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Using System Reliability to Evaluate and Maintain Structural Systems

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ABSTRACT

A reliability approach to evaluate structural performance has gained increased acceptability and usage over the past two decades. Most reliability analyses are based on the reliability of an individual component without examining the entire structural system. These analyses often result in either unnecessary repairs or unsafe structures. This study uses examples of series, parallel, and series-parallel models of structural systems to illustrate how the component reliabilities affect the reliability of the entire system. The component-system reliability interaction can be used to develop optimum lifetime inspection and repair strategies for structural systems. These examples demonstrate that such strategies must be based on the reliability of the entire structural system. They also demonstrate that the location of an individual component in the system has a profound effect on the acceptable reliability of that component. Furthermore, when a structure is deteriorating over time, the reliability importance of various components is also changing with time. For this reason, the most critical component in the early life of the structure may not be the most critical later.

Keywords: system reliability, cost-based optimal maintenance, inspection/repair/replacement planning, probability

1. Introduction

With aging national infrastructures that require billions of dollars to maintain, structural reliability analysis methods have become more relevant, useful and acceptable. These methods have become more efficient as computers become faster and ubiquitous. Application of such methods has the potential to result in lower cost and greater safety as the uncertainties in loads, strengths, and models are better quantified. Most reliability analyses quantify the safety of a single structural component using a single limit state equation. Most engineering structures are complex redundant systems involving multiple limit states. System reliability evaluation can be quite complex and usually involves the contribution of component failure events to the system failure, the post-failure behavior of components, and the statistical correlation between failure events (Hendawi and Frangopol, 1994).

A series system, also called a weakest-link system, fails when any individual member in the system fails. A chain is only as strong as its weakest link - and that's only true if the failure events are perfectly correlated. If a series system is treated as a series of z elements, the probability of failure of the system P_f is written as the probability of a union of events

$$P_f = (\bigcup_{a=1}^{z} \{g_a(x) \le 0\}) \tag{1}$$

Depending on the correlation between the failure modes, the possible range of values for P_f are (Cornell, 1967)

$$\max[P_f(a)] \le P_f \le 1 - \prod_{a=1}^{z} 1 - P_f(a) \tag{2}$$

The lower bound occurs when the failure modes are perfectly correlated ($\rho = 1.0$) and the upper bound when they are statistically independent ($\rho = 0.0$). Ditlevsen (1979)

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developed tighter bounds using joint-event probabilities which accounted for failure mode correlation. Results in this paper are based on the average of the Ditlevsen's bounds for the probability of failure of a series system.

A parallel system, also called a redundant or fail-safe system, requires every individual member in the system to fail for the system to fail. A fail-safe system is at least as strong as its strongest member - and again that requires perfect correlation. The probability of failure of a parallel system is the probability of an intersection of failure events

$$P_f = (\bigcap_{a=1}^{z} \{g_a(x) \le 0\}) \tag{3}$$

The possible range of values are (Ang and Tang, 1984):

$$\prod_{a=1}^{z} (1 - P_f(a)) \le P_f \le \min[P_f(a)] \tag{4}$$

where the lower bound results from mutual independence and the upper bound from perfect correlation. These bounds are often too wide to provide a useful solution. Since the component reliability results in this paper were obtained by reducing all random variables to their equivalent normal distributions, the parallel system results were found by solving the n-dimensional joint standardized distribution integral (Thoft-Christensen and Murotsu, 1986).

$$P(F_1 \cap F_2 \cap \cdots \cap F_z)$$

$$= \int_{\beta_1}^{\infty}\int_{\beta_2}^{\infty}\cdots\int_{\beta_z}^{\infty} \frac{1}{(2\pi)^{z/2}\sqrt{det[\rho_{sys}]}} e^{-1/2\{\beta\}\{\rho_{sys}\}^{-1}\{\beta\}^T} d\{\beta\} \tag{5}$$

where $\{\beta\} = \{\beta_a, \beta_b, \cdots, \beta_z\}$, ρ_{sys} is the system correlation matrix, and z is the number of members in the parallel system.

Many general engineering systems can be modeled as a combination of series and parallel systems. For example, a series of y parallel systems where each parallel system a has z_a components would have a probability of failure expressed as

$$P_f = P(\bigcup_{a=1}^{y} \bigcap_{b=1}^{z_a} \{g_{ab}(x) \le 0\}) \tag{6}$$

Any complex system can be sequentially broken down into simpler equivalent subsystems. The reliabilities of a series subsystem and parallel subsystem are solved individually as described above using the reliabilities and direction cosines at the points of failure of individual components. An example of how a series-parallel system is reduced is shown in Fig. 1. The system shown contains six components. Initially, the two parallel systems are reduced to an equivalent component which forms part of a series system. The series system is then reduced to a single equivalent component. The correlation of the equivalent components is computed using equivalent alpha vectors which are a function of the equivalent direction cosines as described in Estes (1997). These reductions and computations were performed using RELSYS (Reliability of Systems) (Estes and Frangopol, 1998), a computer program developed at the University of Colorado. Any structural system that can be modeled as a combination of series and parallel components can be analyzed.

