

## **A Quantitative Decision-making Analysis Using Fuzzy Theory in Nuclear Power Plants**

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**Abstract.** In general, analysis of the decision problems in nuclear system management involves a simultaneous consideration of various criteria and decision alternatives. Sometimes, it is a complex, unstructured, ill-defined process incorporating the multi-criteria and the data of impreciseness. To cope with this analysis, a fuzzy hierarchical analysis methodology is proposed and demonstrated with a simple example.

**Key Words :** *decision-making, management, multi-criteria, fuzzy, nuclear system.*

### **1. INTRODUCTION**

The decision-making associated with the management of complex systems such as nuclear systems generally requires the consideration of various decision criteria that are sometimes vague, ill-defined and not commensurate. It causes severe difficulties in making a decision when the evaluation criteria are not commensurate each other or the data are imprecise. Because of these difficulties, the decision is often made on the basis of expert subjective judgments. Such a subjectivity introduced into a decision-making evokes a violent dispute in the society of experts now and then. The subjectivity may be regarded as a lack of any clear mathematical basis, logical foundations, assumptions and justifications behind the decision. If it is unavoidable to use the expert subjective judgment, a systematic and logical approach is required to assure the reliability of the decision-making.

The aim of the paper is to present the methodology to the analysis of strategic alternatives in nuclear system management. Given a decision problem, using the methodology, a decision maker can arrange decision criteria and alternatives hierarchically of the given decision problem, evaluate them applying the fuzzy set theory

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to determine the priorities for them, and accomplish a “what-if” analysis to investigate the effect change in ratings.

## 2. ASSUMPTIONS

The basic idea of our methodology is initiated from the fact that a methodology is required to be helpful in performing the following two tasks that are crucial for nuclear system management:

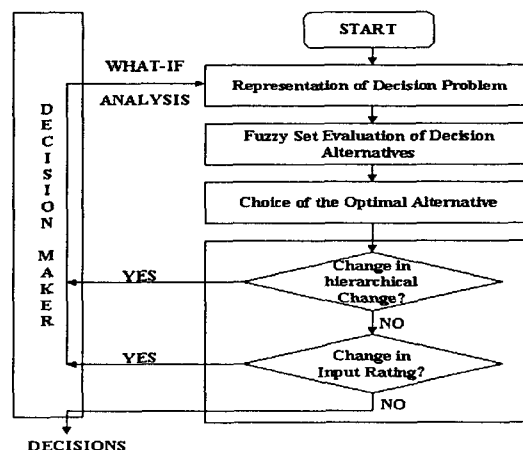
- Formulation of decision goal and criteria of a given decision problem;
- Choice of the optimal alternative to achieve the goal and the criteria.

The main assumptions to build our methodology are as follows:

- The methodology should include a multi-attribute evaluation procedure due to the fact that the decision problem has more than one criterion and more than one possible decision alternatives;
- The methodology should allow for the decision maker to be able to estimate his or her personal opinions and preferences about goal, criteria and alternatives;
- The methodology should allow for both quantitative and qualitative criteria in the analysis easily;
- The methodology should be interactive to be able to react easily on all problem changes, i.e., changes in criteria, alternatives, and their evaluations (judgments).

## 3. OUTLINE OF THE METHODOLOGY

To meet the given assumptions, we propose a fuzzy hierarchical analysis methodology to structure the problem hierarchically and integrate a multi-attribute evaluation procedure based on a fuzzy set theory and a “what-if” hierarchical analysis model. The main element of the methodology is the concept of hierarchical structure.



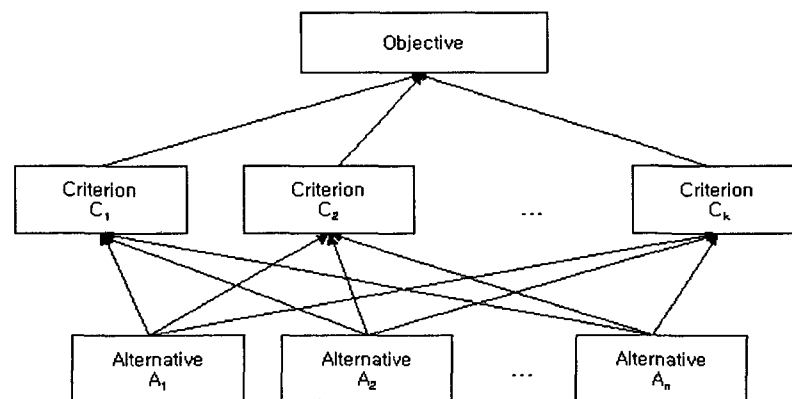
**Figure 1.** Overview of the proposed methodology

