on the back-wall, is given by

$$\overline{\sigma} = \overline{\sigma}_0 - \gamma \overline{\sigma}_c \tag{2}$$

where $\overline{\sigma}_c$ is the averaged stress due to the reflection experienced at the crack surface and $\overline{\sigma}_0$ the stress due to the reflection at the backwall of an uncracked plate. Since the closed crack surfaces contacting each other are expected to cause multiple reflection of the wave between the surfaces, both the amplitude and the phase of the wave in the reflection and transmission are supposed to be affected by the stress closing the crack [14]. The coefficient of wave reflection is thus assumed to be a function of σ_p , the stress closing the crack, as follows:

$$\gamma = \gamma_{\rm a} \exp(i\pi \gamma_{\phi}) \tag{3}$$

$$\gamma_{\rm a} = \frac{C_0}{C_0 - \sigma_{\rm p}} = \frac{1}{1 - \overline{\sigma}_{\rm p}} \tag{4}$$

$$\gamma_{\phi} = \begin{cases} -1 & (-1/C \leq \gamma_{a} \leq 1) \\ C \gamma_{a} & (\gamma_{a} < -1/C) \end{cases}$$
 (5)

where i is $\sqrt{-1}$ and, γ_a and γ_ϕ represent the reflection coefficient concerned with the amplitude and change in phase, respectively. The coefficient C is taken here as a negative constant being independent of the stress closing the crack. The closure stress $\bar{\sigma}_n$ representing an averaged value of the local compressive stresses normalized by the positive constant C₀ takes a negative value for closed cracks and zero for open cracks, and is assumed to be distributed uniformly along the crack surface. It can be pointed out here that the present modeling would not be of much help for the case of cracks having partly closed and partly open region, which may, however, the case for large cracks, and, in practice, is rarely encountered in the case of smaller cracks.

2.4. Inverse Analysis and Quantitative Evaluation

For the evaluation of unknown closed cracks quantitatively by using the present angle-beam approach, an inverse evaluation algorithm has been developed introducing the model of crack closure. In the analysis, crack size a, closure stress $\overline{\sigma}_p$, the information about how tightly the crack is closed, are treated as the most desirable parameters to be determined by solving the inverse problem.

The inverse problem is defined here as follows: given the measured amplitudes of the back-wall echo obtained with the specimen containing the crack, determine simultaneously the unknown parameters of interest through a comparison of the predicted and the measured amplitude of the echo signal. For the sake of numerical comparison on a computer, a theoretical quantity, $\Delta \sigma^{\text{th}}$, corresponding to the experimental quantity, $\Delta \sigma^{\text{ex}}$ in Eq. (1), is introduced, which is likewise defined as the normalized change in the predicted amplitude (whose detail is available in Ref. [16]).

An objective function F is defined by the theoretical and experimental quantities as follows:

$$F = \sum_{i=1}^{n} \left\{ \left(\Delta \sigma^{ex} \right)_{i} - \left(\Delta \sigma^{th} \right)_{i} \right\}^{2}$$
 (6)

where n is the total number of measurements and $(\Delta \sigma^{\rm ex})_{\rm i}$ and $(\Delta \sigma^{\rm th})_{\rm i}$ are the corresponding values of $\Delta \sigma^{\rm ex}$ and $\Delta \sigma^{\rm th}$ obtained in different xpositions (Fig. 4). The value of the objective function is thus calculated by comparing the experimental results with that of the theoretical prediction, and finally, an optimization algorithm (Powell's numerical optimization algorithm)[17] is used to minimize the function F on a personal computer. The best estimate of the true values of the parameters is taken to be the one which minimizes the difference between the predicted and measured amplitudes in the sum square sense. Figure 6 illustrates the flow chart for determining the unknown parameters in the evaluation by analyzing the inverse problem. Evaluated parameters for five fatigue specimens along with the actual crack sizes are presented in Table 1. It has been verified through the 9application that tightly closed fatigue cracks of depth from 1mm the smallest one to 4mm have been evaluated with good accuracy as shown in Table 1 in relation to the stress closing the crack by analyzing an inverse algorithm.

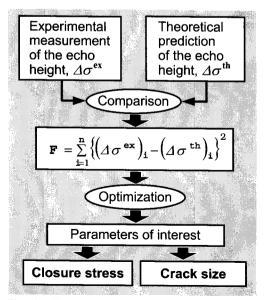


Fig. 6 Flow chart for the analysis of inverse problem

Table 1 Evaluated results based on the analysis of inverse problem of ultrasonic testing and the actual crack size

Specimen	Actual crack depth (mm)	Evaluated crack depth (mm)	Normalized closure stress
F2	1.00	1.00	-0.58
F3	1.76	1.70	-0.57
F4	2.15	2.56	-0.56
F5	2.21	2.05	-0.58
F6	4.00	4.13	-0.50

3. Closely Coupled Probes Potential Drop Technique for NDE of A Small Crack

D-c potential drop (DCPD) technique has recently interested many researchers as a powerful tool for quantitative NDE of cracks. It has been proved that the DCPD technique can evaluate a closed crack [18]. The potential drop technique uses four probes: a set of two probes for current input and output, and another set of two probes for measuring potential drop. While the potential drop is measured near the crack, the current input and output probes are usually located in a large distance from the crack to make a uniform current flow in the region far from the crack; see the work of Johnson [19] as a typical example.

