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Reliability of a k-out-of-n Cold Standby System with Imperfect Switches

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Abstract. A k-out-of-n standby system is considered where all of its components are s-independent and classified either working or cold standby connected with imperfect switches. The probability density function of the life length for this system is established in closed form, when the underlying components have constant failure rates. Also the reliability function of the system is derived. Finally, the reliability functions for one, two and three out of four systems are deduced for perfect or imperfect switches and identical or non-identical constant failure rates for working and standby components.

Key Words : *k-out-of-n standby system, imperfect switches, reliability function, component of type-I, component of type-II.*

1. INTRODUCTION

Many researchers have discussed the problem of modeling reliability for systems with active redundancy property, see for example Chan et al. (1995), Cramer and Kamps (2000), Sarhan and Abouammoh (2001) and the references there in.

In general, the k-out-of-n system is working if at least k of its n components are operating and fails if n-k+1 components fail. Practical examples of k-out-of-n systems are, e.g. an aircraft with four engines which works if at least two out of its four engines remain functioning, or a satellite which will have enough power to send signals if not more than four out of its ten batteries are discharged.

The k-out-of-n systems can be utilized for modeling many other practical situations such as quality control problems, inspection procedures and radar detection problems, see Saperstein (1973, 1975) and Nelson (1978).

In this paper we consider a general form of k-out-of-n system with independent and non-identical components, cold standby and imperfect switches. The probability density and reliability functions of this system are discussed for the cases $n-k=1$ and $n-k>1$.

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As illustrative examples, we derive the probability density and reliability functions of one, two and three out of four systems in general. Special cases for perfect and imperfect switches with identical and non-identical main and standby components are also discussed

The paper is organized as follows. Section 2, gives the basic definitions and assumptions. The probability density and reliability functions of k-out-of-n systems with one or more than one standby component are derived in section 3. The probability density and reliability functions of some particular systems are presented in section 4.

2. DEFINITIONS AND ASSUMPTIONS

Here we consider a k-out-of-n : G standby system. All components are s-independent with constant failure rates. The components are classified into two types, operating and standby. Each standby component is connected with an imperfect switch. The system starts operating with the k main components while the rest n-k components are in cold standby. When a component fails, instantaneously one of the standbys, if there is one, becomes active (operating). Components do not fail simultaneously and there is no repair. Further it is assumed that all groups of main components, standby components and the switches are s-identical. Let λ, μ and α be the failure rates of the main, standby components and the switch, respectively.

The assumptions indicate that the operating components are classified into two types. One of these types is the old operating components and the second is the new operating components. The old operating components are defined as those components that start working with the system operating. The new operating components are defined as those components that enter to the operating mode from the standby mode. Here, we call the old operating components as type-I and the new operating components as type-II. Note that each operating component of type-II is connected with an imperfect switch. So we refer by a component of type-II fails that either the component itself or the switch connected with fails.

The vector (n_1, n_2, n_3) will be used to refer the system components, where n_1 represents the number of components of type-I, n_2 is the number of components of type-II and n_3 is the number of standby components.

To obtain the reliability function of the system we need to derive the probability density function, shortly pdf, of its lifetime. For this reason we devote the following part for firstly obtaining the pdf of k-out-of-n system lifetime. Then using the obtained pdf, the reliability function can be derived.

3. MAIN RESULTS

In the following, we establish a closed form for the pdf of the k-out-of-n system when there exists only one component in the standby mode.

Theorem 3.1 The pdf of k-out-of-n system lifetime, when $n-k=1$, is

$$f(t)=\frac{k\lambda[(k-1)\lambda+\lambda_b]}{\lambda-\lambda_b}\{\exp\{-[(k-1)\lambda+\lambda_b]t\}-\exp\{-k\lambda t\}\}, \tag{3.1}$$

where $\lambda_b=\mu+\alpha$.

Proof. To proof this theorem let us assume that E represents the event [System fails in the interval dt after surviving for time t]. The system will fail if two component fail. But the system components do not fail simultaneously. Then event E can be represented as the intersection of the following two independent events:

Event-1: Occurs when one component fails before the time t and the system works.

