論文
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A Study of the Equivalence Problem in ξ_{Σ}^{0} Class

(ξ_{Σ}^{0} 등급에서의 동치문제 연구)

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ABSTRACT

In this paper, some interesting aspects of Grzegorczyk classes ξ_{Σ}^{n}, n $\geq$0 & Σ= { 1, 2 } of word-theoretic primitive recursive functions are observed including the classes of its corresponding predicates (ξ_{Σ}^{n})$_*$. In particular, the small classes ξ_{Σ}^{n} (n $\leq$ 2) are very incomparable to the corresponding small classes ξ^{n} where ξ^{n} is the number-theoretic Grzegorczyk classes. As one of some interesting aspects of the small classes, we show that the equivalence problem in ξ_{Σ}^{0} is undecidable.

요 약

이 논문에서는 기존의 number-theoretic 순환함수와 연계된 word-theoretic 순환함수 및 술어(predicates)들의 Grzegorzyk 클래스를 논한다. 특히 small 클래스 ξ_{Σ}^{n} (n $\leq$ 2)에서의 특성은 그에 대응하는 number-theoretic small 클래스 ξ^{n} 과는 매우 틀린 특성을 보인다 [2]. 흥미 있는 문제 중의 하나인 ξ_{Σ}^{0} 등급에서의 동치문제는 undecidable 임을 증명한다.

1. Introduction and Some Definitions

In [1], [5] and [6] primitive recursive word-theoretic functions and predicates are well defined. In particular, Asser shows that the class of the primitive recursive word-theoretic functions is essentially the same as that of the primitive recursive number-theoretic functions when word-theoretic functions are naturally interpreted in "number- theoretic" terms[1].

Throughout this paper, Grzegorczyk classes of word-theoretic functions and predicates will be focused.

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1.1 Definition

We shall denote by Σ^* the set of all words (or strings), including the empty string $\wedge$, over an alphabet Σ. Σ+ denotes Σ^* - $\{\wedge\}$.

A word-theoretic function f^{Σ} is f^{Σ}: $(\Sigma^*)^k \rightarrow \Sigma^*$ for some $k \geq 1$. //

Throughout this paper, we restrict our attention to the alphabet $\Sigma=\{1, 2\}$ for simplicity, since most properties we deal with are alphabet invariant for an alphabet with more than one element. Moreover in string manipulation it will be more convenient to interpret words in the number-theoretic terms by means of the dyadic notation.

1.2 Definition

The function $\alpha : \Sigma^* \rightarrow N$, which denotes the numerical interpretation of a word, is defined as follows:

$\alpha(\wedge) = 0$

$\alpha(x1) = 1 + 2 \cdot \alpha(x)$

$\alpha(x2) = 2 + 2 \cdot \alpha(x)$. //

It is easy to see that the function α is bijective. Now let $x \in \Sigma^+$ be

$x= a_k\ a_{k-1} \cdots a_1\ a_0$ where $a_i \in \Sigma^+$, $0 \leq i \leq k$.

Clearly, $\alpha(x) = a_0 + 2 \cdot a_1 + \cdots + 2^k \cdot a_k$ by the above definition.

Conversely, under the dyadic notation each positive integer n is represented by a unique string $x= a_k\ a_{k-1} \cdots a_1\ a_0 \in \Sigma^+$ such that

$n = \sum_{i=0}^{k} a_i \cdots 2^i$ for some $k \geq 0$.

Since $\alpha(x) = n$, without ambiguity we shall view each $n \in N$ as both a number and a string - the latter being the numeral value of x i.e. $\alpha(x)$.

Similarly each word-theoretic function f^{Σ}: $(\Sigma^*)^k \rightarrow \Sigma^*$ corresponds to a unique number-theoretic function $f : N \rightarrow N$ such that $f(n) = \alpha(f^{\Sigma}(\alpha^{-1}(n)))$. Hence, hereafter, we shall drop the superscript Σ and denote both by f(x).

For instance, $\lambda_x[x+1]$ is meaningful even for $x \in \Sigma^*$; in that case $\lambda_x[\alpha^{-1}(\alpha(x) + 1)]$ is understood. In English: **"given a string x first interpret as number and add one. Return the dyadic notation of the result; i.e. return a string".**

We shall now define word-theoretic operations over functions.