The technique using four probes which are in close proximity to each other has been proposed by Saka et al. [20]; that is the closely coupled probes potential drop (CCPPD) technique. It has been shown by treating a 2-D surface crack problem, see Fig. 7, that the sensitivity of DCPD technique is enhanced significantly in comparison with the usual method using a uniform current flow in the region far from the crack.

Furthermore, the CCPPD technique has recently been developed to a 3-D crack as shown in Fig. 8 [21]. Figure 9 shows an apparatus, which enables us to evaluate the depth of a 3-D crack in a moment, based on the CCPPD

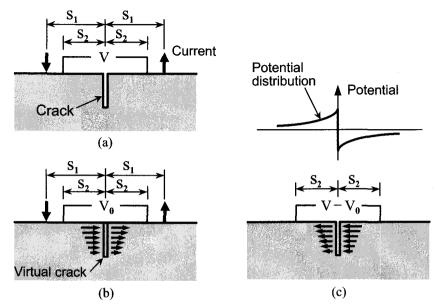


Fig. 7 Superposition in the electric crack problem: problem (a) is divided into two sub-problems of (b) and (c)

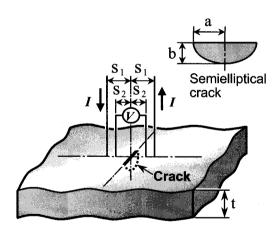


Fig. 8 A 3-D surface crack and closely coupled probes

technique. Table 2 illustrates some examples of evaluated results in comparison with the actual crack depths, where the distance between current input and output probes was 3 mm and that of potential measuring probes was 2 mm. One can see good agreement of the evaluated crack depths with the actual values.

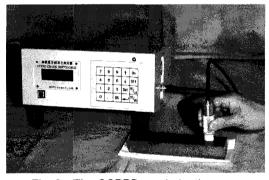


Fig. 9 The CCPPD crack depth gage

4. On NDE of Materials Degradation

Material is known to spend most of cycles in fatigue life for the process of damage accumulation which happens prior to crack occurrence. Once a crack appears, the material breaks with relatively less cycles. NDE of early fatigue damage is a big subject which has recently been investigated by many researchers. This subject requires us to develop highly sophisticated techniques. The purpose of NDE of

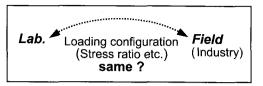
Table 2	Evaluated results by using CCPPD
	technique and the crack size observed
	on the fractured surface

Specimen	Actual crack length (mm)	Actual crack depth (mm)	Evaluated crack depth (mm)
B-1	15.8	3.4	3.0
B-2	16.1	3.7	3.7
B-3	11.2	2.1	2.3
B-4	11.3	1.8	1.9
B-5	10.8	1.6	1.7
B-6	10.8	1.8	1.6
B-7	10.2	1.3	1.1
B-8	18.1	4.3	4.1
B-9	17.6	4.2	3.9

early fatigue damage is on earlier estimation of the remaining life of inspected material. This estimation will be performed in the following procedure:

- (1) The relationship between NDE quantity such as ultrasonic wave velocity or attenuation, for examples, and N/N_f is obtained, where N is fatigue cycle and N_f the cycle to failure. The relationship is obtained by using laboratory specimens.
- (2) The NDE quantity is measured on an actual structure in industry.
- (3) By substituting the measured value of the NDE quantity into the relationship obtained in step (1), one can get the value of N/N_f for the actual structure. Then by using known value of N, the value of N_f and the remaining life, $N_f N$, can be obtained.

However, it is noted that for making the above-stated estimation reliable, the loading configuration such as stress ratio for laboratory specimen in step (1) should be the same as the configuration for the actual structure in step (2). Different loading configurations result in different relationships of NDE quantity with N/N_f as shown in Fig. 10. Also N_f takes different values for different loading configurations. In this way, the estimation of the remaining life is affected by loading configuration and it is not



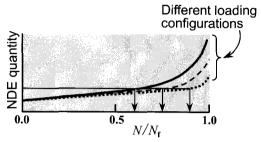


Fig. 10 NDE of early fatigue damage

easy to simulate in laboratory the loading configuration for the actual structure. Results obtained by using laboratory specimens with a loading configuration which is different from the field conditions cannot be used to estimate the remaining life.

On the other hand, NDE of a small crack which happens as a result of materials degradation is independent of loading configuration, where laboratory experiment is directly connected to field measurement. If the critical size of small crack is given, it can be used as a measure of fatigue life. Hence one can say that key strategy is just to perform reliable NDE of a small crack and to relate reasonably of small a crack evaluated nondestructively to the critical one.

5. Conclusion

Two typical techniques developed recently for sensitive NDE of small cracks were described, which treated an isolated crack. Material sometimes causes multiple cracks due to fatigue. In order to predict service lives of structures containing multiple cracks, it is necessary to use suitable methods to evaluate the cracks quantitatively. It is noted that a technique has been developed for NDE of 3-D multiple cracks [22].

Acknowledgement

The author would like to thank Mr. T. Shōji and Mr. A. S. Akanda of Tohoku University for their help with the preparation of the manuscript.

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