At the top of such a hierarchical structure, the most general goal or objective of the interest is placed – the so-called overall purpose. The goal is decomposed into the decision criteria at the lower level of the hierarchy. Each of them in turn is decomposed into a set of sub-criteria at the next lower level. The process of the hierarchical structuring is continued down to the most specific alternatives that are placed at the lowest level of the hierarchy. The general overview of the proposed methodology is shown in Figure 1, which includes 3 main steps: 1) representation of decision problem, 2) fuzzy set evaluation of alternatives, and 3) choice of the optimal alternative.

### 3.1 Representation of Decision Problem

The starting point to solve the decision problem is to define the problem. In our methodology, it consists of 3 activities as follows: 1) identifying a decision goal and a set of decision alternatives; 2) identifying a set of decision criteria; and 3) building a hierarchical structure of decision problem under consideration.

Given a decision problem, the decision goal and the decision alternatives can be easily identified. The decision goal is a final objective and the alternatives are the tools to achieve the goal. The goal can be represented with natural words or numerical values according to the characteristics of the problem. If  $n$  decision alternatives are identified from the given problem, the set of the alternatives is defined as follows:  $A = \{A_i \mid i = 1, 2, \dots, n\}$ . If  $k$  fuzzy decision criteria are identified, the set of the criteria is defined as follows:  $C = \{C_t \mid t = 1, 2, \dots, k\}$ . After the identification of decision goal, alternatives, and decision criteria, the problem can be represented as hierarchical structure, called decision tree as shown in Figure 2. Each component of the structure can be further decomposed into subdivisions, if needed.



**Figure 2.** An Example of hierarchical structure

### 3.2 Fuzzy Set Evaluation of Alternatives

This step includes 3 activities: 1) choosing sets of the preference ratings for the importance weight of the decision criteria and those for the alternatives versus the criteria;

2) evaluating the importance weight of each criterion and the fuzzy ratings for the appropriateness of the alternatives versus the criteria; and 3) aggregating the weights of the criteria and the fuzzy ratings of the alternatives.

In general, the sets of the preference ratings consist of 3 elements: the linguistic variable  $x$  representing the importance weight of each criteria under consideration and the fuzzy ratings for the appropriateness of the alternatives versus the criteria; term set  $T(x)$  representing the ratings of linguistic variables; and membership function corresponding to each element of term set. As an example, the preference ratings for the importance weights of the decision criteria are defined as follows:  $T(\text{importance}) = \{\text{very low, low, medium, high, very high}\}$ .

Using the sets of the preference ratings in the previous step, a certain ratings are assigned to the decision criteria and the alternatives, respectively, by decision maker. After the assignment of the ratings, the membership function will be matched to each rating for arithmetic operation. In our approach, the triangular fuzzy numbers are used as membership functions corresponding to the elements in term set. The reason of using triangular fuzzy number is that it is intuitively easy to be used by the decision maker. The triangular fuzzy number is denoted as follows:

$$f_M(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b, \\ \frac{(x-c)}{(b-c)}, & b \leq x \leq c, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where  $a$ ,  $b$ , and  $c$  are real numbers.

Let  $W_t$  be the importance weight of decision criterion  $C_t$ ,  $S_{it}$  be the fuzzy ratings of the appropriateness of alternative  $A_i$  for decision criterion  $C_t$ , and  $F_i$  be the fuzzy appropriateness index [1] for alternative  $A_i$ , which represents the degree of appropriateness for the alternative that is obtained by aggregating  $S_{it}$  and  $W_t$ .

Many methods have been proposed to aggregate the decision maker's assessments, for example, mean, median, max, min, and mixed operators [2]. Among the operators, we will use the mean operator to aggregate the decision maker's assessments since the average operation is the most commonly used aggregation method. Using the mean operator,  $F_i$  is given by

$$F_i = \left(\frac{1}{k}\right) [(S_{i1} \otimes W_1) \oplus (S_{i2} \otimes W_2) \oplus \dots \oplus (S_{ik} \otimes W_k)] \quad (2)$$

Substituting  $S_{it}$  and  $W_t$  with triangular fuzzy numbers, that is,  $S_{it} = (o_{it}, pit, qit)$  and  $W_t = (at, bt, ct)$ ,  $F_i$  is approximated as

$$F_i \cong (Y_i, Q_i, Z_i) \quad (3)$$

with

$$Y_i = \left(\frac{1}{k}\right) \cdot \sum_i o_{it} \cdot a_t$$

$$Q_i = \left(\frac{1}{k}\right) \cdot \sum_i p_{it} \cdot b_t$$

$$z_i = \left(\frac{1}{k}\right) \cdot \sum_t q_{it} \cdot c_i$$

for  $i=1,2,\dots,n$  and  $t=1,2,\dots,k$ .