Event-2: Occurs when the remaining components function for the balance of the interval $(0, t)$ and one of them fails in dt.

Let us denote by t_1 to the lifetime of the component fails first. Then t-t_1 will be the operating time of the component fails later, see Figure 1.

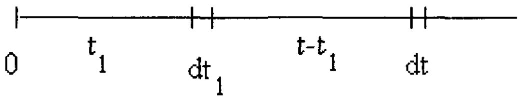

Figure 1. The system surviving period.

Let A_1 be the event that one component survives for t_1 and fails in dt_1 and A be the event that the remaining components survive for t-t_1 and one of them fails in dt. The event A_1 occurs when a component of type-I fails at time t_1 and the remaining k-1 components are surviving. But the event A may occur if either: (*i*) a component of type-I fails at t-t$_1$ and the rest k-2 components of type-I and one component of type-II are surviving, or (*ii*) the component of type-II fails at t-t_1 and the rest k-1 components are surviving.

Therefore, using the lack of memory property for the exponential distribution, one can verify that

$$\begin{aligned}P(A_1)&=k\lambda\exp\{-k\lambda t_1\},\\ P(A)&=[(k-1)\lambda+\lambda_b]\exp\{-[(k-1)\lambda+\lambda_b](t-t_1)\}\end{aligned} \tag{3.2}$$

But the system pdf is given by

$$f(t)=\int_{t_1=0}^{t}P(A_1)P(A)dt_1 \tag{3.3}$$

Substituting from (3.2) into (3.3), the pdf becomes

$$f(t)=k\lambda[(k-1)\lambda+\lambda_b]\exp\{-[(k-1)\lambda+\lambda_b]t\}\int_{t_1=0}^{t}\exp\{-(\lambda-\lambda_b)t_1\}dt_1 \tag{3.4}$$

Solving the integral in the right side of Eq. (3.4), one can easily write $f(t)$ as given by Eq. (3.1) that completes the proof.

The following corollaries give the reliability functions of the k-out-of-n systems with imperfect or perfect switches and non-identical or identical components when the system has only one component in the standby mode.

Corollary 3.2 The reliability function of k-out-of-n system when $n-k=1$ is given by

$$R(t)=\frac{1}{\lambda-\lambda_b}\{k\lambda\exp\{-[(k-1)\lambda+\lambda_b]t\}-[(k-1)\lambda+\lambda_b]\exp\{-k\lambda t\}\}, \quad (3.5)$$

where $\lambda_b=\mu+\alpha$.

Proof. The proof can easily be reached by substituting from Eq. (3.1) into the relation between pdf and reliability function given by

$$R(t)=\int_t^\infty f(u)du. \quad (3.6)$$

Corollary 3.3 The reliability function of k-out-of-n standby system with perfect switch when $n-k=1$ is given by

$$R(t)=\frac{1}{\lambda-\mu}\{k\lambda\exp\{-[(k-1)\lambda+\mu]t\}-[(k-1)\lambda+\mu]\exp\{-k\lambda t\}\}. \quad (3.7)$$

Proof. Setting $\alpha=0$ in Eq. (3.5) we obtain Eq. (3.7).

Corollary 3.4 The reliability function of k-out-of-n standby system with imperfect switch but identical components, when $n-k=1$, is given by

$$R(t)=\frac{1}{\alpha}\{(k\lambda+\alpha)\exp\{-k\lambda t\}-k\lambda\exp\{-(k\lambda+\alpha)t\}\}. \quad (3.8)$$

Proof. Setting $\mu=\lambda$ in Eq. (3.5) we get Eq. (3.8).

Corollary 3.5 The reliability function of k-out-of-n standby system with perfect switch and identical components, when $n-k=1$, is given by

$$R(t)=(1+k\lambda)\exp\{-k\lambda t\}. \quad (3.9)$$

Proof. Take the limit of R(t), given by Eq. (3.8), when $\alpha\to 0$ we can derive Eq. (3.9).

The following theorem gives the pdf of k-out-of-n standby system when the system has more than one component in standby mode. From now and henceforth we shall use $f(z,n_1,n_2,n_3)$ to denote the probability that one of the system components fails after surviving for time z but the rest components are surviving given that the system components are (n_1, n_2, n_3).

