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# PREDICTION OF SOUND RADIATION FROM TIRE TREADBAND VIBRATION

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**ABSTRACT**–The noise generated from a treadband mechanism of a tire has been the subject of this research. In particular, the treadband has been treated as an infinite tensioned beam resting on an elastic foundation which includes damping. The main objective is here to predict the sound power generated from a system mentioned above by locating harmonic point forces representing the excitation of treadband at the contact patch. It is possible to predict the sound power radiated from this structure by wavenumber transformation techniques. To find out the minimum radiated sound power, All parameters were varied. Thus this model can be used as a tire design guide for selecting parameters which produce the minimum noise radiation.

**KEY WORDS :** Harmonic point forces, Relative sound power, Sound radiation, Sound intensity, Tire, Wavenumber ratio

## 1. INTRODUCTION

The model of tread element may be used to predict the force exerted on the treadband by contacting the road. Here, this tread element model is assumed to be an infinite beam model so that treadband can be treat as an infinite elastic beam. The elastic foundation represents the sidewall stiffness supporting the treadband. Wavenumber transformation techniques make it possible to predict the sound power radiated by such a structure.

In this paper, the problems in sound radiation of elastic beam under the action of harmonic point forces moving at subsonic speeds is studied. Further the reaction due to light fluid loading on the vibratory response of the beam is taken into account. Here, the beam is assumed to occupy the plane $z=0$. The material of beam and the elastic foundation are also assumed to be lossless and governed by the law of Bernolli-Euler beam theory including a tension force (T), damping coefficient (C), and stiffness of foundation ($k_s$).

Mogilevskii 1981 studied the problems related to sound radiation from beams under the action of a moving harmonic point force in the absence of an elastic foundation. Keltie and Peng 1985 computed the sound power produced by a point-forced elastic beam, and obtained quantitative measures of the power produced by the flexural nearfield and the propagating portions of the beam's response.

The non-dimensional sound power is derived by integrating of the surface intensity distribution over the entire beam. The expression for sound power is integrated numerically and the results are examined as a function of Mach number, M, the wavenumber ratio, $\gamma$, and stiffness factor, $\psi$.

All parameters may be varied to allow for the identification of optimal values (i.e., the set of parameters which result in minimum radiated sound power).

Hence, this model lends itself to the problem of treadband sound power minimization. Also the method of superposition suggests that an arbitrary periodic point load may be applied. Such an input causes altering the sequence of the tread elements and pitch length, which provides another tool for the minimization of radiated sound power.

The purpose of this paper is to explain the response of a sound power over a number of non-dimensional parameters describing forcing velocity, treadband tension, treadband stiffness, treadband damping, and foundation stiffness.

## 2. FORMULATION OF SOUND POWER

In this section, a mathematical modeling and the key procedures to predict the sound power radiated by such a structure will be described. Under the assumption that an infinite beam occupies the plane $z=0$ and the beam is excited by a harmonically oscillating point force moving in the x-direction at the velocity $v_o$, as shown in Figure 1.

The space where $z>0$ is filled with the air. The infinite

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beam represents an "unrolled" tire. The sound radiated from the treadband on either side of the contact patch. The equation of motion for the beam is:

$$D\frac{\partial^4 u}{\partial x^4}+\rho_s A\frac{\partial^4 u}{\partial t^2}-T\frac{\partial^2 u}{\partial x^2}+\left(C\frac{\partial u}{\partial t}+k_s U\right)$$
$$= F_o\delta(x-\upsilon_o t)e^{j\omega t}-p(x,z=0,t) \quad (1)$$

Where $u(x, t)$ is the transverse displacement of the beam, $\omega$ is the circular driving frequency, D is the flexural stiffness of the beam; $\rho_s A$ is the mass per unit length of the beam, T is the axial tension force; C is the foundation damping coefficient; $k_s$ is the foundation stiffness, $F_o$ is the input force amplitude, p is the acoustic pressure induced by the surface motion, and $\delta(x)$ is the Dirac delta function respectively. The pressure distribution induced in the air by the vibration beam is denoted by $p(x, z, t)$ satisfying the wave equation in two dimensional space, and is given by

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c_o^2}\frac{\partial^2}{\partial t^2}\right]=0 \quad (2)$$