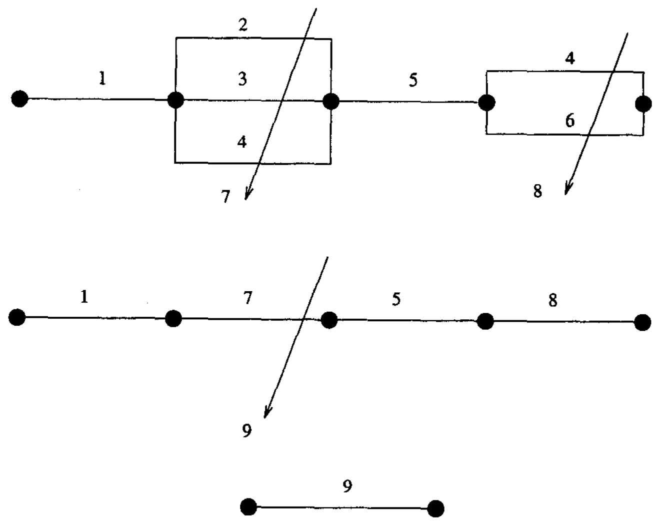

Fig. 1. Reduction of a series-parallel system to an equivalent single component.

2. Series System

Fig. 2 shows a seven bar determinate truss which is modeled as a series system where the failure of a single bar will result in the failure of the entire system. The cross-sectional areas are defined for the bottom chords (A_1), the diagonals (A_2), and the top chord (A_3). The limit state equations based on equilibrium are:

$$g(1) = g(2) = R - 0.5(Q/A_1) = 0 \tag{7}$$

$$g(3) = g(4) = g(5) = g(6) = R - \frac{\sqrt{2}}{2}(Q/A_2) = 0 \tag{8}$$

$$g(7) = R - (Q/A_3) = 0 \tag{9}$$

where R is the resistance of the bars and Q is the load on the truss. The resistance R is a normally distributed random variable with a mean value of 2.0 and a standard deviation

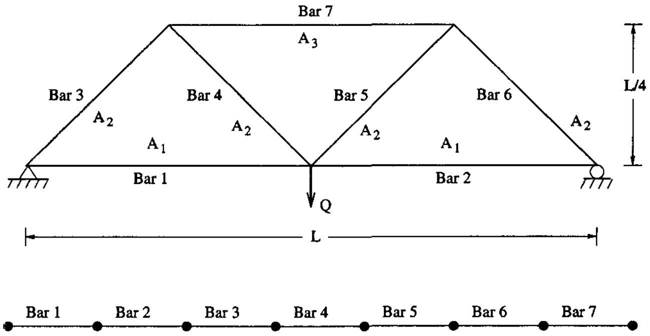

Fig. 2. Seven bar statically determinate truss modeled as a series system.

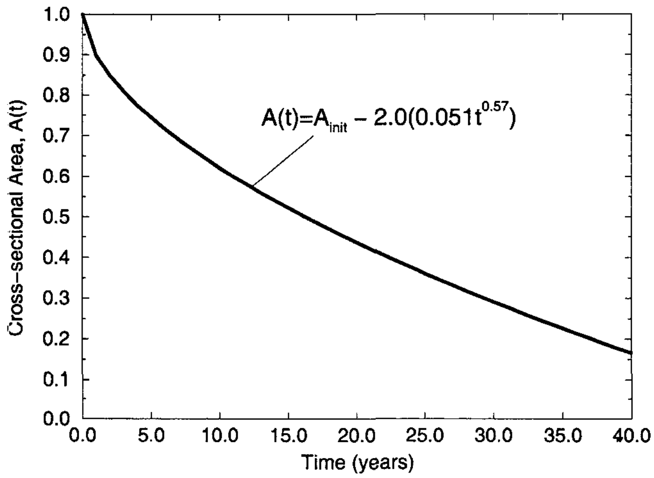

Fig. 3. Deterioration of cross-sectional area over time.

of 0.2, denoted as N[2.0, 0.2]. The distribution parameters for the normally distributed load Q are N[1.0, 0.1]. It is assumed that the resistances of the bars are perfectly correlated and that the bar capacities are the same in both tension and compression. As time passes, bars are deteriorating. The deterioration is an exponential loss of cross-section over time expressed as (Estes, 1997):

$$A(t) = A_{init} - 2.0(0.051t^{0.57}) \tag{10}$$

where $A(t)$ is the area of the bar at any time t and A_{init} is the initial area of the bar prior to deterioration. The deterioration function (10) is shown in Fig. 3 where only half the original area remains after about 17 years. The useful life of the truss is 70 years. The cost of a repair is the sum of a variable cost ($C_{var} = 5.0$) which is charged for each bar repaired and a fixed cost ($C_{fix} = 5.0$) which is charged each time a repair is made. The cost model was deliberately kept simple to observe the trends in the problem. The model could easily be modified to include damage intensity and time value of money (Frangopol *et al.*, 1997).

The minimum allowable system reliability index is arbitrarily chosen as $\beta_{min} = 2.0$. The choice of β_{min} would typically be a complex decision which requires a balancing of initial cost, lifetime maintenance cost, and failure cost as described in Frangopol and Moses (1994). Inspections are conducted every two years and if the results indicate that the reliability of the structure will fall below β_{min}, then remedial action in the form of a repair of at least one component is needed to improve the reliability of the system. In this approach, a component reliability threshold $\beta_{threshold}$ is chosen. When the reliability of the system falls below 2.0 all components whose individual reliabilities fall below $\beta_{threshold}$ are repaired. Again, for simplicity, it is assumed that the repair work is perfect. This implies that the decayed reliability level is restored to the original level. This can be achieved by replacing damaged components with new components. The value of $\beta_{threshold}$ is varied until an optimum solution is obtained based on minimum total cost.