Theorem 3.6 The pdf of k-out-of-n system lifetime, when $n-k>1$, is

$$f(t)=\int_{t_{n-k}=0}^{t}\prod_{j=1}^{n-k-1}\int_{t_j=0}^{t_{j+1}}P(A_1)\sum_{i_{n-k}=1}^{2}\cdots\sum_{i_3=1}^{2}\sum_{i_2=1}^{2}P(A_{1i_2})P(A_{1i_2i_3})\ldots P(A_{1i_2i_3\ldots i_{n-k}})f_{1i_2i_3\ldots i_{n-k}}(\tau)\prod_{j=1}^{n-k}dt_j, \quad (3.10)$$

where

$$f_{i_1i_2\ldots i_{n-k}}(\tau)=\begin{cases}\left[k\lambda-d_{i_1i_2\ldots i_m}(\lambda-\lambda_b)\right]\exp\left\{-\left[k\lambda-d_{i_1i_2\ldots i_m}(\lambda-\lambda_b)\right]\tau\right\} & \text{if } k-d_{i_1i_2\ldots i_m}\geq 0,\\ 0 & \text{otherwise,}\end{cases}$$

$$P(A_{i_1i_2\ldots i_m1})=\begin{cases}\left(k-d_{i_1i_2\ldots i_m}\right)\lambda\exp\left\{-\left[\left(k-d_{i_1i_2\ldots i_m}\right)\lambda+d_{i_1i_2\ldots i_m}\lambda_b\right]\tau_{m+1}\right\} & \text{if } k-d_{i_1i_2\ldots i_m}>0,\\ 0 & \text{otherwise,}\end{cases}$$

$$P(A_{i_1i_2\ldots i_m2})=d_{i_1i_2\ldots i_m}\lambda_b\exp\left\{-d_{i_1i_2\ldots i_m}\lambda_b\tau_{m+1}\right\}\times\begin{cases}\exp\left\{-\left(k-d_{i_1i_2\ldots i_m}\right)\lambda\tau_{m+1}\right\} & \text{if } k-d_{i_1i_2\ldots i_m}>0,\\ 1 & \text{if } k-d_{i_1i_2\ldots i_m}=0,\\ 0 & \text{if } k-d_{i_1i_2\ldots i_m}<0,\end{cases}$$

$$P(A_1)=k\lambda\exp\{-k\lambda t_1\},\quad d_{i_1i_2\ldots i_m}=\#\left\{i_j : i_j=1, j=1,2,\ldots,m\right\},$$

and

$$\tau=t-t_{n-k},\ \tau_i=t_i-t_{i-1},\quad i=1,2,\ldots,n-k;\ t_0=0. \quad (3.11)$$

Proof. Since the system fails if *n-k+1* of its components fail. Components do not fail simultaneously. Then the system surviving period (*0*, *t*) can be divided into *n-k+1* subintervals, as shown in Figure 2. We assume that at the end of each subinterval one of the system components fails after surviving for the length of that subinterval.

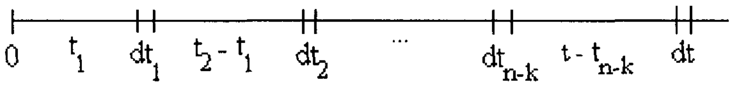

Figure 2. The system surviving period

Next we note the following:

At time t_1 : there is only one possible event, say A_1. This event may occur when one component of type-I fails. Given this event has occurred, the system components become $(k-1, 1, n-k-1)$. The system pdf is given by

$$f(t)=\int_{t_1=0}^{t}P(A_1)f(t-t_1,k-1,1,n-k-1)dt_1, \quad (3.12)$$

where $P(A_1)=k\lambda\exp\{-k\lambda t_1\}$.

