이중 셔틀 자동창고 시스템의 주행시간 모델에 관한 연구

- A Study on Travel Time Model of double shuttle AS/RS -

김 채 수*
Chae-Soo Kim *

신 성 환**
Seong-Whan Shinn**

ABSTRACT

This paper considers automated storage and retrieval systems with double shuttle. We developed the expected travel time model based on the first come first served rule. An heuristic procedure by retrieval order sequencing was presented which aimed to improve the operation efficiency taking advantage of "No cost zone". Through sensitivity study, we evaluated the performance of the double shuttle system working on the four command cycle.

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* School of Business and Economics, Halla University

** Department of Industrial Engineering, Halla University

1. Introduction

Automated storage and retrieval system (AS/RS) is characterized by using a storage/retrieval (S/R) machine, palletized loads and storage racks, and controlled by centralized computer systems.

Compared with traditional warehouse, AS/RS has many benefits such as savings in labor cost, improved material flow and inventory control, high flow-space utilization, a lower incidence of misplacement or theft, and an improved throughput level.

Maximal benefits of such a system are dependent upon the optimal design of the system (the number of stacker cranes, their horizontal and vertical speeds, the length and height of the storage racks, etc.), and the optimal scheduling of the system. Our current research effort considers optimal scheduling of automated warehousing systems, with eventual feedback to the optimal design of such systems.

Much of the previous AS/RS related research papers have dealt with the storage assignment and retrieval problem in conventional unit load system. Grave, Hausman and Schwarz [7] considered three kinds of storage assignment policies (random, full turnover based and class based) in terms of the expected S/R machine travel time. Later they [6] extended the work to include interleaving, that is, the sequencing of storage and retrieve requests, and they [3] simulated the scheduling policy. Bozer and White [8] studied the similar problem where the rack is rectangular in time and computed the expected travel times for random storage assignment under both single and dual command cycles. Relaxing the assumption that the rack is square in time, Hwang, Koh, and Jang [1] determined the optimal rack shape factor for class based storage assignment.

For retrieval sequencing in conventional unit load AS/RS where several retrieval requests are available and dual command cycles are performed. Han, McGinnis, Shieh and White [4] developed a heuristic sequencing rule taking advantage of "No Cost Zone" concept, and they approximated expected travel times.

Alternative I/O locations and rack configurations are considered by Bozer and White [8]. Also, corresponding expected travel time models were derived by them. Additionally, they examined various dwell point strategies for the S/R machine.

All of those papers, they are assumed that the AS/RS is for conventional unit load, that is, the S/R machine handles one unit at a time.

In this study, we consider the AS/RS with double shuttle which be able to treat two unit at a time. In unit load system, dual command cycle is composed of one storage and one retrieval, whereas, in double shuttle system, two storage and two retrievals. Hwang, Kim and Koh [2] considered double shuttle in carousel system which is also a kind of AS/R system.

With two shuttles, the S/R machine is able to perform two storage and two retrieval operations in a cycle. A cycle is a sequence of operations, beginning and ending with the S/R machine at the I/O station. We will call this type of operations four command (FC) cycle.

We suggest two kinds of operating rules for FC cycle. One is First Come First Served (FCFS) rule, the other is an heuristic procedure which exploit the concept of "No Cost Zone" to improve the efficiency of double shuttle.

This paper is organized as follows : Section 2 presents the basic assumptions and basis of model development. Some notations and normalizaton concept are addressed in section 3. In section 4, we develop the expected cycle time models for four commands under the FCFS operating rule. respectively. We suggest a heuristic procedure by retrieval order sequencing to improve operation efficiency in section 5. Finally, concluding remarks appear in section 6.

2. Basic assumption

In accordance with Hausman et al. [7], the following assumptions are made:

(1) Each pallet holds only one part number or item type.

(2) All storage locations are the same size, as are the pallets themselves..

(3) The system is served by a S/R machine with double bucket (double shuttle) which carries two units load at a time.

(4) The S/R machine can move simultaneously in horizontal and vertical direction (Chebyshev travel)

(5) The rack length and height, and the movement speed of the S/R machine are known.

(6) The pickup/deposit (P/D) time of the S/R machine is constant for all cycles. For convenience P/D times are ignored. The P/D time is generally independent of the rack shape and the travel velocity of the S/R machine.