Initially the cross-sectional areas of the bars were chosen so that each component had the same reliability when the truss was placed in service ($A_1 = 1.0$; $A_2 = \sqrt{2}$; $A_3 = 2.0$). Because the bars have different sizes, the reliability of the bars will differ over time as the same depth of deterioration

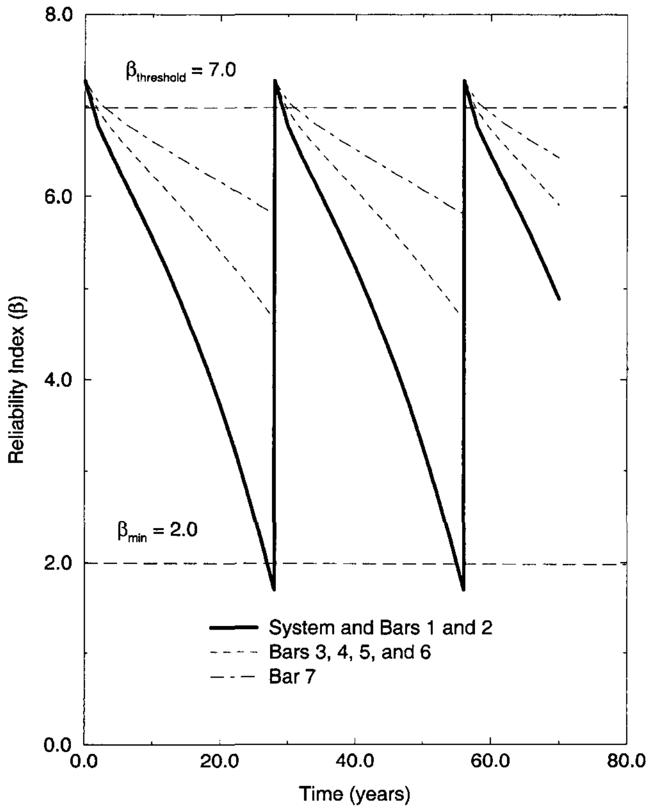

Fig. 4. Repair plan for a seven bar statically determinate truss, $\beta_{threshold} = 7.0$; *Cost* = 80.

has a different effect on these bars. Fig. 4 shows the results when $\beta_{threshold} = 7.0$ which causes every bar to be replaced whenever the system reliability falls below β_{min} and results in a total lifetime cost of 80. Repairs were required after 28 and 56 years of service.

Lowering $\beta_{threshold}$ to 4.0 as shown in Fig. 5 results in repairs after 28 and 50 years of service. Only bars 1 and 2 are replaced during the first repair; bars 1 through 6 are replaced on the second repair and bar 7 is never replaced. The resulting cost is 50 which is the optimum solution. When $\beta_{threshold}$ was lowered to 2.6 (see Fig. 6) a special repair at year 56 was required for bars 1 and 2. These bars did not get repaired after the 50 year inspection which resulted in additional repair cost. In this case, the resulting lifetime cost is 55. If the reliability of any individual member was allowed to fall below $\beta_{threshold} = 2.6$, the system reliability fell below $\beta_{threshold} = 2.0$ value which illustrates the fallacy of evaluating individual components in isolation. Because of the series nature of the system, every component could be well above the minimum prescribed system reliability and the reliability of the system as a whole would be unacceptable. A truss which started its deterioration with all bars having equal initial areas was

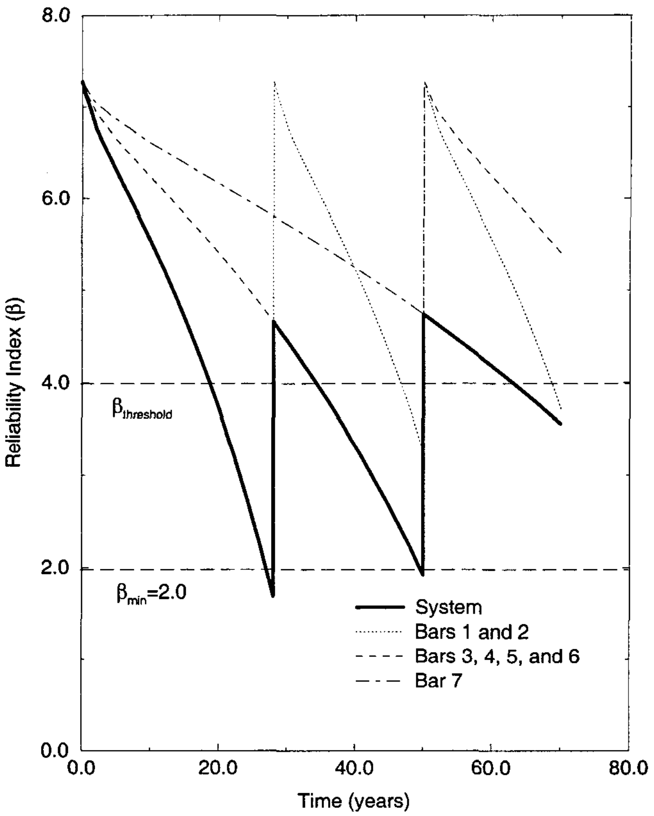

Fig. 5. Repair plan for a seven bar statically determinate truss, $\beta_{threshold} = 4.0$; *Cost* = 50.