At time t_2 : there are two possible events, say A_{11} and A_{13}. The event A_{11} may occur when a component of type-I fails given that the event A_1 has occurred at time t_1. Given the event A_{11} has occurred, the system components become $(k-2,\ 2,\ n-k-2)$. The event A_{12} may occur when a component of type-II fails given that the event A_1 has occurred at time t_1. Given the event A_{12} has occurred, the system components become $(k-1,\ 1,\ n-k-2)$. The system pdf can be written as

$$f(t)=\int_{t_2=0}^{t}\int_{t_1=0}^{t_2}P(A_1)\left[P(A_{11})f_{11}(t-t_2)+P(A_{12})f_{12}(t-t_2)\right]dt_1\,dt_2\,, \tag{3.13}$$

where $P(A_{11})=(k-1)\lambda\exp\{-[(k-1)\lambda+\lambda_b]\tau_2\}$,
$P(A_{12})=\lambda_b\exp\{-[(k-1)\lambda+\lambda_b]\tau_2\}$,
$f_{11}(t-t_2)=f(t-t_2,k-2,2,n-k-2)$,
$f_{12}(t-t_2)=f(t-t_2,k-1,1,n-k-2)$, $\tau_2=t_2-t_1$.

At time t_3: there are four possible events, say A_{111}, A_{112}, A_{121} and A_{123}. The event A_{111} may occur when a component of type-I fails given that the event A_{11} has occurred at time t_3. Given the event A_{111} has occurred, the system components become $(k-3,\ 3,\ n-k-3)$. The event A_{112} may occur when a component of type-II fails given that the event A_{11} has occurred at time t_3. Given the event A_{112} has occurred, the system components become $(k-2,\ 2,\ n-k-3)$. The event A_{121} may occur when a component of type-I fails given that the event A_{12} has occurred at time t_3. Given the event A_{121} has occurred, the system components become $(k-2,\ 2,\ n-k-3)$. The event A_{122} may occur when a component of type-II fails given that the event A_{12} has occurred at time t_3. Given the event A_{122} has occurred, the system components become $(k-1,\ 1,\ n-k-3)$. The system pdf can be written as

$$\begin{aligned}f(t)=\int_{t_3=0}^{t}\int_{t_2=0}^{t_3}\int_{t_1=0}^{t_2}&P(A_1)\{P(A_{11})[P(A_{111})f_{111}(t-t_3)+P(A_{112})f_{112}(t-t_3)]+\\&+P(A_{12})[P(A_{121})f_{121}(t-t_3)+P(A_{122})f_{122}(t-t_3)]\}dt_1\,dt_2\,dt_3\end{aligned} \tag{3.14}$$

where
$P(A_{111})=(k-2)\lambda\exp\{-[(k-2)\lambda+2\lambda_b]\tau_3\}$, $P(A_{112})=2\lambda_b\exp\{-[(k-2)\lambda+2\lambda_b]\tau_3\}$,
$P(A_{121})=(k-1)\lambda\exp\{-[(k-1)\lambda+\lambda_b]\tau_3\}$, $P(A_{122})=\lambda_b\exp\{-[(k-1)\lambda+\lambda_b]\tau_3\}$,
$f_{111}(t-t_3)=f(t-t_3,k-3,3,n-k-3)$, $f_{112}(t-t_3)=f(t-t_3,k-2,2,n-k-3)$,
$f_{121}(t-t_3)=f(t-t_3,k-2,2,n-k-3)$, $f_{122}(t-t_3)=f(t-t_3,k-1,1,n-k-3)$,
and $\tau_3=t_3-t_2$.

Generally at time $t_m, 1<m\le n-k$: There are 2^{m-1} possible events, say $A_{i_1i_2\ldots i_m}$ where $i_j=1,2,\ \forall\, j=1,2,\ldots,m$. The event $A_{i_1i_2\ldots i_m}$ occurs when a component of type-I (type-II) for $i_m=1$ ($i_m=2$) fails given that the event $A_{i_1i_2\ldots i_{m-1}}$ has occurred at time t_{m-1}. That is, $A_{i_1i_2\ldots i_m}$ denotes the event that system components survive for the period of interval (t_{m-1}, t_m) and a component of type-I (type-II), for $i_m=1$ ($i_m=2$) fails at t_m given that the system components

are $(k-d_{i_1i_2\ldots i_m}-1, d_{i_1i_2\ldots i_{m-1}}, n-k-m+1)$. Given that the event $A_{i_1i_2\ldots i_m}$ has occurred the system components become $(k-d_{i_1i_2\ldots i_m}, d_{i_1i_2\ldots i_m}, n-k-m)$. Then the system pdf becomes