Where $c_o$ is the sound speed in the acoustic medium. The boundary condition at $z=0$ is given by

$$\rho_o\frac{\partial^2 u}{\partial t^2}=-\frac{\partial p}{\partial z}|_{z=0} \quad (3)$$

Where $r_o$ is the mass density of the acoustic medium. By applying the spatial Fourier Transformation, the equation becomes

$$FT(f(x))\int_{-\infty}^{\infty}f(x)e^{j\xi x}dx$$

Where $\xi$ is the wave number variable. In conjunction with the boundary condition, the time averaged radiated sound power may be obtained by integrating the surface of acoustic intensity distribution over the entire beam. The force function in the wavenumber domain may be written as

$$\hat{F}(\xi,t)=F_o e^{(\xi\upsilon_o+\omega)t} \quad (4)$$

This form implies that in the wavenumber domain, both the transformed displacementhat $U(\xi,t)$ and pressure $p(\xi,z,t)$ will have the common factor $e^{j(\xi\upsilon_o+\omega)t}$.

That is,

$$\hat{U}(\xi,t)=U(\xi)e^{j(\xi\upsilon_o+\omega)t} \quad (5a)$$

$$\hat{p}(\xi,z,t)=p(\xi,z)e^{j(\xi\upsilon_o+\omega)t} \quad (5b)$$

By substituting equation (5a) and (5b) into the beam equation (1) and the acoustic equation (2), it is easily found that

$$U(\xi)=\frac{F_o}{Z_a+Z_b} \quad (6)$$

$$p(\xi,z=0)=Z_a U(\xi) \quad (7)$$

Where,

$$Z_a=\frac{j\rho_o(\xi\upsilon_o+\omega)^2}{\sqrt{(k_o+M\xi)^2-\xi}}$$

$$Z_b=[D\xi^4-\rho_s A(\xi\upsilon_o+\omega)^2+T\xi^2+K_s]+j[(\xi\upsilon_o+\omega)C]$$

Solving the wave equation,

$$\left[\frac{\partial^2}{\partial z^2}-\frac{(\xi\upsilon_o+\omega)^2}{c_o}-\xi^2\right]p(x,z)=0 \quad (8)$$

$$\left[\frac{\partial^2}{\partial z^2}+((k_o+M\xi)^2-\xi^2)\right]p(\xi,z)=0 \quad (9)$$

Therefore, pressure $p(\xi, z)$ is

$$p(\xi,z)=p(\xi,z=0)e^{-jk_z z} \quad (10)$$

$$k_2=\begin{cases}-j\sqrt{\xi^2-(K_o+M\xi)^2} : & \xi^2>(K_o+M\xi)^2\\ \sqrt{(K_o+M\xi)^2-\xi^2} : & \xi^2<(K_o+M\xi)^2\end{cases} \quad (11)$$

Where, M is the Mach number which is the same as $V/c_o$, and $K_o$ is the acoustic wavenumber equal to $\omega/c_o$. Surface intensity distribution $I(x)$ is

$$I(x)=\frac{1}{2}Re[P(x)V^*(x)] \quad (12)$$

Where, $P(x)$ is surface pressure and $V^*(x)$ is a conjugate form of surface velocity. By integrating the surface intensity distribution over entire beam, sound power can be obtained from the following formula for a unit width.

$$W=\frac{1}{2}Re[\int_{-\infty}^{\infty}P(x)V^*(x)dx] \quad (13)$$

$$W=\frac{1}{2}Re$$

$$\left[\int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}P(\xi_1)e^{-j\xi_1 x}d\xi_1\right]\cdot\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}V^*(\xi_2)e^{j\xi_2 x}d\xi_2\right]dx\right] \quad (14)$$

Where, $\xi_1$, $\xi_2$ are dummy variables. There can be written as follows:

$$W=\frac{1}{4\pi}Re$$

$$\left[\int_{-\infty}^{\infty}[\int_{-\infty}^{\infty}P(\xi_1)d\xi_1]\cdot[\int_{-\infty}^{\infty}V^*(\xi_2)d\xi_2]\cdot\left[\frac{1}{2\pi}e^{-j(\xi_1-\xi_2)x}\right]dx\right] \quad (15)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-j(\xi_1-\xi_2)x}dx=\delta(\xi_1-\xi_2) \quad (16)$$