<u>**Primitive recursion on notation**</u>

Let the function g, h_1, h_2 be given. We say that a function f is defined from g, h_1, h_2 by primitive recursion on notation if and only if for all $\bar{x}$, y

$f(\bar{x}, \wedge) = g(\bar{x})$

$f(\bar{x}, y1) = h_1(\bar{x}, y, f(\bar{x}, y))$

$f(\bar{x}, y2) = h_2(\bar{x}, y, f(\bar{x}, y))$ where $\bar{x} = (x_1, x_2, \cdots, x_n)$. //

Let's consider the successor function $s_i(x) \overset{def}{=} xi$(or ix). The $s_i(x) = ix$(resp. xi) is called the i-th right successor (resp. the i-th left successor). It is easy to see that the successor function $s_i(x)$ is right (resp. left) primitive recursion on notation.

We will see that left and right primitive recursion on notation are equivalent under some reasonable assumptions. We proceed for a while with right recursion on notation.

1.3 Definition

Primitive Recursive Word-theoretic function (PRW)

PRW is the smallest class containing the initial functions:

$z(x) = \Lambda$, $s_1(x) = x1$, $s_2(x) = x2$,

$U^i_n(\bar{x}) = x_i$

and being closed under the following operations:

(A) composition

(B) primitive recursion on notation.//

Example. The function $conc(x, y) = xy \overset{def}{=}$ "concatenation of x and y" is in PRW since the function can be obtained from primitive recursion on notation, i. e.

$conc(x, \Lambda) = U^1_1(x)$

$conc(x, y1) = s_1(U^3_3(x, y, conc(x, y)))$

$conc(x, y2) = s_2(U^3_3(x, y, conc(x, y)))$. //

The above example is equivalent to the following scheme:

$conc(x, \Lambda) = x$

$conc(x, y1) = s_1(conc(x, y))$

$conc(x, y2) = s_2(conc(x, y))$.

Word-theoretic Grzegorczyk hierarchy of primitive recursive functions

We shall introduce the word-theoretic Grzegorczyk hierarchy defined in [5]. Before doing this, we shall define a sequence of "growth functions" on the strings and the notion of "limited recursion on notation".

1.4 Definition

A version of Ackermann functions [10]

A sequence of Ackermann functions $A_n: (\Sigma^*)^2 \rightarrow \Sigma^*$, $n \in N$ is defined by

$A_0(x, y) = y1$

$A_1(x, \Lambda) = x$

$A_2(x, \Lambda) = \Lambda$

$A_n(x, \Lambda) = 1$ if $n \geq 3$

for $n \geqq 0$,

$A_{n+1}(x, y1) = A_n(x, A_{n+1}(x, y))$

$A_{n+1}(x, y2) = A_n(x, A_{n+1}(x, y))$.//

1.5 Definition

Limited (right) recursion on notation

Let the functions g, h_1, h_2, j be given. We say that a function f is defined form g, h_1, h_2, j by limited recursion on notation iff

$f(\bar{x}, \Lambda) = g(\bar{x})$

$f(\bar{x}, y1) = h_1(\bar{x}, y, f(\bar{x}, y))$

$f(\bar{x}, y2) = h_2(\bar{x}, y, f(\bar{x}, y))$

$| f(\bar{x}, y) | \leq | j(\bar{x}, y) |$

where $|x|$ denotes the length of $x \in \Sigma^*$.//

Note that under limited recursion on notation, the length of $f(\bar{x}, y)$ must be bounded by that of $j(\bar{x}, y)$.

1.6 Definition

Word-theoretic Grzegorczyk hierarchy ξ^n_Σ

ξ^n_Σ, $n \geq 0$ is the smallest class containing the initial functions:

$z(x)$, $s_1(x)$, $s_2(x)$, $U^i_n(\bar{x})$, $A_n(x, y)$

and being closed under the following operations:

(A) composition

(B) limited recursion on notation.//

Example. λxy [cond(x, y, z)] is in ξ^0_Σ, where cond(x, y, z) $\overset{def}{=}$ "if x=$\wedge$ then y else z ".