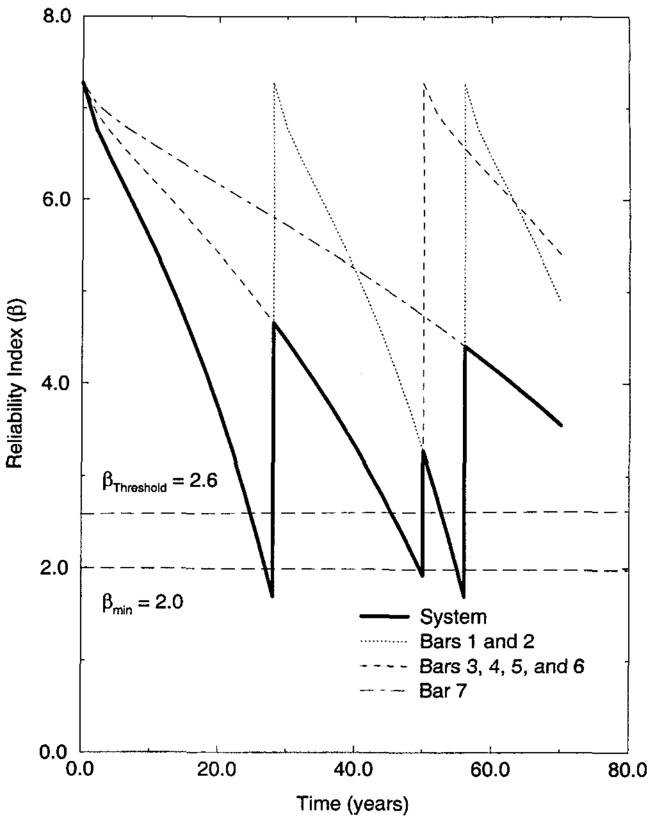

Fig. 6. Repair plan for a seven bar statically determinate truss, initial equal importance of bars; $\beta_{threshold} = 2.6$; *Cost* = 55.

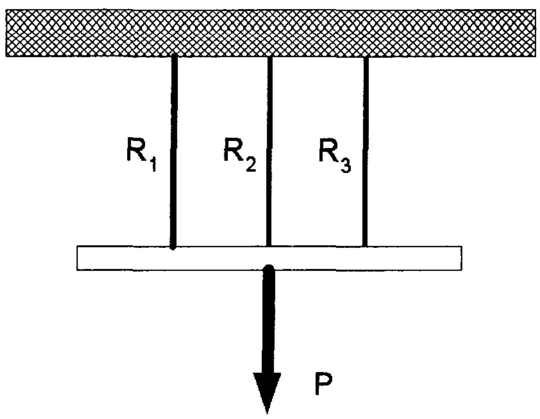

Fig. 7. A three bar parallel system.

also examined in Estes (1997) and the results were similar.

3. Parallel System

The three bar system shown in Fig. 7 was analyzed. In this case, all three bars must fail for the system to fail. The same useful life of 70 years, the same deterioration model, and the same minimum system reliability of $\beta_{min} = 2.0$ were imposed. The load P and bar resistances R are nor-

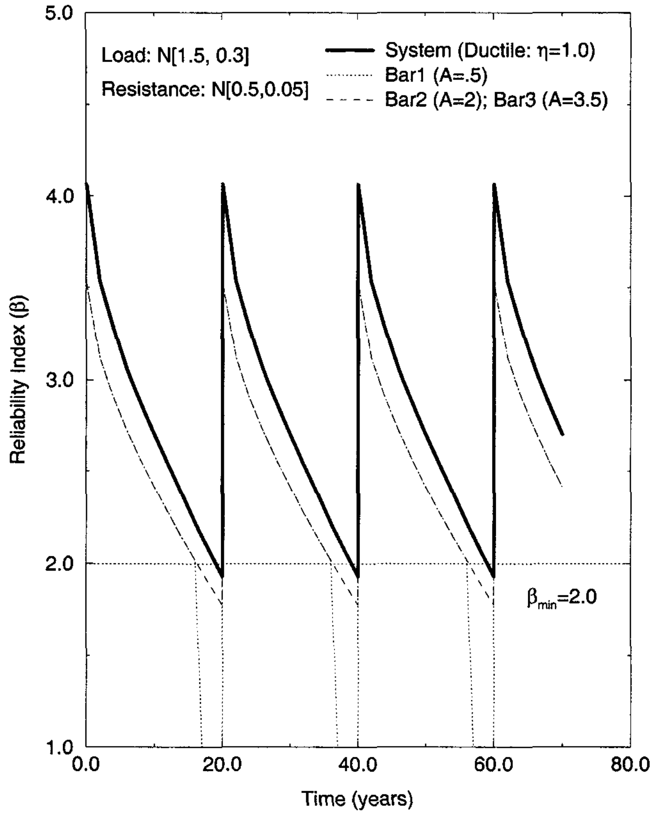

Fig. 8. Repair plan for a ductile three bar parallel system with unequal bar areas $A_1 = 0.5$, $A_2 = 2.0$, $A_3 = 3.5$, load: N[1.5, 0.3], resistance: N[0.5, 0.05], uncorrelated resistances.