Thererfore,

$$W = \frac{1}{4\pi} Re[\int_{-\infty}^{\infty} P(\xi) V^*(\xi) d\xi] \tag{17}$$

Surface pressure and a conjugate form of surface velocity in wavenumber domain are

$$P(\xi, z=0) = \frac{j\rho_o(\xi V+\omega)^2}{\sqrt{(K_o+M\xi)^2-\xi^2}} U(\xi) \tag{18}$$

$$V^*(\xi) = j(\xi V+\omega) U^*(\xi) \tag{19}$$

By substituting equation of surface pressure and conjugate velocity into a sound power spectrum, the sound power can be obtained as follows:

$$W = \frac{\rho_o}{4\pi} Re\left[\int_{-\infty}^{\infty} \frac{(\xi V+\omega^3)}{\sqrt{(K_o+M\xi)-\xi^2}} |U(\xi)|^2 d\xi\right] \tag{20}$$

Referring to Equation (11), it is seen that the denominator of the integral in Equation (20) is real only over a restricted interval of the integration range. Specializing to the case of subsonic motion of the traveling force, the limits within which $k_z$ is real are given by

$$\xi_1 = \frac{-K_o}{1+M} \le \xi \le \frac{K_o}{1-M} = \xi_2 \tag{21}$$

Let $\xi \to K_o\zeta$. (for dimensionless)

$$W = \frac{\rho_o F_o^2}{4\pi(\rho_s A)^2 \omega} \Pi \tag{22}$$

The non-dimensional sound power radiated from the beam is given by

$$\Pi = \int_{\zeta_1}^{\zeta_2} \frac{\alpha^3\sqrt{\alpha^2-\zeta^2}d\zeta}{[(\gamma^4\zeta^4+2T_1\gamma^2\Psi\zeta^2-\alpha^2+\Psi^2)\sqrt{\alpha^2-\zeta^2}]^2 + [2\beta\Psi\alpha\sqrt{\alpha^2-\zeta^2}+\alpha_o\alpha^2/\gamma]} \tag{23}$$

Where $\Pi$ is the non-dimensional sound power obtained by multiplying the dimensional power by the factor $4\pi\omega\rho_s^2A^2/\rho_oF_o^2$. In the integral, $\xi_1=-1/(1+M)$ and $\xi_2=1/(1-M)$ are the shifted limits of the integration range; $M=v_o/c_o$ is the Mach number of the moving force; $\alpha=1+M\zeta$; $\gamma=K_o/k_b$ is the ratio of the acoustic wavenumber to the bending wavenumber; $k_b=(\rho_sA\omega_2/D)^{1/4}$ is the free bending wavenumber, and $\alpha_o=\rho_oc_l/\rho_sAc_o\sqrt{12}$ is the fluid loading factor. $\rho_o$ is the volume density of the air, $c_l$ is the longitudinal wave speed of the beam material and $\Psi$ is foundation stiffness factor. $T_1$ $(T/(2\sqrt{k_sD}))$ is the axial tension factor, and $\beta=C/(2\sqrt{\rho_sAk_s})$ is foundation damping factor.

## 3. NUMERICAL RESULTS AND DISCUSSION

The curves presented show the variation of beam response and radiated sound power over a range of the various non-dimensional parameters. Identifying these parameters of passenger vehicle tires will be the subject of further investigation. To investigate the effects of the stiffness factor ($\psi$) and tension (T) for the radiated sound power level, the sound power was calculated as a function of the variables, $\psi$, with a few different values of the force Mach number, M, and for a constant values of the wavenumber ratio, $\gamma$. The sound power radiated from a beam under the action of one point forces is typically represented by the curves in Figures 1.

The relative sound power level versus stiffness and Mach number for the air loading is shown in Figure 2. For a case of $M=0$, the most prominent feature is relatively high radiation peaks emerge around the value of $\psi=1.0$ for higher frequency. This phenomenon can be called a resonance radiation. As the Mach number increases, two different radiated sound power peaks are build up. For example, if the Mach number $M=0.5$, then one of the peaks is located in the range of $\psi=1.0$ and the other is in $\psi>1.0$. For the higher frequency range, the sound power level is decreased if $\psi>1.5$ as shown in figure, while the range of increases over 1.0 as the driving frequency gets lower. The figure shows the sound power level increased effectively on the compressive forces