$$f(t)=\int\limits_{t_m=0}^{t}\prod_{j=1}^{m-1}\int\limits_{t_j=0}^{t_{j+1}}P(A_1)\sum_{i_m=1}^{2}\cdots\sum_{i_3=1}^{2}\sum_{i_2=1}^{2}P(A_{1i_2})P(A_{1i_2i_3})\ldots P(A_{1i_2i_3\ldots i_m})f_{1i_2i_3\ldots i_{n-k}}(t-t_m)\prod_{j=1}^{m}dt_j, \quad (3.15)$$

The process is continued until the failure *n-k*. This failure is possible at time t_{n-k}. This is because at the end of the subinterval (t_{n-k-1}, t_{n-k}) the system components become $(k-d_{i_1i_2\ldots i_{n-k}}, d_{i_1i_2\ldots i_{n-k}}, 0)$. Therefore, the system will fail at the next failure of one of the remaining system components that may fail at the end of the last subinterval (t_{n-k}, t). It means that the relation given by Eq. (3.10) can be derived from Eq. (3.15) by setting $m=n-k$. This completes the proof.

4. ILLUSTRATIVE EXAMPLES

To illustrate how one can use the previous theoretical results, we derive the pdf and reliability functions of *1*-out-of-*4*, *2*-out-of-*4* and *3*-out-of-*4* system with identical or non-identical components and perfect or imperfect switches.

Corollary 4.1 The pdf of *1*-out-of-*4* cold standby system with imperfect switches is given by

$$f(t)=\frac{\lambda\lambda_b^3\exp[-\lambda_b t]}{\lambda-\lambda_b}\left\{\frac{t^2}{2}-\frac{t}{\lambda-\lambda_b}+\frac{1-\exp\{-(\lambda-\lambda_b)\}t}{(\lambda-\lambda_b)^2}\right\}. \quad (4.1)$$

Proof. Substituting $k=1$ and $n-k=3$ into Eqs. (3.10) and (3.11), we get

$$f(t)=\int\limits_{t_3=0}^{t}\int\limits_{t_2=0}^{t_3}\int\limits_{t_1=0}^{t_2}P(A_1)\sum_{i_3=1}^{2}\sum_{i_2=1}^{2}P(A_{1i_2})P(A_{1i_2i_3})f_{1i_2i_3}(\tau)dt_1\,dt_2\,dt_3, \quad (4.2)$$

where $P(A_1)=\lambda\exp\{-\lambda t_1\}, \tau=t-t_3$;

a. $d_1=1$, $k-d_1=0$ then $P(A_{11})=0,\ P(A_{12})=\lambda_b\exp\{-\lambda_b\tau_2\}$;
b. $d_{11}=2$, $k-d_{11}=-1$ then $P(A_{111})=0,\ P(A_{112})=0,\ f_{111}(\tau)=0, f_{112}(\tau)=0,$;
c. $d_{12}=1$, $k-d_{11}=0$ then $P(A_{122})=\lambda_b\exp\{-\lambda_b\tau_3\}\ P(A_{121})=0,,\ f_{121}(\tau)=0,$ $f_{122}(\tau)=\lambda_b\exp\{-\lambda_b\tau\}$,

Namely, Eq. (4.2) reduces to

$$f(t)=\int\limits_{t_3=0}^{t}\int\limits_{t_2=0}^{t_3}\int\limits_{t_1=0}^{t_2}P(A_1)P(A_{12})P(A_{122})f_{122}(\tau)dt_1\,dt_2\,dt_3, \quad (4.3)$$

That is,

$$f(t)=\lambda\lambda_b^3\exp\{-\lambda_b t\}\int\limits_{t_3=0}^{t}\int\limits_{t_2=0}^{t_3}\int\limits_{t_1=0}^{t_2}\exp\{-(\lambda-\lambda_b)t_1\}\,dt_1\,dt_2\,dt_3, \quad (4.4)$$

Solving the integral in the right side of Eq. (4.4), one can derive *f(t)* as given by Eq. (4.1).