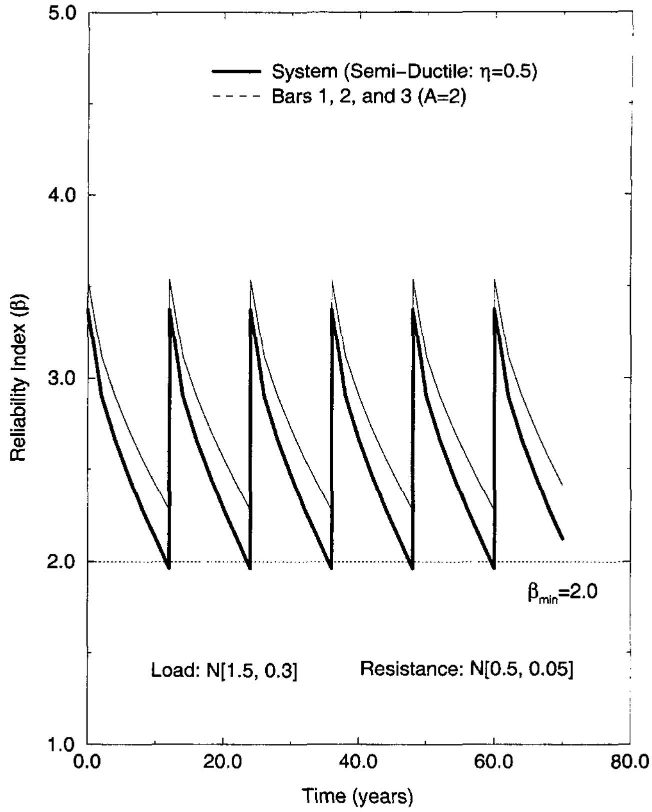

Fig. 9. Repair plan for a semi-ductile three bar parallel system with equal bar areas A = 2.0; load: N[1.5, 0.3], resistance: N[0.5, 0.05], no correlation.

mally distributed, N(1.5,0.3) and N(0.5,0.05), respectively. All variables are uncorrelated. Fig. 8 shows the results for a parallel system with different bar areas (A_1 = 0.5; A_2= 2.0;A_3 = 3.5). Despite different cross sectional areas, the individual bars all have identical component reliabilities due to load redistribution which eliminated the usefulness of $\beta_{threshold}$. The reliabilities of all bars will remain equal regardless of which member gets repaired. For a ductile system, the individual component reliabilities are all allowed to fall below β = 2.0 while the reliability of the system remains above β_{min} = 2.0. In fact, bar 1 with the smallest area completely vanishes and the reliability of the system is still acceptable.

The same system was analyzed for different material behaviors as shown in Fig. 9. A post-elastic behavior factor η is introduced which reports post-elastic capacity and ranges from η = 1.0 (ductile material) to η = 0.0 (brittle material). As the material becomes more brittle (see Fig. 10) the individual component reliabilities must be higher than η = 2.0 to maintain a minimum system reliability of 2.0.

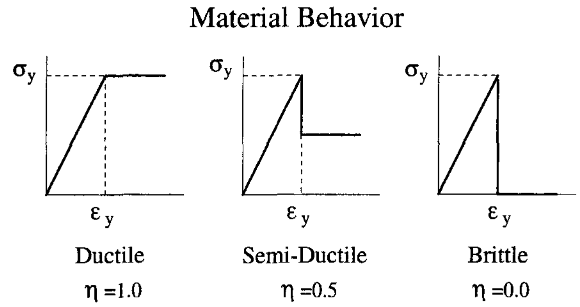

Fig. 10. Material behavior using a post-elastic behavior factor η.

4. Series-Parallel System

The three bar indeterminate truss shown in Fig. 11 is modeled as a series-parallel system where the failure of any two bars will cause failure of the system. Let the event "Bar 2|1" indicate failure of bar 2 given that bar 1 has already failed, and "Bar 1" indicate failure of bar 1. The useful life is 70 years and the allowable system reliability level is β_{min} = 2.0. The resistances of the three bars are: $R_{bar1} \rightarrow N[15, 1.5]$, $R_{bar2} \rightarrow N[15, 1.5]$, $R_{bar3} \rightarrow N[10, 1.0]$.

The post-elastic behavior factor (see Fig. 10) is again taken into account.

The load on the truss is P with parameters N[20, 4.0]. The limit state equations which describe the components in the series-parallel model are:

(a) Prior to any bars failing (Bar 1, Bar 2, Bar 3)

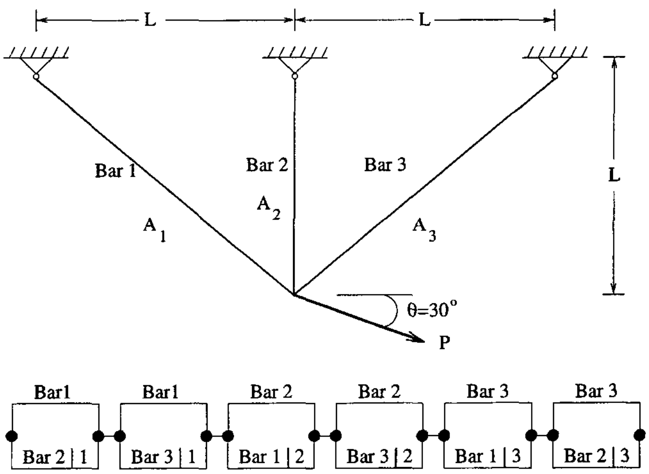

Fig. 11. Three bar indeterminate truss modeled as a series-parallel system.