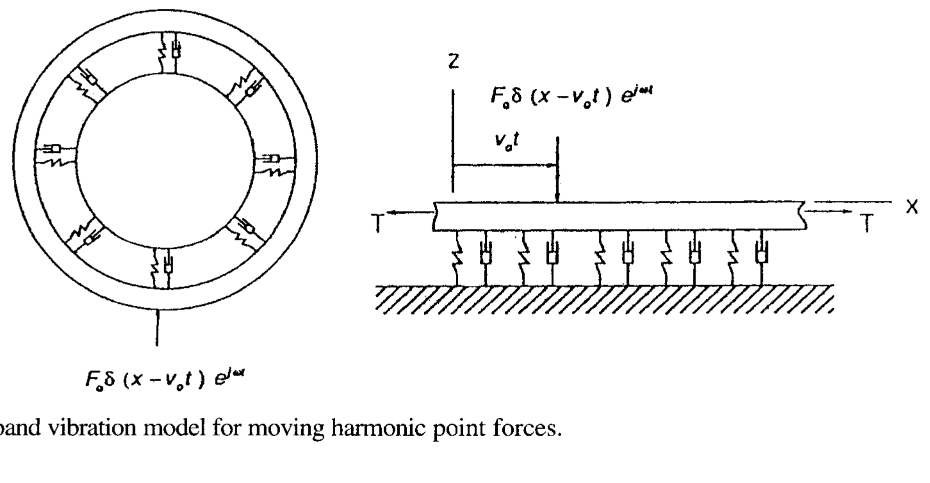


Figure 1. Treadband vibration model for moving harmonic point forces.

