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Optimal Designs of Complete Diallel Crosses

Kuey Chung Choi *
Department of Computer Science and Statistics
Chosun University, Kwangju 501-759, Korea

Abstract. Two general methods of construction leading to several series of universally optimal block designs for complete diallel crosses are provided in this paper. A method of constructing variance balance designs is also given.

Key Words : *general combining ability, variance balanced designs, nested rows and columns designs.*

1. INTRODUCTION

Diallel crosses are commonly used to study the genetic properties of inbred lines in plant and animal breeding experiments. Suppose there are p inbred lines and let a cross between lines i and j be denoted by (i, j), $i < j = 1, 2, \ldots, p$. Let n_c denote the total number of distinct crosses in the experiment. Our interest lies in comparing the lines with respect to their general combining ability *(gca)* parameters. We consider complete diallel crosses which involve all possible crosses among p parental lines with $n_c = p(p-1)/2$.

Gupta and Kageyama (1994) introduced the method of constructing block designs for diallel crosses using p lines as the treatments of the design. The previous authors e.g. Singh and Hinkelmann (1988), Agarwal and Das (1990), Singh and Hinkelmann (1995), used all the n_c crosses as the treatments of the design leading to several replications of each cross for optimality. Gupta and Kageyama (1994) proved that the existence of a nested balanccd incomplete block design, the nest having block size two, implies the existence of a universally optimal (Kiefer, 1975) block design for diallel crosses. They also provided two general series of optimal designs. The orthogonal blocking of diallel cross plans was investigated by Gupta et al. (1995). In an orthogonal design, no loss of efficiency on the comparisons of interest is incurred due to blocking. Subsequently, Dey and Midha (1996), Das,

* *E-mail address : kjchoi@mail.chosun.ac.kr*

Dey and Dean (1998), and Das, Dean and Gupta (1999) constructed further designs using Gupta and Kageyama's method.

The purpose of this paper is to provide two general methods of construction leading to several series of optimal designs. A method of constructing variance balanced designs is also provided.

2. CONSTRUCTIONS OF OPTIMAL DEGINES

Two general methods of obtaining universally optimal designs are presented in this section. We first give below a series of designs in $p-1$ blocks of size p each, where p denotes the number of lines.

Theorem 2.1 There exists an optimal diallel cross design with parameters p, $b = p-1$, $r=2$, $k=p$. The ith block of the design is obtained by developing the cross $(0,i)$ mod (p). $i = 1,2,\ldots,b$.

The optimality of designs obtained using the above theorem follows from Gupta, Das and Kageyama (1995).

Example 2.1 For $p = 6$, the above theorem yields the following design, where rows give the block contents:

{ (0,1), (1,2),(2,3),(3,4),(4,5),(5,0) }
{ (0,2), (1,3),(2,4),(3,5),(4,0),(5,1) }
{ (0,3), (1,4),(2,5),(3,0),(4,1),(5,2) }
{ (0,4), (1,5),(2,0),(3,1),(4,2),(5,3) }
{ (0,5), (1,0),(2,1),(3,2),(4,3),(5,4) }

For instance, the first block contains the crosses {(0,1), (1,2), (2,3), (3,4), (4,5), (5,0)}, where the lines are coded as 0,1,2,3,4,5, and for $i > j$, is replaced by (j,i).

We now give a method of construction using balanced incomplete block designs with nested rows and columns (BIBRC). We first state the following definition.

Definition: An arrangement of p treatment in b blocks each containing k' treatments arranged in q_1 rows and q_2 columns is called a BIBRC or BIBRC $\{p, b, r', q_1, q_2, \lambda\}$ if

i. each treatment appears r' times,

ii. each treatment appears in each block at most once,

iii. $q_1N_1N_1' + q_2N_2N_2' - NN' = aI_p + \lambda J_p$

for some integers a and λ, where N,N_1,N_2 are the treatment-block, treatment-row, and treatment-column incidence matrices respectively, I_p is the $p \times p$ identity matrix and J_p is the $p \times p$ matrix of ones. A BIBRC in which each of the component designs

N_1 and N_2 is a balanced incomplete block design is called a Series A design (Agrawal and Prasad, 1983)

Theorm 2.2 The existence of a Series A BIBRC $\{p,\ b,\ r',\ q_1 = 2,\ q_2,\ \lambda\}$ implies the existence of an optimal diallel cross design with parameters p, b, $r = r'/(p-1)$, $k = q_2$.