We can see by showing the following operation:

cond($\wedge$, y, z) = y

cond(x1, y, z) = z

cond(x2, y, z) = z

$|\lambda xy\ [\,cond(x, y, z)\,]\ | \le |\ \lambda y\ [\ s_1^z(y)\ ]\ |$

where $s_1^z(y)=s_1(s_1(\cdots(s_1(y))\cdots)$.

z times

The function $s_1^z(y)$ is simply obtained in ξ^0_Σ through composition.//

2. Preliminaries and Some Results

In the definition of the above classes, **limited right recursion on notation** is used. We can easily show that ξ^0_Σ is also closed under **limited left recursion on notation** as follows. Observe that the left-successor $\widehat{s}_1(x)=1x$ (resp. $\widehat{s}_2(x)=2x$) belongs to ξ^0_Σ since

$\widehat{s}_1(\wedge)=1$ resp. $\widehat{s}_2(\wedge) = 2$

$\widehat{s}_1(x1)= s_1(\ \widehat{S}_1(x))$ $\widehat{s}_2(x1)= s_1(\ \widehat{S}_2(x))$

$\widehat{s}_1(x2)= s_2(\ \widehat{s}_1(x))$ $\widehat{s}_2(x2) = s_2(\ \widehat{S}_2(x))$

$|\ \widehat{s}_1(x)| \le |A_0(x, x)|| \ \widehat{s}_2(x)| \le |A_0(x, x)|$.

In general, we will now show that ξ^0_Σ, n ≥ 0 is also preserved the same property.

2.1 Proposition

ξ^n_Σ, n$\ge$0 is closed under limited left recursion on notation.

Proof: *We shall define an auxiliary function* rev(x) $\overset{def}{=}$ "the word x in reversed order" by a scheme in ξ^0_Σ as follows:

rev($\wedge$) = $\wedge$

rev(x1) = $\widehat{s}_1$(rev(x))

rev(x2) = $\widehat{s}_2$(rev(x))

| rev(x) | $\le$ | A_0(x, x) | .

Hence rev $\in \xi^n_\Sigma$, n $\ge$0.

Let us assume that a function f is defined from the functions g, h_1, h_2, j by limited left recursion on notation. To see that function f can be also defined by limited right recursion on notation, we shall define a function $\hat{f}$ by the scheme of (right) recursion on notation as follows:

$\hat{f}(\bar{x},\wedge) = g(\bar{x})$

(A) $\hat{f}(\bar{x}, y1) = \widehat{h}_1(\bar{x}, y, \hat{f}(\bar{x}, y))$

$\hat{f}(\bar{x}, y2) = \widehat{h}_2(\bar{x}, y, \hat{f}(\bar{x}, y))$

where $\widehat{h}_i(\bar{x}, y, z) \overset{def}{=}$

$h_i(\bar{x}$ rev(y), z) i = 1, 2 .

Now we will show by induction on the formation of y that f($\bar{x}$, y) = $\hat{f}$($\bar{x}$, rev(y)) holds for all $\bar{x}$, y. It is true when y = $\wedge$.

We assume f($\bar{x}$ y)= $\hat{f}$($\bar{x}$, rev(y)).

For 1y :

f($\bar{x}$, 1y) = h_1($\bar{x}$, y, f($\bar{x}$, y))

= h_1($\bar{x}$, rev(rev(y)), $\hat{f}$($\bar{x}$, rev(y)))

/* by assumption */

= $\widehat{h}_1$($\bar{x}$, rev(y), $\hat{f}$($\bar{x}$, rev(y)))

/* by scheme (A) */

= $\hat{f}$($\bar{x}$, rev(y)1)

= $\hat{f}$($\bar{x}$, rev(1y)).

Similarly for 2y , f($\bar{x}$, 2y) = $\hat{f}$($\bar{x}$, rev(2y)).

Also, | $\hat{f}$($\bar{x}$, rev(y)) | $\le$ | j($\bar{x}$, y) | since |

It follows that $|\ \mathfrak{I}(\overline{x}, y)\ | \leq |\ j(\overline{x}, rev(y))\ |$.