**3.3. Choice of the Optimal Alternative**

This step includes 3 activities: 1) prioritization of decision alternatives; 2) choice of the alternative with highest priority as the optimal; and 3) ‘what-if’ analysis with the change in the input ratings.

The prioritization of the aggregated assessments is required to rank the alternatives. Since the aggregated assessments are represented as triangular fuzzy numbers, the method to rank the fuzzy triangular numbers is required. There are some methods ranking fuzzy numbers [3-5]. In this paper, the total integral value method [5] is used because of the easiness to use. Let the total integral value for triangular fuzzy number  $F = (a, b, c)$ , be defined as

$$I_r^\alpha(F) = \left(\frac{1}{2}\right) [\alpha c + b + (1 - \alpha)a] \tag{5}$$

Here,  $\alpha$  is called an index of optimism that represents the degree of optimism of the decision maker. A larger value of  $\alpha$  indicates a higher degree of optimism. For given fuzzy numbers  $F_i$  and  $F_j$ , if  $IT\alpha(F_i) < IT\alpha(F_j)$ , then  $F_i < F_j$ ; if  $IT\alpha(F_i) = IT\alpha(F_j)$ , then  $F_i = F_j$ ; and if  $IT\alpha(F_i) > IT\alpha(F_j)$ , then  $F_i > F_j$ . Since the larger  $F_i$  means the higher appropriateness to the criteria, hence, the decision alternative with the largest total integral value is regarded as the optimal decision alternative in this paper.

In final, ‘what-if’ analysis is a kind of sensitivity analysis. The decision maker can change the input ratings and observe the effect of changes in ratings on the entire hierarchy.

The procedures in this approach are summarized in Table 1.

**4. CASE STUDY**

As a case study, the approach is applied to select the most appropriate option to improve the operation reliability of a task under multiple decision criteria. The same decision-making process that was once performed [6] has been re-analyzed with this methodology.

The decision problem is briefly described. The task is assigned to the operator to control coolant flow rate supplied to a water-cooling tank. The detail of the task is as follows:

- Objective: Settle the coolant flow rate at a certain appropriate level  $F_s$ .
- State Indication: The inlet flow rate  $F_i$ , the inlet temperature  $T_i$ , and the outlet temperature  $T_o$  of coolant are indicated on a control panel
- Restriction: The difference between the inlet and the outlet temperature,  $T_i$  and  $T_o$  must be kept within a certain limit  $\Delta T$ .

- Manipulation Facility: The valve opening is controlled by the electric motor rotation corresponding to the rotation angle of a rotary switch on a control panel.
- Operator: Currently, only one operator performs this task.