A design of the above theorem is obtained by taking the treatments as p lines and forming the crosses between the two lines in each column of the BIBRC. The optimality of the resulting diallel cross design follows from Gupta and Kageyama(1994).

Example 2.2 Using the BIBRC {13,39,12,2,2,1} given in Example 1 of Uddin(1992), the following diallel cross design for $p = 13$ can be obtained;

{(10,2),(1,6)},{(10,6),(2,1)},{(10,9),(3,8)},
{(10,7),(5,4)},{(10,1),(6,2)},{(10,4),(7,5)},
{(10,3),(8,9)},{(10,8),(9,3)},{(11,3),(1,7)},
{(11,9),(2,4)},{(11,7),(3,1)},{(11,2),(4,9)},
{(11,6),(5,8)},{(11,8),(6,5)},{(11,1),(7,3)},
{(11,5),(8,6)},{(11,4),(9,2)},{(12,4),(1,8)},
{(12,5),(2,3)},{(12,2),(3,5)},{(12,8),(4,1)},
{(12,3),(5,2)},{(12,7),(6,9)},{(12,9),(7,6)},
{(12,1),(8,4)},{(12,6),(9,7)},{(13,5),(1,9)},
{(13,7),(2,8)},{(13,6),(3,4)},{(13,3),(4,6)},
{(13,9),(5,1)},{(13,4),(6,3)},{(13,8),(7,2)},
{(13,2),(8,7)},{(13,1),(9,5)},{(10,12),(13,11)},
{(10,11),(12,13)},{(10,13),(11,12)},{(7,9),(12,6)}

For $i > j$ cross (i, j) is replaced by (j, i).

Several families of Series A BIBRC designs with $q_1 = 2$ have been given by Uddin(1992). Using these families, the following optimal diallel cross designs on the actual construction of Series A BIBRC designs.

i. For $s = 4t - 1$ a prime or a prime power, there exists an optimal diallel cross design with parameters $p = s^2 + s + 1$, $b = s(s^2 + s + 1)$, $r = 1$, $k = 2$.
ii. For $s = 4t + 1$ a prime or a prime power, there exists an optimal diallel cross design with parameters $p = s^2 + s + 1$, $b = 2s(s^2 + s + 1)$, $r = 2$, $k = 2$.
iii. For $s = 12t - 7$ with the finite field of order s having 3 as its primitive root, there exists an optimal diallel cross design with parameters $p = s^2 + s + 1$, $b = \frac{1}{4}s(s+1)(s^2 + s + 1)$,$r = 1$,$k = 2$.
iv. For $s = 2t + 1$ a prime or a prime power, there exists an optimal diallel cross design with parameters $p = s^2 + s + 1$, $b = \frac{1}{2}s(s+1)(s^2 + s + 1)$, $r = 2$, $k = 2$.

3. BALANCED DEGIGNS USING MOLs

A method of constructing variance balanced designs using a pair of mutually orthogonal Latin squares (MOLs) is presented in this section. Each of the crosses is replicated two times in the resulting designs.

Theorem 3.1 The existence of a pair of mutually orthogonal Latin squares of order p implies the existence of a variance balanced diallel cross design with parameters $p,\ b = p,\ r = 2,\ k = p - 1$.

To construct a design of the above theorem, we first obtain a Greaco-Latin square using a pair of MOLs. The ith block of the diallel cross design is then obtained from the ith column of the Greaco-Latin square, $i = 1, 2, \ldots, p$. Note that pairings of the type (j, j), $j = 1, 2, \ldots, p$. are not considered and are therefore deleted from the design. Futher, the pair of MOLs chosen should be such that the resulting Greaco-Latin square has exactly one pairing of the type (j, j) in each of its columns.

Since a pair of such MOLs can always be constructed when p is a prime power, we have the following.

Corollary 3.1 For p a prime or a prime power, there exists a variance balanced diallel cross.

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