Since $rev(y) \in \xi^{n}_{\Sigma}$ so does $j(\overline{x}, rev(y))$ and since $\mathfrak{I}$ was defined with right recursion, $\mathfrak{I} \in \xi^{n}_{\Sigma}$.

Hence $f(\overline{x}, y) = f(\overline{x}, rev(y)) \in \xi^{n}_{\Sigma}$.

Therefore ξ^{n}_{Σ} is closed under limited left recursion on notation when $n \geq 0$. //

By the above proposition , we are able to use freely both limited left and right recursion on notation in ξ^{n}_{Σ} , $n \geq 0$.

2.2 Proposition

The following table shows in which classes some important functions are contained:

Functions	level n of ξ^{n}_{Σ}
(1) $z(x)$, $s_1(x)$, $s_2(x)$, $U^{i}_{n}(\overline{x})$	0
(2) $id(x) \overset{def}{=}$ "identity function"	0
(3) $rev(x) \overset{def}{=}$ "the word in reversed order"	0
(4) $init(x) \overset{def}{=}$ "the word after removing the rightmost letter of x"	0
(5) $last(x) \overset{def}{=}$ " the rightmost letter of x "	0
(6) $\lvert x\rvert \overset{def}{=}$ " the length of x " ($\lvert\Lambda\rvert = \Lambda$)	0
(7) $\lambda xy\,[\,cond(x, y, z)\,]$	0
(8) $1 \dot{-} x$, $x \dot{-} 1$, $x \dot{-} 1$	0
(9) $conc(x, y) \overset{def}{=}$ "concatenation of x and y"	1
(10) $add(x, y) = x + y$, $mult(x, y) = x \cdot y$	1

Proof: We leave the above proof to reader. The detailed proof is shown in [2].//

Definition 2.1

A predicate (relation) $R(\overline{x})$ is in ξ^{n}_{Σ} iff there is a function f in ξ^{n}_{Σ} such that $R(\overline{x}) \equiv f(\overline{x}) = \Lambda$.

$(\xi^{n}_{\Sigma})_*$ stands for the class of predicates of ξ^{n}_{Σ} .//

We define the characteristic function χ_R for a predicate $R(\overline{x})$ by

$$\chi_R(\overline{x}) = \begin{cases} \Lambda, & \text{if } R(\overline{x}) \text{ is true} \\ 1, & \text{otherwise} \end{cases}$$

Corollary 2.3

For every $n \geq 0$, a predicate $R(\overline{x})$ is in $(\xi^{n}_{\Sigma})_*$ if and only if its characteristic function χ_R is in ξ^{n}_{Σ}.

Proof: Let $R(\overline{x})$ be in $(\xi^{n}_{\Sigma})_*$. Then by definition there is a function f in ξ^{n}_{Σ} such that $R(\overline{x}) \equiv f(\overline{x}) = \Lambda$. The characteristic function $\chi_R(\overline{x}) = 1 \dot{-} (1 \dot{-} f(\overline{x})) \in \xi^{n}_{\Sigma}$ since $f(\overline{x})$ is. Conversely, let $\chi_R(\overline{x}) \in \xi^{n}_{\Sigma}$. Since $R(\overline{x}) \equiv \chi_R(\overline{x}) = \Lambda$, it follows that $R(\overline{x}) \in (\xi^{n}_{\Sigma})_*$.//

Proposition 2.4

$(\xi^{n}_{\Sigma})_*$ for $n \geq 0$ is closed under the boolean operations $\neg$, $\vee$, &, $\rightarrow$ and $\leftrightarrow$.

Proof: Let $R(\overline{x})$ and $Q(\overline{x})$ be in $(\xi^{n}_{\Sigma})_*$. Also consider the corresponding characteristic unctions χ_R, χ_Q for $R(\overline{x})$ and $Q(\overline{x})$. It is clear that the functions $\chi_{\neg R}$ and $\chi_{R \vee Q}$ defined by

$\chi_{\neg R}(\overline{x})$ = if $\chi_R(\overline{x}) = \Lambda$ then 1 else Λ

$\chi_{R \vee Q}(\overline{x})$ = if $\chi_R(\overline{x}) = \Lambda$ then Λ else $\chi_Q(\overline{x})$ are the characteristic functions in ξ^{n}_{Σ}, n

≥ 0 for the predicates $\neg R(\bar{x})$ and $R(\bar{x}) \vee Q(\bar{x})$. Hence $(\xi^n_\Sigma)_*$, $n \geq 0$ is closed under all boolean operations since all other operations can be defined by means of $\neg$, $\vee$.//

Example: The predicates $x = \wedge$, $x = 1$ and $x = 2$ are in $(\xi^0_\Sigma)_*$.