**Table 1.** Synopsis of the approach

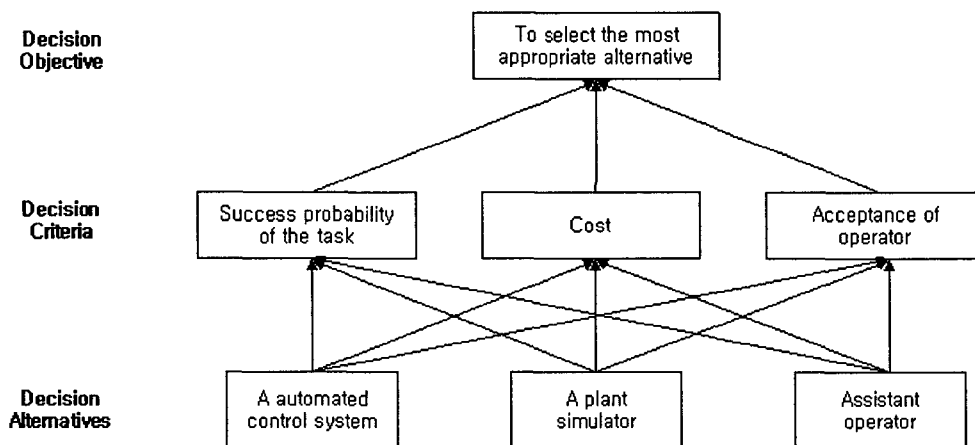
Steps	Activities	Main Tool
Representation of Decision Problem	<ul style="list-style-type: none"> <li>● Identifying a decision goal and a set of decision alternatives, <math>A=\{A_i\}</math></li> <li>● Identifying a set of decision criteria, <math>C=\{C_t\}</math></li> <li>● Building a hierarchical structure of decision problem</li> </ul>	<ul style="list-style-type: none"> <li>● Decision tree</li> </ul>
Fuzzy Set Evaluation of Decision Alternatives	<ul style="list-style-type: none"> <li>● Choosing sets of the preference ratings for the importance weight of the decision criteria and those for the alternative versus criteria</li> <li>● Evaluating the importance weight of each criterion <math>C_t</math> and the fuzzy ratings for the appropriateness of alternative <math>A_i</math> versus the criteria</li> <li>● Aggregating the weights of the decision criteria and the fuzzy ratings of the decision alternatives</li> </ul>	<ul style="list-style-type: none"> <li>● Linguistic variables, Triangular fuzzy number</li> <li>● Fuzzy mean operator</li> </ul>
Choice of the Optimal Alternative	<ul style="list-style-type: none"> <li>● Prioritization of decision alternatives using the aggregated assessments</li> <li>● Choice of the alternative with highest priority as the optimal</li> <li>● 'What-if' analysis changing input ratings</li> </ul>	<ul style="list-style-type: none"> <li>● Total integral value method</li> </ul>

This task enforces an operator to check multiple process signals and to manipulate a facility concurrently. Because of the complexity of this task, the frequency of operation error occurrence is known to be significant. Consequently the administrative managers of this plant decided to improve the operation reliability of this task by implementing the most appropriate countermeasure. The identified alternatives are as follows: 1) introduce a reliable automated control system that the operator should just set a target flow rate once and monitor accident machine faults (A1); 2) introduce a plant simulator to train the operator periodically (A2); and 3) employ an assistant operator to monitor the process state (A3). Also, the decision criteria are identified as follows: 1) success probability of the task in each alternative (C1); 2) cost required by each alternative (C2); and 3) acceptance of operators for each alternative (C3).

Given the above decision problem, the actual steps taken in this study are summarized as follows:

**Step 1. Representation of Decision Problem**

- (1) The decision goal is to select the most appropriate countermeasure to improve the operation reliability. Three decision alternatives are identified as follows:  $A = \{A1, A2, A3\}$ , where  $A1 =$  a automated control system,  $A2 =$  a plant simulator, and  $A3 =$  assistant operator.
- (2) Then, the decision-making criteria are as follows:  $C = \{C1, C2, C3\}$  where  $C1 =$  success probability of the task in each option,  $C2 =$  cost required by each option, and  $C3 =$  acceptance of operators for each option
- (3) The hierarchical structure of this problem is shown in Fig. 3.



**Figure 3.** Hierarchical structure of an example problem

**Step 2. Fuzzy Evaluation of Decision Alternatives**

- (1) With the linguistic variables which represent the importance weights of the criteria and the fuzzy ratings for the appropriateness of the alternatives versus the criteria, the term sets are assigned as follows:  $T(\text{importance}) \equiv W = \{VL, L, M, H, VH\}$ , where  $VL =$  very low,  $L =$  low,  $M =$  medium,  $H =$  high, and  $VH =$  very high; and  $T(\text{appropriateness}) \equiv S = \{VP, P, F, G, VG\}$ , where  $VP =$  very poor,  $P =$  poor,  $F =$  fair,  $G =$  good, and  $VG =$  very good. The membership function corresponding to each element of each term set is represented by the appropriate corresponding triangular fuzzy number, as shown in Table 2.

**Table 2.** Triangular fuzzy number corresponding to each linguistic value

Linguistic Variable	VL VP	L P	M F	H G	VH VG
Fuzzy Number	(0, 0, 0.25)	(0, 0.25, 0.5)	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)	(0.75, 1, 1)

- (2) The importance rating for each decision criterion and the appropriateness rating for each decision alternative with respect to each criterion are assigned and summarized in Tables 3 and 4, respectively.
- (3) The fuzzy appropriateness index is calculated for each decision alternative as shown in the last column of Table 4.