Substituting from (4.1) into (3.6) and solving the getting integral, one can derive the reliability function of *1*-out-of-*4* cold standby system with imperfect switches in the following form

$$R(t)=\frac{\lambda\lambda_b^3}{\lambda-\lambda_b}\left\{\left[\frac{t^2}{2\lambda_b}+\frac{(\lambda-2\lambda_b)t}{\lambda_b^2(\lambda-\lambda_b)}+\frac{\lambda^2-3\lambda\lambda_b+3\lambda_b^2}{\lambda_b^3(\lambda-\lambda_b)^2}\right]\exp\{-\lambda_b t\}-\frac{\exp\{-\lambda t\}}{\lambda(\lambda-\lambda_b)^2}\right\}. \quad (4.5)$$

One can deduce the following special cases, from Eq. (4.5):

1) Setting $\mu=\lambda$, then the reliability function of *1*-out-of-*4* cold standby system with identical components and imperfect switches becomes

$$R(t)=\frac{\lambda(\lambda+\alpha)^3}{\alpha^3}\left\{\frac{1}{\lambda}-\left[\frac{t^2}{2}+\frac{(\lambda+2\alpha)t}{\alpha(\lambda+\alpha)}+\frac{\lambda^2+3\alpha(\lambda+\alpha)}{\alpha^2(\lambda+\alpha)^2}\right]\frac{\alpha^2\exp\{-\alpha t\}}{\lambda+\alpha}\right\}\exp\{-\lambda t\}. (4.5)$$

2) Setting $\alpha=0$, then the reliability function of *1*-out-of-*4* cold standby system with non-identical components and perfect switches becomes

$$R(t)=\frac{\lambda\mu^3}{\lambda-\mu}\left\{\left[\frac{t^2}{2\mu}+\frac{(\lambda-2\mu)t}{\mu^2(\lambda-\mu)}+\frac{\lambda^2-3\lambda\mu+3\mu^2}{\mu^3(\lambda-\mu)^2}\right]\exp\{-\mu t\}-\frac{\exp\{-\lambda t\}}{\lambda(\lambda-\mu)^2}\right\}. \quad (4.6)$$

3) Setting $\mu=\lambda$ and $\alpha\to 0$, then the reliability function of *1*-out-of-*4* cold standby system with identical components and perfect switches becomes, which agrees with the result presented by Grosh ((1989), p. 168)

$$R(t)=\left\{1+\frac{\lambda t}{1!}+\frac{(\lambda t)^2}{2!}+\frac{(\lambda t)^3}{3!}\right\}\exp\{-\lambda t\}. \quad (4.7)$$

Corollary 4.2 The pdf of *2*-out-of-*4* cold standby system with imperfect switches is given by

$$f(t)=\frac{2\lambda\lambda_b}{(\lambda-\lambda_b)^2}\{\lambda\exp\{-2\lambda_b t\}+(2\lambda+\lambda_b)\exp\{-2\lambda t\}+$$
$$+[(\lambda^2-\lambda_b^2)t-3\lambda-\lambda_b]\exp\{-(\lambda+\lambda_b)t\}\}. \quad (4.8)$$

Proof. Substituting *k=2* and *n-k=2* into Eqs. (3.10) and (3.11), we get

$$f(t)=\int_{t_2=0}^{t}\int_{t_1=0}^{t_2}P(A_1)\sum_{i_2=1}^{2}P(A_{1i_2})f_{1i_2}(\tau)\,dt_1\,dt_2\,, \quad (4.9)$$

where $P(A_1)=2\lambda\exp\{-2\lambda t_1\},\tau=t-t_2$;

a. $d_1=1$, $k-d_1=1$ then $P(A_{11})=\lambda\exp\{-(\lambda+\lambda_b)\tau_2\}$, $P(A_{12})=\lambda_b\exp\{-(\lambda+\lambda_b)\tau_2\}$;

b. $d_{11}=2$, $k-d_{11}=0$ then $f_{12}(\tau)=(\lambda+\lambda_b)\exp\{-(\lambda+\lambda)\tau\}$, $f_{11}(\tau)=2\lambda_b\exp\{-2\lambda\tau\}$, ;