By showing the corresponding characteristic functions for each predicate, we can prove "the predicate $x = \wedge$" as follows: The characteristic function $\chi_\wedge$ is obtained in ξ^0_Σ by

$\chi_\wedge(\wedge) = \wedge$

$\chi_\wedge(x1) = 1$

$\chi_\wedge(x2) = 1$

$|\chi_\wedge(x)| \leq |s_1(x)|$.

Similarly, we can show that the other predicates x=1 and x=2 are in $(\xi^0_\Sigma)_*$. by easily providing the corresponding characteristic functions.//

The following definition is extremely useful in dealing with our final result in the small class ξ^0_Σ (resp. $(\xi^0_\Sigma)_*$).

Definition 2.2

Let x, y be words over Σ^*. x is said to be "a part of y", written xPy if y=uxv for some words u, v $\in \Sigma^*$. We say that "x begins y"(written xBy), "x ends y" (written xEy) if y=xv, y=ux respectively for some words u, v $\in \Sigma^*$. Note that u, v can be empty.

Further, $(\exists y)_{Bz}$, $(\exists y)_{Ez}$, $(\exists y)_{Pz}$ stand for "there is a $y \in \Sigma^*$ which begins z, ends z and is a part of z such that" respectively. Similarly, we also define for $(\forall y)$. //

Proposition 2.5

For every $n \geq 0$, if $R(\bar{x}, y)$ is in $(\xi^n_\Sigma)_*$ then $(\exists y)_{Bz} R(\bar{x}, y)$ is in $(\xi^n_\Sigma)_*$.

Proof: Let $\chi_R(\bar{x}, y)$ be the characteristic function for $R(\bar{x}, y)$. We shall define the characteristic function $\hat{\chi}(\bar{x}, z)$ for $(\exists y)_{Bz} R(\bar{x}, y)$ by the following way:

$\hat{\chi}(\bar{x}, \wedge) = \chi_R(\bar{x}, \wedge)$

$\hat{\chi}(\bar{x}, z1) =$ if $\hat{\chi}(\bar{x}, z) = \wedge$ then $\wedge$ else $\chi_R(\bar{x}, z1)$

$\hat{\chi}(\bar{x}, z2) =$ if $\hat{\chi}(\bar{x}, z) = \wedge$ then $\wedge$ else $\chi_R(\bar{x}, z2)$

$|\hat{\chi}(\bar{x}, z)| \leq |s_1(z)|$

Then the function $\hat{\chi}(\bar{x}, z)$ is in ξ^n_Σ if $\chi_R(\bar{x}, y) \in \xi^n_\Sigma$ when $n \geq 0$.

Hence $(\exists y)_{Bz} R(\bar{x}, y)$ is in $(\xi^n_\Sigma)_*$ $n \geq 0$. //

Clearly, $(\forall y)_{Bz} R(\bar{x}, y)$ is also in $(\xi^n_\Sigma)_*$, $n \geq 0$ because of $(\forall y)_{Bz} R(\bar{x}, y) = \neg (\exists y)_{Bz} R(\bar{x}, y)$. In the similar way, we can show that $(\forall y)_{Ez} R(\bar{x}, y)$ and $(\forall y)_{Pz} R(\bar{x}, y)$ are in $(\xi^n_\Sigma)_*$ if $R(\bar{x}, y)$ is in ξ^n_Σ (the detailed proof in [2]).

Proposition 2.8

The following functions or predicates are in ξ^0_Σ or $(\xi^0_\Sigma)_*$ respectively.