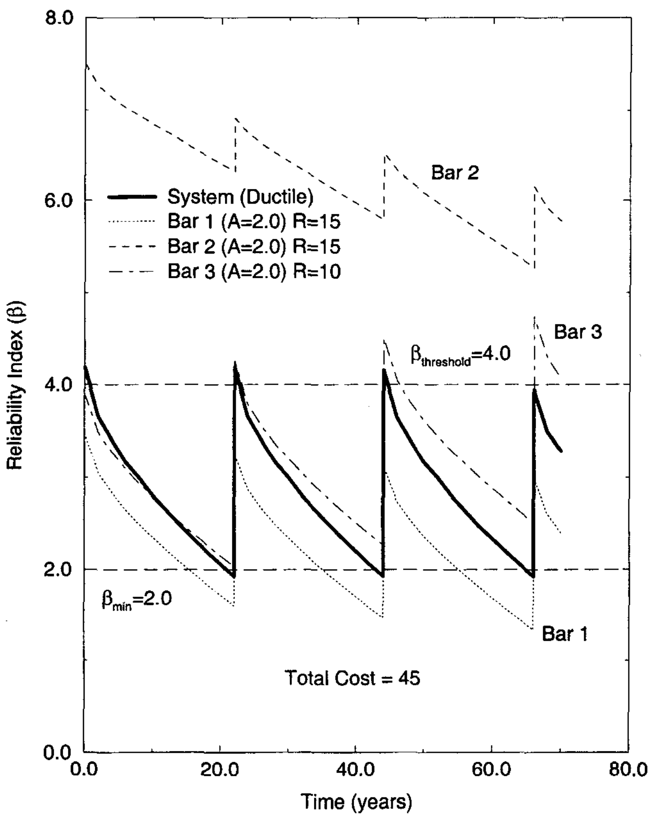

Fig. 12. Repair plan for a ductile three bar indeterminate truss with equal bar areas; $\beta_{threshold} = 4.0$; $Cost = 45$.

$$g(1) = R_1(2.0A_1A_3 + \sqrt{2}(A_1A_2 + A_3A_2)) - \sqrt{2}PA_1(\cos\theta + \sin\theta) - 2.0PA_2\cos\theta = 0 \quad (11)$$

$$g(2) = R_2(2.0A_1A_2 + \sqrt{2}(A_1A_2 + A_3A_2)) - \sqrt{2}P((A_1 - A_3)cos\theta + (A_1 + A_3)\sin\theta) = 0 \quad (12)$$

$$g(3) = R_3(\sqrt{2}A_1A_3 + A_1A_2 + A_3A_2) + P(A_3\sin\theta - A_3\cos\theta - \sqrt{2}A_2\cos\theta) = 0 \quad (13)$$

(b) Given failure of bar 1 (Bar 2|1, Bar 3|1):

$$g(4) = R_2A_2 - P(\sin\theta + \cos\theta) + \sqrt{2}\eta R_1A_1 = 0 \quad (14)$$

$$g(5) = R_3A_3 - P\sqrt{2}\cos\theta + \eta R_1A_1 = 0 \quad (15)$$

(c) Given failure of bar 2 (Bar 1|2, Bar 3|2):

$$g(6) = R_1A_1 - \sqrt{2}/2P(\sin\theta + \cos\theta) + \eta R_2A_2 = 0 \quad (16)$$

$$g(7) = R_3A_3 - \sqrt{2}/2P(\cos\theta - \sin\theta) + \eta R_2A_2 = 0 \quad (17)$$

(d) Given failure of bar 3 (Bar 1|3, Bar 2|3):

$$g(8) = R_1A_1 - P\sqrt{2}\cos\theta + \eta R_3A_3 = 0 \quad (18)$$

$$g(9) = R_2A_2 - P(\sin\theta + \cos\theta) + \sqrt{2}\eta R_3A_3 = 0 \quad (19)$$

Using the same cost model described earlier, the optimum repair strategy ($\beta_{threshold} = 4.0$, Cost = 45) for a ductile truss is shown in Fig. 12 where bars 1 and 3 are replaced three times during the life of the structure and bar 2 is never replaced. Larger values of $\beta_{threshold}$ caused bar 2 to be needlessly replaced. Attempting to lower $\beta_{threshold}$ to the point where bar 3 does not get replaced resulted in a repair plan where the minimum system safety could not be maintained for the life of the structure.

In Fig. 12, the reliability of Bar 1 is permitted to fall below the minimum reliability of the system $\beta_{min} = 2.0$ because of the redundant nature of the (ductile) system. Even though bar 2 is never replaced, the reliability of bar 2 is increased as other bars are replaced and are thus able to assume more of the load on the structure. Similarly as bar 2 deteriorates, the load distribution is altered. This decreases the reliability of bar 1 but increases the reliability of bar 3.

If the components of the truss shown in Fig. 11 are brittle, the optimum solution is shown in Fig. 13 where $\beta_{threshold} = 4.0$ and Cost = 90. With no post-elastic capacity, no individual component reliability was allowed to fall

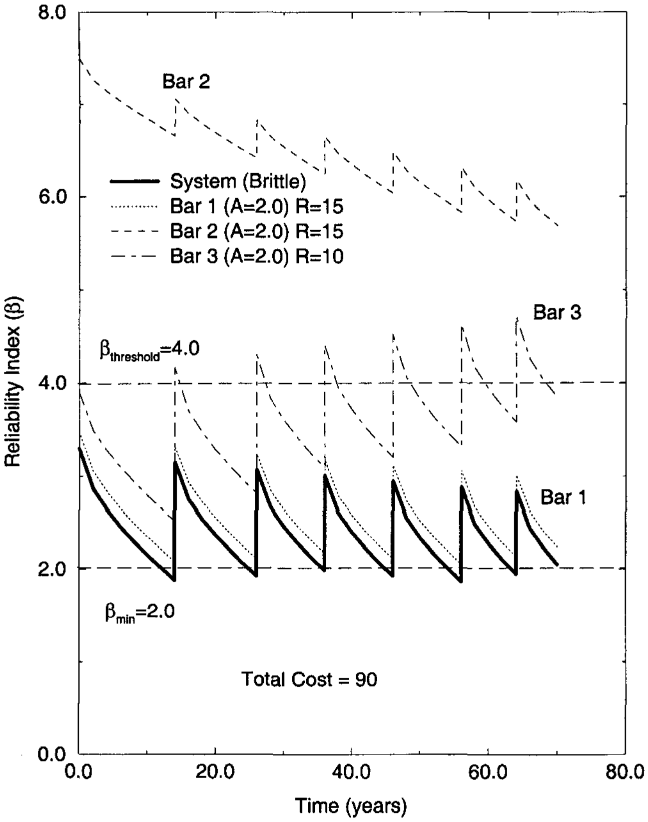

Fig. 13. repair plan for a brittle three bar indeterminate truss with equal bar areas; $\beta_{threshold} = 4.0$; $Cost = 90$.