(7) The rack is considered to be a continuous rectangular face where the input/output (I/O) point is located at the lower left hand corner.

(8) The system is square in time.

3. Rack Normalization and Notations

The rack face is assumed to be a continuous rectangle in the time with known dimensions. As the previous result of Bozer and White [8], the rack face will be "normalized" by the horizontal travel time, t_h and vertical travel time t_v by T, where

s_h= horizontal travel speed

s_v= vertical travel speed

L = rack length

H = rack height

t_h= time to reach the end of the rack

t_v= time to reach the top of the rack

T = Max{ t_h, t_v} = normalization constant

b = Min{ t_h, t_v} / T

The factor b has bean referred to as the "shape factor" for the rack. It will be assumed without loss of generality that $t_h \le t_v$, and thus the rack has dimensions 1 x b as shown in Figure 1. Note that if $t_h = t_v$ then the rack is said to be squared in time.

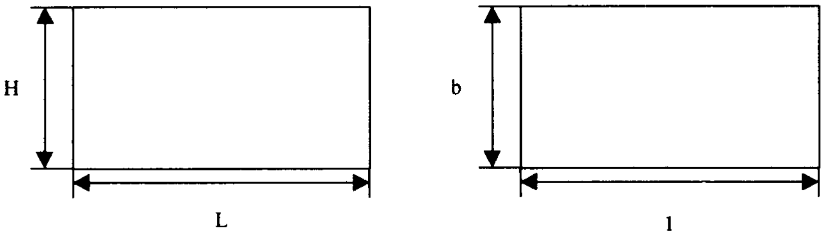

Figure 1. Rack normalization

4. Expected Four Command Cycle Time

In a FC cycle (refer to Figure 2), the shuttles of the S/R machine leave the I/O station with two storage loads and move to a desired empty rack location P_{s_1}, where one of the loads is stored. After storing the load, both the shuttles proceed to the nearest retrieval location, P_{r_1}, where a load is retrieved by the empty shuttle and the second storage load is stored in the empty rack location, P_{s_2}, created by the retrieval. Note that $P_{r_1} = P_{s_2}$ Then the shuttles proceed to another retrieval location, P_{r_2}, and execute a retrieval. Finally, the shuttles move to the I/O point.

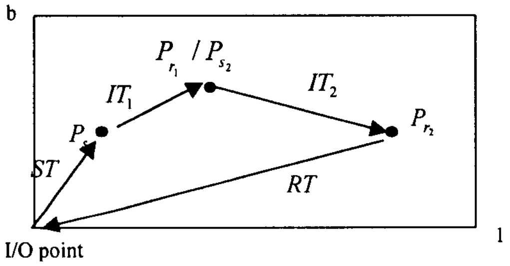

Figure 2. Four command cycle

The four command cycle time FC in the system can be expressed as

$$FC = ST + IT_1 + IT_2 + RT \tag{1}$$

where ST = the shuttle movement time from I/O point to the first storage point,
IT_1 = the first interleave time from P_{s_1} to P_{s_2},
IT_2 = the second interleave time from P_{s_2} to P_{r_2},
and RT= the shuttle movement time from P_{r_2} to the I/O station.

To find the expected FC cycle time, we have to know the expected value for the first interleave time (IT_1). For the other travel time (ST, IT_2, RT), we can utilize the result of Bozer and White [8] for single shuttle system.

Let TB_{ij} be the travel between time from ith order point to jth order point,

$$IT_1 = \text{Min}\ \{ TB_{P_{s1}P_{r1}},\ TB_{P_{s1}P_{r2}} \} \tag{2}$$

$$\begin{aligned}\Pr(IT_1 \le z) &= \Pr\left\{Min(TB_{p_{s1},p_{r1}}, TB_{p_{s1},p_{r2}}) \le z\right\}\\ &= 1 - \Pr\left\{Min(TB_{p_{s1},p_{r1}}, TB_{p_{s1},p_{r2}}) \ge z\right\}\\ &= 1 - \Pr(TB_{p_{s1},p_{r1}} \ge z)\Pr(TB_{p_{s1},p_{r2}} \ge z)\end{aligned} \tag{3}$$

$$\Pr(IT_1 \le z) = \begin{cases} 1-\left[1-(2z-z^2)(\frac{2z}{b}-\frac{z^2}{b^2})\right]^2 , & 0 \le z \le b \\ 1-\left[1-(2z-z^2)\right]^2 , & b < z \le 1 \end{cases} \tag{4}$$