**Table 3.** Importance rating for each decision-making criterion

Criterion	Success probability	Cost	Acceptance
Importance Rating	Very high	Low	Low

**Table 4.** Appropriateness rating and fuzzy appropriateness index for each alternative

Alternative	Appropriateness rating			Fuzzy appropriateness index
	Success probability	Cost	Acceptance	
Automated system	Very good	Poor	Poor	(0.1875,0.3750,0.5000)
Plant simulator	Good	Very good	Fair	(0.1250,0.3750,0.6250)
Assistant operator	Very good	Fair	Good	(0.1875,0.4375,0.6250)

### **Step 3. Choice of the Optimal Alternatives**

- (1) For three different values of  $\alpha$ , the total integral values for fuzzy appropriateness indices are calculated for each decision alternative as shown in Table 5.
- (2) The total integral values are also ranked and summarized in the parentheses of Table 5. One may notice that regardless of optimism index value of the decision maker, the first rank of decision alternatives does not change except the ranks of alternatives A1 and A2 change in the optimistic case. Hence, the employment of assistant (i.e., A3) is identified as the most appropriate countermeasure to improve the operation reliability.
- (3) As a 'what-if' analysis, we change the importance rating for each decision criterion as shown in 6 and observe the effects of the change in input ratings. And the corresponding fuzzy appropriateness index is summarized in the last column of Table 7. Also, the total integral values are summarized in Table 8. This analysis also shows that the employment of assistant (i.e., A3) is identified as the most appropriate countermeasure regardless of the change in decision criteria.



**Table 5.** Total integral value and rank for various values of optimism index  $\alpha$

Alternative	Optimism index value of decision maker		
	Moderate ( $\alpha=0.5$ )	Pessimistic ( $\alpha=0.0$ )	Optimistic ( $\alpha=1.0$ )
A1	0.3594 (2)	0.2813 (2)	0.4375 (3)
A2	0.3334 (3)	0.2500 (3)	0.5000 (2)
A3	0.3855 (1)	0.3125 (1)	0.5313 (1)

**Table 6.** Importance rating for each decision-making criterion

Criterion	Success probability	Cost	Acceptance
Importance rating	High	Medium	Medium

**Table 7.** Appropriateness rating and fuzzy appropriateness index for each alternative

Alternative	Appropriateness rating			Fuzzy appropriateness index
	Success probability	Cost	Acceptance	
Automated system	Very good	Poor	Poor	(0.1875,0.4167,0.4584)
Plant simulator	Good	Very good	Fair	(0.2084,0.5000,0.7709)
Assistant operator	Very good	Fair	Good	(0.1875,0.5417,0.7709)

**Table 8.** Total integral value and rank for various values of optimism index  $\alpha$

Alternative	Optimism index value of decision maker		
	Moderate ( $\alpha=0.5$ )	Pessimistic ( $\alpha=0.0$ )	Optimistic ( $\alpha=1.0$ )
A1	0.3696 (3)	0.1980 (3)	0.4376 (3)
A2	0.4949 (2)	0.3542 (2)	0.6355 (2)
A3	0.5103 (1)	0.7292 (1)	0.6563 (1)

### 5. DISCUSSION AND CONCLUSION

In this paper, a simple and easy-to-use approach using the fuzzy set theory is proposed to aid the evaluation of the degree of appropriateness for each alternative in the decision-making process of nuclear system management under uncertain environment. The applicability of this method has been demonstrated through an example of a decision problem to improve operation reliability of a plant task. The main features of the proposed method are summarized as follows;

- A reasonable and consistent solution for a decision problem under various mutually conflicting criteria is systematically obtained.

- The foundations behind the decision are visualized to allow re-examinations and adjustments for better consensus formulation

The case study presented in this study is rather simple and straightforward. However, the approach could be more comprehensive with some further efforts in the future to resolve the issues as follows:

- How to assign the membership grades of a fuzzy set to represent a linguistic variable: It is always the starting point to assign the membership grades for any fuzzy set analysis, and the membership grades should represent the linguistic variable as realistically as possible. Prior to actual assignment, it is recommended to perform a series of sensitivity study to identify the impact of varying the membership grades.
- How to perform arithmetic operations in the fuzzy set analysis: Although extensive researches have been done to develop the basic concept of the fuzzy set theory, it often happens that individual practitioners have their own intuitive notions about how concepts of arithmetic operation should be applied. There is a need to generalize the arithmetic operation.

Once these issues are well resolved, the use of fuzzy sets would be generally accepted as one of good decision-making techniques with respect to nuclear system management.

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