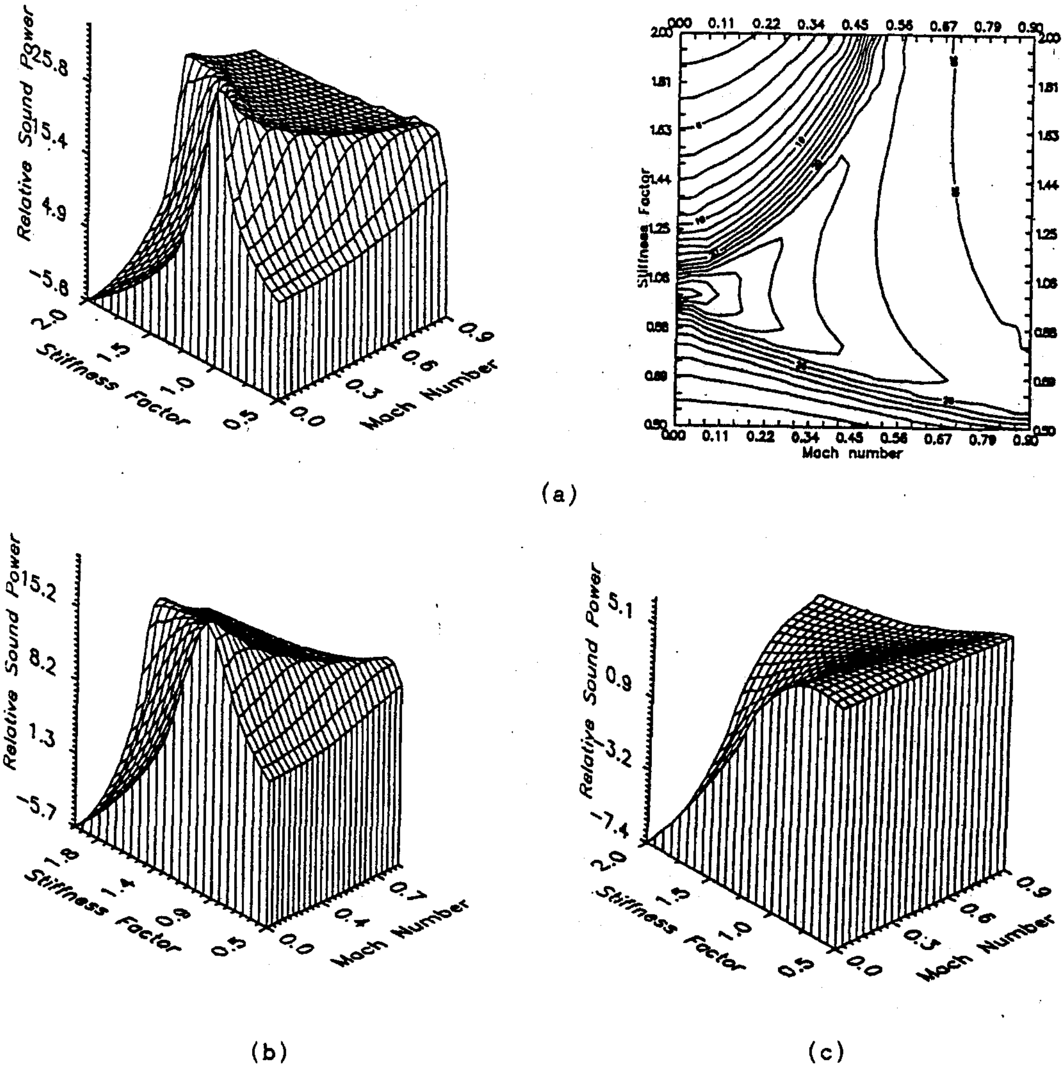


Figure 2. Relative sound power level versus stiffness factor and mach number.
(a) $T_1 = 0.4$, $\beta = 0.01$, $\gamma = 0.2$, (b) $T_1 = 0.4$, $\beta = 0.1$, $\gamma = 0.2$ and (c) $T_1 = 0.4$, $\beta = 0.5$, $\gamma = 0.2$

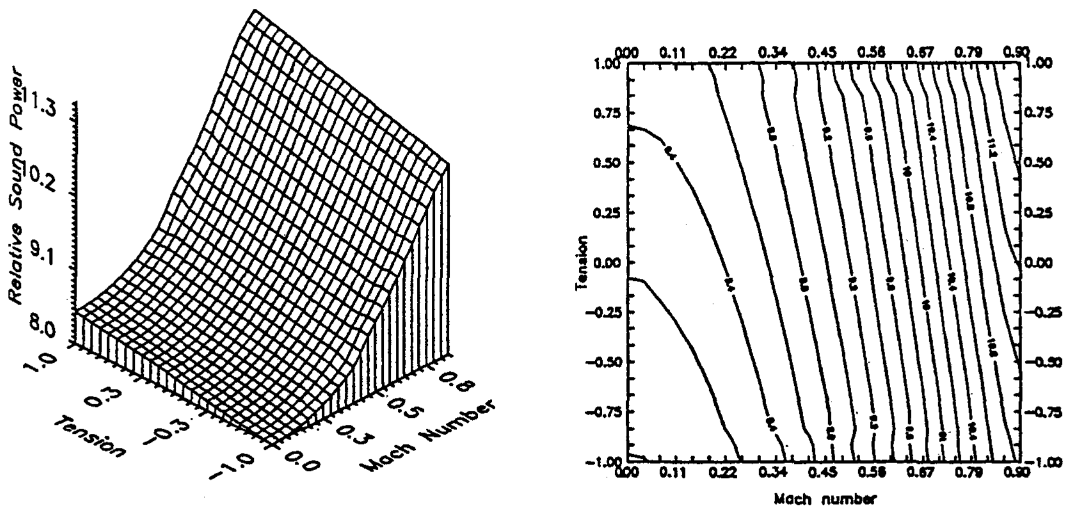


Figure 3. Relative sound power level versus tension and mach number. ($\beta = 0.1$, $\gamma = 0.2$, $\psi = 0.5$)