Also, the pdf can be summarized as

$$h_{IT_1}(z) = \begin{cases} 2\left\{1-(2z-z^2)(\frac{2z}{b}-\frac{z^2}{b^2})\right\}\left\{(2-2z)(\frac{2z}{b}-\frac{z^2}{b^2})+(2z-z^2)(\frac{2}{b}-\frac{2z}{b^2})\right\} & 0 \le z \le b \\ 2\left\{1-(2z-z^2)\right\}(2-2z) & b < z \le 1 \end{cases} \tag{5}$$

Expected travel time model for the first interleave operation (IT_1) is

$$E(IT_1) = \int_0^1 z\, h_{IT_1}(z)\,\mathrm{dz}$$

$$= \frac{1}{5}+\frac{b^2}{3}-\frac{31b^2}{105}+\frac{5b^4}{42}-\frac{11b^5}{630} \tag{6}$$

Thus, expected FC cycle time is

$$E(FC) = (\frac{1}{2}+\frac{b^2}{6})+(\frac{1}{5}+\frac{b^2}{3}-\frac{31b^2}{105}+\frac{5b^4}{42}-\frac{11b^5}{630})+(\frac{1}{3}+\frac{b^2}{6}-\frac{b^3}{30})+(\frac{1}{2}+\frac{b^2}{6})$$

$$= \frac{23}{15}+\frac{5b^3}{6}-\frac{23b^3}{70}+\frac{5b^4}{42}-\frac{11b^5}{630} \tag{7}$$

Now, we describe the comparison with the expected dual command operation for single shuttle system to illustrate the efficiency of double shuttle system. Let e represent time to pick up or deposit a load, the round trip time to perform one transaction (storage plus retrieval) in dual command for the single shuttle system is given as $E(DC) + 4e$. On the other hand, for the double shuttle system, the round trip time is presented as $E(FC) + 6e$, because the pickup or deposit operation by double shuttle at the I/O point can be performed simultaneously at one time.

Therefore, the throughput per unit time, α for single shuttle and β for double shuttle are

$$\alpha = 2 / [E(DC) + 4e] \tag{8}$$

$$\beta = 4 / [E(FC) + 6e] \tag{9}$$

Figure 3 shows that the impact of P/D time on throughput increase for the case of T = 2(min), b = 0.8, 1.0. As the P/D time increases, the throughput decreases steeply, and is relatively sensitive.

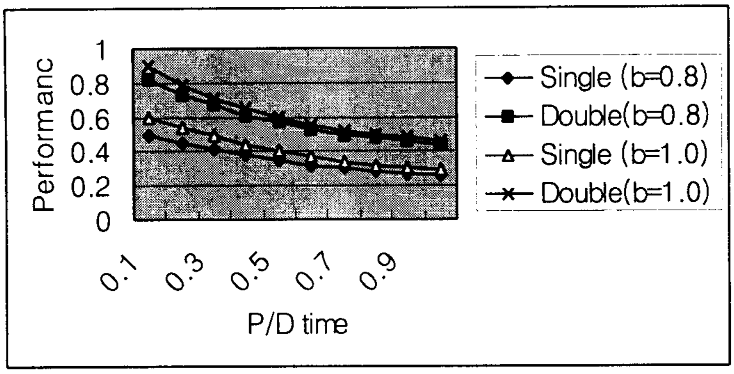

Figure 3. Single shuttle vs. double shuttle

5. A Heuristic Procedure for double shuttle System

A common practice, reflected in the studies, is that both storage and retrieval requests are processed in FCFS manner. The FCFS assumption is reasonable for storage, since most AS/R systems are interfaced with a conveyor loop for input and output. In this case, there is no capability for changing the sequence of loads presented for storage. However, for retrievals, the FCFS assumption is less compelling because retrieval requests are nothing more than messages, typically in some control computer. Hence, there is no intrinsic reason why they cannot be rearranged into any convenient sequence.

The problem of optimally sequencing a given list of retrievals is probably hard. In the case of a single open location for the initial storage operation, the sequencing problem is equivalent to the well known travelling salesman problem (TSP) [5]. The TSP is one of the class of so-called Np-complete problems, which is strong evidence that no efficient solution algorithm can be developed. Thus the retrieval sequencing problem with one open location is difficult to solve. Further, it is no easier to solve if there are multiple open locations.