below the reliability of the system and more lifetime repairs were required. A higher $\beta_{threshold}$ value resulted in fewer lifetime repairs which reduced the fixed cost, but having to replace component 2 caused an even greater increase in the variable cost. Again, the redistribution of load caused by the deterioration of component 2 increased the reliability of component 3, reduced the reliability of component 1, and decreased the reliability of the system. As expected, maintaining the reliability of the brittle structure was much more expensive than maintaining the reliability of the ductile structure.

Further investigations involved changing the direction of the load, changing the resistance or initial area of the bars, varying post-elastic properties, and varying the correlation among resistances. Correlation between resistances is interesting in a series-parallel system because increased resistance correlation improves the reliability of a series system but decreases the reliability of a parallel system. Whether resistance correlation hurt or benefited the system reliability varied depending on the specific case being considered. The optimum solution, for example, for a truss with semi-ductile components ($\eta = 0.5$) occurred when the correlation between the resistances was $\rho = 0.5$ (Estes, 1997).

Fig. 14. Photograph of Bridge E-AH-17.

5. Series-Parallel System: Existing Highway Bridge

The truss examples were deliberately simplistic to illustrate the relevant principles in an efficient manner. The concept of using system reliability to optimize a lifetime repair strategy was extended to an existing highway bridge (Bridge E-17-AH) located in Colorado (Estes, 1997; Estes and Frangopol, 1999). The principles used in the previous simplistic examples did not change. Significant effort and data were needed to identify costs, deterioration models, repair options, and limit states of a real-world structure. Bridge E-17-AH is a simply-supported, three-span, four-lane steel girder structure as shown in Fig. 14. The deck is reinforced concrete and the steel girders are standard rolled shapes. The interior span supports are reinforced concrete pier columns with a pier cap and the exterior abutments are concrete piles cased in steel.

Sixteen component failure modes ranging from moment failure of the slab and girders to crushing of the abutments and shear failure of the pier cap were identified and analyzed. The limit state equations for these failure modes use 24 random variables which include material strength uncertainty, dimension uncertainty, and model uncertainty. Random variable parameters were obtained from the existing literature. The nine girders were classified as exterior (two girders) interior (five girders), and interior-exterior (two girders) The live load model used for the calibration of the 1994 AASHTO LRFD Design Specification was used to account for uncertainty associated with vehicle traffic across the bridge.

The bridge was modeled as a series-parallel system where the superstructure will not fail until three adjacent girders fail (Estes and Frangopol, 1999). The model was simplified by ignoring irrelevant failure modes, accounting