Therefore, Eq. (4.2) reduces to

$$f(t)=2\lambda\lambda_b\int_{t_2=0}^{t}\int_{t_1=0}^{t_2}\exp\{-(\lambda-\lambda_b)t_1\}\{2\lambda\exp\{-(\lambda-\lambda_b)t_2-2\lambda_b t\}+$$
$$+(\lambda+\lambda_b)\exp\{-(\lambda+\lambda_b)t\}\}\,dt_1\,dt_2\,, \qquad (4.10)$$

Solving the integral in the right side of Eq. (4.10), one can derive *f(t)* as given by Eq. (4.8)

Now to derive the reliability function of *2*-out-of-*4* cold standby system with imperfect switches we substitute from (4.8) into (3.6) and solving the getting integral, we hence get the following form

$$R(t)=\frac{2\lambda\lambda_b}{(\lambda-\lambda_b)^2}\left\{\frac{\lambda\exp\{-2\lambda_b t\}}{2\lambda_b}+\frac{(2\lambda+\lambda)\exp\{-2\lambda t\}}{2\lambda}+[(\lambda-\lambda_b)t-2]\exp\{-(\lambda-\lambda_b)t\}\right\}. \qquad (4.11)$$

One can derive the following particular cases, from Eq. (4.11):

1) Setting $\mu=\lambda$, then the reliability function of *2*-out-of-*4* cold standby system with identical components and imperfect switches becomes

$$R(t)=\frac{2\lambda(\lambda+\alpha)}{\alpha^2}\left\{\frac{\lambda\exp\{-2\alpha t\}}{2(\lambda+\alpha)}+\frac{3\lambda+\alpha}{2\lambda}-[\alpha t+2]\exp\{-\alpha t\}\right\}\exp\{-2\lambda t\}. \qquad (4.12)$$

2) Setting $\alpha=0$, then the reliability function of *2*-out-of-*4* cold standby system with non-identical components and perfect switches becomes

$$R(t)=\frac{2\lambda\mu}{(\lambda-\mu)^2}\left\{\frac{\lambda\exp\{-2\mu t\}}{2\mu}+\frac{(2\lambda+\mu)\exp\{-2\lambda t\}}{2\lambda}+[(\lambda-\mu)t-2]\exp\{-(\lambda-\mu)t\}\right\}. \qquad (4.13)$$

3) Setting $\mu=\lambda$ and $\alpha\to 0$, then the reliability function of *2*-out-of-*4* cold standby system with identical components and perfect switches becomes

$$R(t)=\left\{1+\frac{2\lambda t}{1!}+\frac{(2\lambda t)^2}{2!}\right\}\exp\{-2\lambda t\}. \qquad (4.14)$$

***Corollary** 4.3* The reliability function of *3*-out-of-*4* cold standby system with imperfect switches is given by

$$R(t)=\frac{1}{\lambda-\lambda_b}\{3\lambda\exp\{-(2\lambda+\lambda_b)t\}-(2\lambda+\lambda_b)\exp\{-3\lambda t\}\}. \qquad (4.15)$$

Proof. Substituting *k=3* into Eq. (3.5), we get directly Eq. (4.15).

One can easily deduce the following particular cases from Eq. (4.15):

1) Setting $\mu=\lambda$, then the reliability function of *3*-out-of-*4* cold standby system with identical components and imperfect switches becomes

$$R(t) = \frac{1}{\alpha}\{3\lambda + \lambda_b - 3\lambda \exp\{-\alpha t\}\}\exp\{-3\lambda t\}. \qquad (4.16)$$

2) Setting $\alpha = 0$, then the reliability function of *2*-out-of-*4* cold standby system with non-identical components and perfect switches becomes

$$R(t) = \frac{1}{\lambda - \mu}\{3\lambda \exp\{-\mu t\} - (2\lambda + \mu)\exp\{-\lambda t\}\}\exp\{-2\lambda t\}. \qquad (4.17)$$

3) Finally, setting $\mu = \lambda$ and $\alpha \to 0$, then the reliability function of *3*-out-of-*4* cold standby system with identical components and perfect switches becomes

$$R(t) = \{1 + 3\lambda t\}\exp\{-3\lambda t\}. \qquad (4.18)$$

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