(1) Tally (x) $\stackrel{def}{=}$ "x is composed of 1's only or empty"

(2) ones $= 1^{|x|}$ $\stackrel{def}{=}$ "concatenation of 1's as many times as |x|'s" ($1^\wedge = \wedge$)

(3) sub(x, y) $\stackrel{def}{=}$ $1^{|x| \dot{-} |y|}$

(4) $|x| = |y|$, $|x| \neq |y|$, $|x| < |y|$

(5) xBy, xEy, xPy

(6) z = conc (x, y)

Proof: The detailed proof in [2]. //

3. The Equivalence Problem in ξ^0_Σ

The Kleene T-predicate T(z, x, y) is defined in Computability Theory [7] as follows:

T(z,x,y) ≡ "the Turing Machine coded by the number z when presented with input x has a computation coded by the number y".

Definition 3.1 [7]

The class of S-rudimentary predicates is defined by a finite number of operations of rules (1) - (4) as follows:

(1) The ternary predicate z= conc (x,y) is S-rudimentary.
(2) Any explicit transform[1] of an S-rudimentary predicate is S-rudimentary.
(3) If $R(\bar{x})$, $S(\bar{x})$ are S-rudimentary, then so are $\neg R(\bar{x})$, $R(\bar{x}) \vee S(\bar{x})$ and $R(\bar{x})$ & $S(\bar{x})$.
(4) If $R(\bar{x}, y)$ is S-rudimentary, then so are $(\exists y)_{Bz} R(\bar{x}, y)$, $(\exists y)_{Ez} R(\bar{x}, y)$, $(\exists y)_{Pz} R(\bar{x}, y)$. //

Theorem 3.1: The T-predicate is S-rudimentary. [7]

Proposition 3.2

$(\xi^0_\Sigma)_*$ contains the S-rudimentary predicates.

Proof: By substitution rule, it is easy to see that $(\xi^0_\Sigma)_*$ is closed under explicit transform. Furthermore we have already shown that $(\xi^0_\Sigma)_*$ is closed under rules (3) & (4), and also $\lambda xyz [\![z = conc(x, y)]\!] \in (\xi^0_\Sigma)_*$. Thus $(\xi^0_\Sigma)_*$ includes the S-rudimentary predicates. //

Corollary 3.3

$(\xi^0_\Sigma)_*$ contains the T-predicate.

Proof: It follows immediately from theorem 3.1and proposition 3.2. //

Theorem 3.4

The equivalence problem in ξ^0_Σ is undecidable.

Proof: Let $\chi_T(z, x, y)$ be the characteristic function of T (z, x, y). By the corollary 3.2, $\chi_T \in (\xi^0_\Sigma)_*$. Consider the set of functions

$V = \{\lambda y [\![\chi_T(x, x, y)]\!] \}_{x \geq 0} \cup \{\lambda y [\![1]\!] \}$.

The problem $\lambda y [\![\chi_T(x, x, y)]\!] = \lambda y [\![1]\!]$ is one instance of equivalence problem in V, hence also in ξ^0_Σ (since $V \subset \xi^0_\Sigma$). But the above problem is equivalent to "T (x, x, y) false for all y" i.e. to $\Phi x (x) \nearrow$[2] which is known to be non-recursive (actually not even recursively enumerable). Thus the equivalence problem in ξ^0_Σ is undecidable. //

4. Conclusions

In this paper, we investigated the classes of primitive recursive word-theoretic functions (resp. predicates) ξ^n_Σ (resp. $(\xi^n_\Sigma)_*$) including the undecidable problem of ξ^0_Σ. In [2], it was shown that $\xi^n_\Sigma = \xi^n$ for $n \geq 3$ but the small classes ξ^n_Σ ($n \leq 2$) are incomparable to the corresponding small

1) We say that $R(\bar{x}_n)$ is an explicit transform of $Q(\bar{y}_m)$ if there are $\zeta_1, \zeta_2, ..., \zeta_m$ such that $R(\bar{x}_n)$ is true iff $Q(\bar{\zeta}_m)$ is true where for each i = 1,2,...,m either $\zeta_i = x_j$, j=1,2,...,n or ζ_i is a constant..

2) $\Phi_x(x) \nearrow$ means that the x-machine with input x computes forever.

classes of ξ^{n}. Specially, [2] suggests many difficult and several open problems requiring the further study in $(\xi^{n}_{\Sigma})_{*}$ for $n \le 2$.

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