Suppose that there is a retrieval list which can be as large as desired, and the number of orders in retrieval block is even number. In this heuristic procedure, we take advantage of the concept of no cost zone of Han et. el [4] depicted as in Figure 4. The cross-hatched area is referred to as the no cost zone since a storage or retrieval operation within the region is essentially free with regard to travel time. Also, in this case, the FC cycle is said to have *zero effective travel between.*

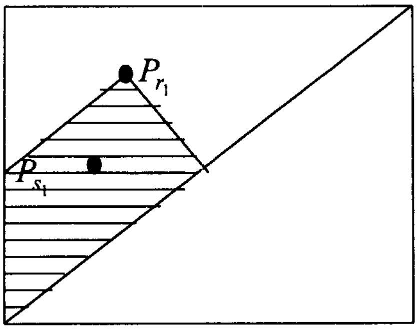

Figure 4 No cost Zone

To explain this heuristic algorithm easier, we need some additional notations.

n = number of orders in retrieval block
m = number of open locations
R = set of retireval orders in a block
S = set of open locations
r_i= ith retrieval order
s_i= ith storage order

The heuristic procedure can be describe as follows:

Step 1) Select an order pair, $r_i \in R$ and $s_i \in S$, with zero effective travel between, if not, with the minimal sum of the travel between time.

Step 2) Perform an operation, storing in s_1, retrieving from r_1 and storing in r_1 just before retrieved.

Step 3) $R=R-\{ r_1\} S=S-\{ s_1\}$

Step 4) Select $r_2 \in R$, with the minimal sum of the travel between time and the time to return.

Step 5) Perform an operation, retrieving from r_2 and returning to the I/O point.

Step 6) $R=R-\{ r_2\}S=S+\{ r_2\}$

Step 7) If the set of retrieval order R is not empty, then go to the Step 1. Otherwise, stop.

The simulation result of heuristic procedure is provided by random number generation and tabulated in Table 1, which illustrates the impact of block size on expected travel time for variant number of open locations.

We could find an important observation from the simulation that the reasonable size of the retrieval order block is about within 20 orders.

Table 1 The Result of Heuristic Procedure

n \ m	1	3	5	7	9	11	13	15
2	2.180	2.007	1.938	1.850	1.848	1.847	1.838	1.833
4	2.072	1.827	1.825	1.795	1.767	1.757	1.723	1.693
6	1.980	1.830	1.766	1.738	1.721	1.720	1.720	1.693
8	1.956	0.801	1.742	1.735	1.686	1.669	1.660	1.667
10	1.921	1.744	1.722	1.679	1.660	1.654	1.631	1.630
12	1.919	1.746	1.711	1.650	1.648	1.622	1.623	1.607
14	1.881	1.728	1.668	1.648	1.615	0.618	1.603	1.595
16	1.866	1.725	1.667	1.636	1.611	1.611	1.602	1.586
18	1.832	1.726	1.669	1.628	1.604	1.589	0.580	1.570
20	1.838	1.706	1.651	1.626	1.613	1.588	1.588	1.556
22	1.820	1.705	1.656	1.592	1.599	1.574	1.571	1.543
24	1.816	1.706	1.678	1.591	1.595	1.565	1.546	1.531
26	1.808	1.690	1.625	1.585	1.587	1.565	1.546	1.537
28	1.807	1.688	1.613	1.582	1.582	1.557	1.547	1.539
30	1.782	1.688	1.614	1.582	1.560	1.550	0.525	1.528

6. Conclusions

In this paper, we dealt with a double shuttle AS/R system which performs four transactions at a round trip. The mathematical model for the four command cycle time of the double shuttle system was derived. Also, we compared the performance of the system in comparison with the traditional single shuttle system. As previously mentioned, we could observe double shuttle system is very efficient in the view of performance.

In addition, we suggested a heuristic procedure for operating the double shuttle system more efficiently using the No-cost-zone concept. If we can select the retrieval order or open location in the No-cost-zone, the travel time for the order in No-cost-zone cannot be considered. The lowest possible limit for the double shuttle system can be reached to the single command cycle time which means that the first storage point and the second retrieval point are both in no cost zone.

7. References

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