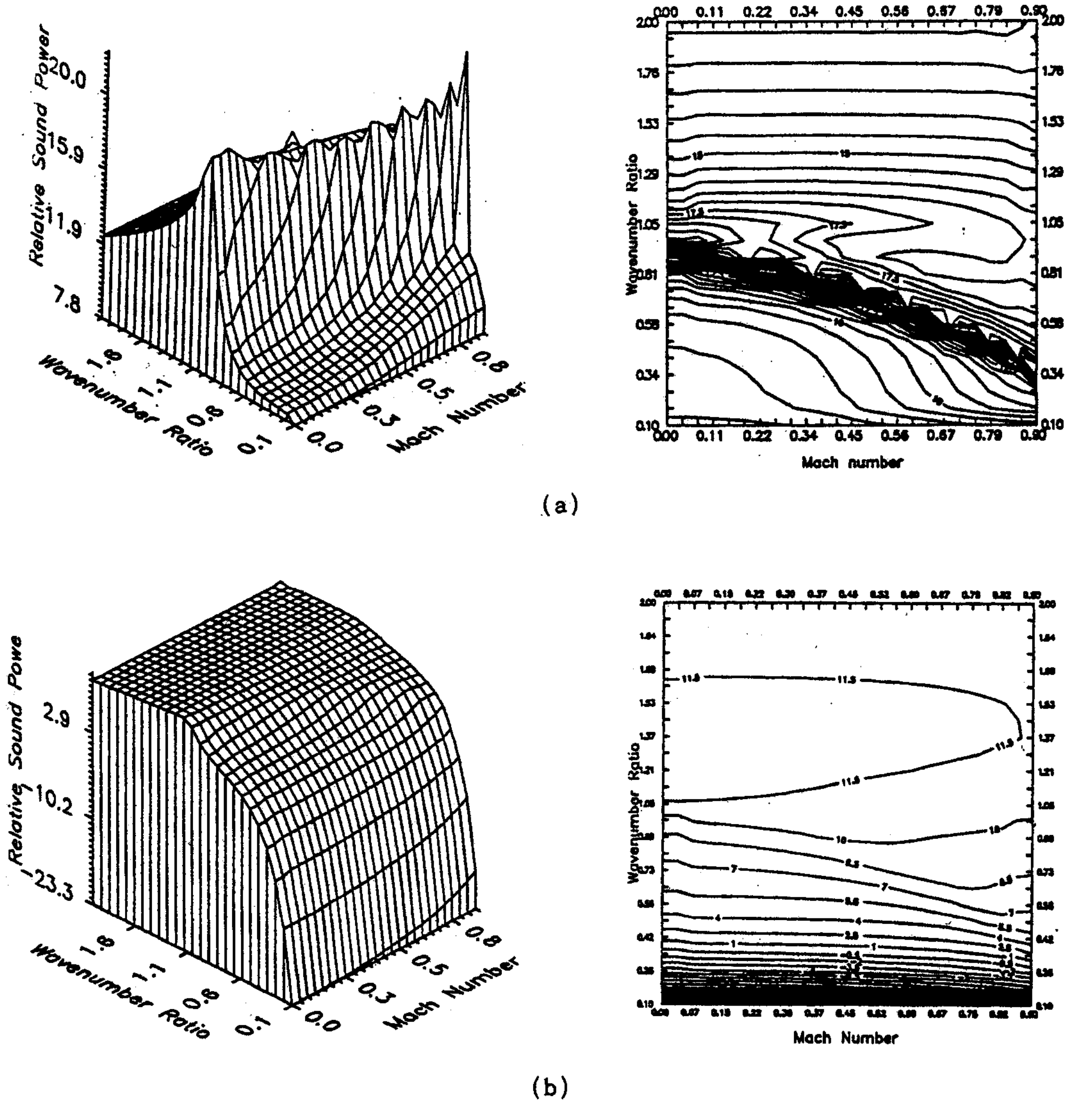


Figure 4. Relative sound power level versus wavenumber and mach number.
(a) Air loading $T = 0.0$, $\beta = 0.1$, $\gamma = 0.5$ and (b) Water loading $T = 0.0$, $\beta = 0.1$, $\gamma = 0.5$

rather than tensile forces. It is found that the forces due to the Mach number and tensile affect the location of the radiated sound power peak.

## 4. CONCLUSIONS

The effects on the sound power emitted by harmonic point forces moving on infinite elastic beams is investigated and also the effects of foundation stiffness, tension, damping and Mach number on the radiated sound power are investigated in this paper, the following conclusions can be drawn:

(a) The values of stiffness factor($\psi$) give an important effect on the radiated sound power levels. For the case of $M=0$, a resonance radiation and a coincidence peak are produced close to the value of $\psi = 1.0$. This phenomenon attributed to the doppler shift effect.

(b) When tensile force is applied, the sound power gets larger than the case of compressive force is applied.

(c) As the Mach number increases, the strong coincidence radiation peak for $M=0$ located at $\gamma = 1.0$ and the coincidence peak changes to range of $\gamma<1$ accordingly. The expression for the radiated sound power makes it possible to find parameter regions where the radiated sound power is minimized. Thus, in principle this model may form the foundation of a design guide for choosing tire parameter so as to minimize noise radiation.

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