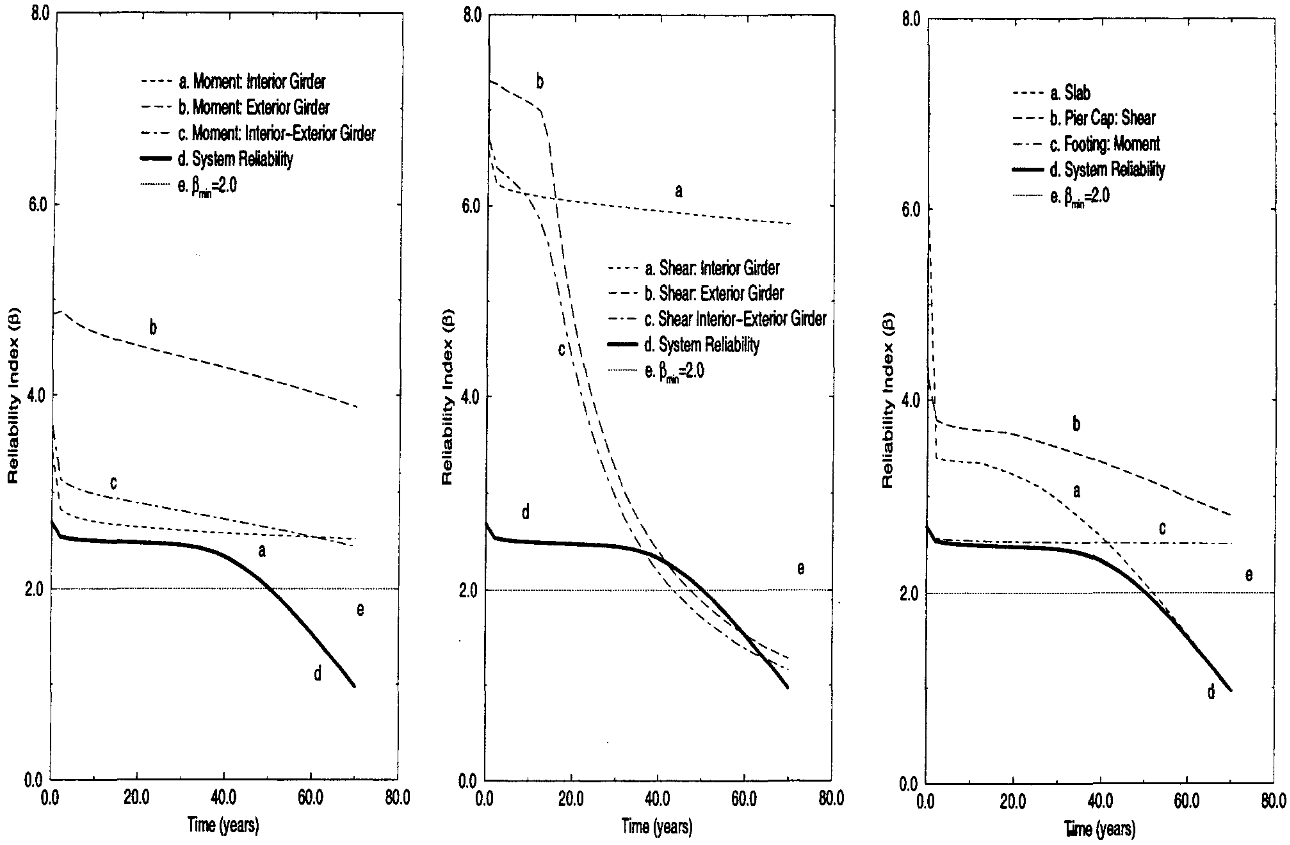

Fig. 15. System and component reliabilities over time for Bridge E-17-AH.

for symmetry and assuming perfect correlation between spans.

The reliability of the system decreases over time as the live load increases and the structure deteriorates. The slab and the pier cap are exposed to road salts where the chloride penetration causes corrosion of the reinforcement after a critical concentration is reached (Thoft-Christensen *et al.*, 1997; Frangopol *et al.*, 1997). The steel girders are corroding (Albrecht and Naeemi, 1984) which reduces the web area and plastic section modulus over time. The deterioration introduces new random variables into the limit state equations which include diffusion rates, chloride surface concentration and corrosion parameters.

Fig. 15 shows the reliabilities of the nine relevant failure modes in the series-parallel model of the bridge along with the reliability of the bridge system over time. The reliability of the girders with respect to moment are initially much lower than those of the girders with respect to shear. The corrosion of the thin web however causes the shear capacity and thus the reliability with respect to shear to diminish at a much faster rate. After 40 years, the reliability of the girders with respect to shear falls below the moment reliability. Because of the parallel nature of the superstructure, the girder reliability with respect to shear is able to fall below $\beta = 2.0$ without violating $\beta_{min} = 2.0$ for the system reliability.

The slab, pier cap and footing moment are all in the series portion of the system and must therefore always reflect component reliabilities higher than that of the system. The slab reliability is initially the highest of the three and the reliability of the column footing dictates the reliability of the system in the early life of the bridge. The slab is deteriorating more quickly than either the pier cap or the footing. As a result, it is the slab which causes the system reliability to fall below $\beta_{min} = 2.0$ at year 50 and illustrates why repairing the slab is an effective repair option.

Fig. 15 illustrates that the reliability of a system depends on the series-parallel model of the system and its deterioration. The component with the lowest reliability may not be the most important component and does not necessarily control the reliability of the system. The most important component early in the life of the structure may not be the most important component during the later periods. It is difficult to predict the reliability of the system even if the reliabilities of all of the components are known and therefore a repair strategy based solely on component reliabilities would be inefficient and potentially unsafe.

In consult with the Colorado DOT experts and cost documents (CDOT 96), five repair options and their associated costs were developed. Inspections are performed every two years and corrective action must be taken if the system reliability falls below β_{min}. This information allows for an optimum lifetime repair strategy to be developed. The optimum lifetime repair strategies for this bridge accounting for various service lives, different interest rates, alternative series-parallel models, and revised deterioration models is presented in detail in Estes and Frangopol (1999).

6. Conclusions

The real potential and benefit of using reliability methods is not realized until a structure is analyzed as an entire system. Given the uncertainties involved and the importance of failure mode correlation in the results, a similar analysis cannot be done deterministically. The actual safety of a structural system cannot be accurately assessed until the interconnection and relationship between the components are understood. Genuine cost savings can be realized as unnecessary repairs are identified and resources are allocated to where they are most needed.

While computer availability and power have made the complex reliability computations easy to perform, the accuracy of the reliability analysis is only as good as the input data which is often not readily available. Determining the statistical parameters for material strengths, model uncertainty factors, loads, deterioration models, and human error requires considerable research and sometimes still boils down to an educated guess. Exact reliability solutions are often not possible in closed form. First and second order approximations are often used. Advanced Monte Carlo simulation methods are valid options, but given the low probability of failure associated with most structures, require caution, experience, and, in general, substantial CPU-time to obtain valid results. Diverse probability distributions are reduced to equivalent normal distributions at a failure point and correlation estimates are particularly difficult to obtain. The errors associated with these approximations are often magnified at the system level, especially when the limit state equations are highly non-linear. Despite these difficulties, there are many common engineering situations where these techniques provide highly accurate and helpful solutions. These methods demonstrate tremendous potential when applied to life-cycle cost maintenance and management of civil infrastructure systems (Frangopol *et al.,* 2000, 2001).

One area that is ripe for future research is obtaining values for β_{design} and β_{min} where β_{design} is the reliability that a new structure is designed to achieve when it is initially placed in service and β_{min} is the minimum reliability that a structure may deteriorate to before some remedial action is required. In this paper, the structures examined were already designed and β_{min} was arbitrarily chosen as 2.0. In reality, these two values are critical to defining what may happen to a structure over its life, the amount of maintenance it will require, how much it will cost to build, and what risk society is willing to accept. Such considerations are not trivial and a wise choice of these two values has the potential of substantial cost savings over the life of the structure. While tremendous work remains, system reliability methods demonstrate tremendous advantage over deterministic methods in understanding and analyzing the importance of individual members to the lifetime performance of the overall structural system.

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