VSS 이론을 활용한 최소위상 비선형 시스템에 대한 강인성연구

Robust Nonlinear Control for Minimum Phase Dynamic System by Using VSS

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요 약

본 논문에서는 불확실성을 갖는 비선형 시스템에 제어기를 설계하기 위해서 출력궤환 선형화 방법을 통하여 비선형 시스템을 선형시스템으로 변환하고 슬라이딩 모드 제어기법을 사용하여 파라미터들을 갱신하고 이를 통하여 제어입력을 갱신하는 강인성제어기를 제안하고자 한다. 비선형시스템은 최소위상시스템이고, 시스템의 상대차수는 $r < n$ 이며, 제로 다이나믹스는 안정하다고 가정한다. 제안된 제어기에 대해서 대역적 점근적 안정도가 보장됨을 보였으며, 제안된 제어기에 대해서 컴퓨터 시뮬레이션을 통하여 타당성을 검증하였다.

Abstract

In this paper, we proposed the robust control scheme for a class of nonlinear dynamical systems using output feedback linearization method. The presented control scheme is based on the VSS. We assume that the nonlinear dynamical system is minimum phase, the relative degree of the system is $r < n$ and zero dynamics is stable. It is also shown that the global asymtotically stability is guaranted. And we verified that the proposed control scheme is the feasible through a computer simulation.

Keywords : Robust controller, parameter update law, nonlinear plant, VSS

I. Introduction

Nonlinear control has emerged as an area of extensive research activity recently.[1]-[2]. An important class of problems in this area concerns the study of disturbance inputs for an analysis and controller synthesis purposes. Feedback stabilization of nonlinear systems at a specified equilibrium is a central topic in control theory and it has been a subject of research by many authors, e.g., see[3]-[5]. The works of Artstein, Sontag-Sussman[6], and Vidyasagar[7] are among the most significant contributions in the study of stabilization using Lyapunov-like techniques. The robust control approach has been developed for the effective control of uncertain linear/nonlinear dynamical systems. The robust control technique does not require the exact functional natures and the accurate parameter values of the system. The robust control scheme including the variable structure control is based on the construction of the control effort overcoming the uncertainty. Therefore, this control scheme needs a priori knowledge of the uncertainty bounds. When dealing with minimum phase nonlinear systems, a stable/unstable decomposition is usually used and the controller must contain only the part with stable inverse. Therefore, the problem of synthesizing control algorithms for plants with unstable zero dynamics is very important. Recently, Hauser et al.[8] proposed a new control design based on an approximate linearized model. For many nonlinear systems, uncertainties are common in control practice. So the design of a robust controller that deals with a nonlinear system with significant uncertainties is an important subject. In particular, the systematic design of output - feedback linearizable system without matched conditions have been conducted and three main extensions have been proposed such as adaptive control[9], Lyapunov based control[10]-[11], and variable structure control[12]. Many design tasks, such as tracking and disturbance rejection problems, often use high gain feedback to asymptotically achieve the desired specifications[13]. Basically, above of all the minimum phase condition must be considered. In this paper, a robust control scheme for a class of nonlinear dynamical system is proposed by using output-feedback linearization method proposed by Isidori et al.. The presented control scheme is based on the VSS concept proposed by Utkin and Itkis. In this control scheme, we assume that the nonlinear dynamical system is minimum phase, i.e., the relative degree of the system is $r < n$ and the zero dynamics is stable. It is also shown that the global asymptotically stability is guaranted under the proposed control scheme. The paper is organized as follows. In section II, mathematical tools is presented and discussed output feedback linearization. In section III, VSS controller is presented and its stability analysis is shown in the Lyapunov sense. In section IV, the feasibility of the proposed control scheme is verified through a computer simulation.

II. Mathematical Tools

Given f, a C^∞ vector field on R^n, and h, a C^∞ scalar field on R^n the Lie derivative of h with respect to $f$is defined as the inner product of the gradient of h with f

$$L_f h = \langle dh, f \rangle = \sum_{i=1}^{n} \frac{\partial h}{\partial x_i} f_i . \qquad (1)$$

We can see that $L_f h$ is also a C^∞ scalar field on R^n. Higher order Lie derivatives can be defined inductively as follows:

$$L_f^k h = L_f(L_f^{k-1} h), \quad k=2,3,\cdots$$

Given f, g a C^∞ vector fields on R^n, the Lie bracket $[f, g]$ is a vector field defined by

$$[f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g \qquad (2)$$

where $\frac{\partial f}{\partial x}$ and $\frac{\partial g}{\partial x}$ are the Jacobians.

$[f, g]$ is also a C^∞vector field on R^n. One can define iterated Lie brackets $[f, [f, g]]$, $[f, [f, [f, g]]]$ etc. The following notation is standard:

$$ad_f^0 g = g$$
$$ad_f^1 g = [f, g]$$
$$ad_f^2 g = [f[f, g]]$$
$$\vdots$$
$$ad_f^k g = [f, ad_f^{k-1} g]$$

The purpose of this section is to show how single-input single-output nonlinear systems can be locally given, by means of a suitable change of coordinates in the state space, a normal form of special interest, on which several important properties can be elucidated. The point of departure of the whole analysis is the notation of relative degree of the system, which is formally described in the following way.

The single-input single-output nonlinear system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t)$$
$$y(t) = h(x(t)) \qquad (3)$$

is said to have relative degree r at a point x^o If

(i) $L_g L_f^k h(x) = 0$ for all x in a neighborhood U of x^o and all $k < r-1$

(ii) $L_g L_f^{r-1} h(x) \neq 0$

where the state x is assumed to belong to an open neighborhood U of R^n, $f(x)$ and $g(x)$ are smooth vector fields on R^n, $u \in R$, $y \in R$, and (i.e., the origin is an equilibrium point). This ma $f(0)=0$thematical property of relative degree can be summarized in the following theorem.

Theorem 2.1 The nonlinear system (3), with $f(x)$ and $g(x)$ being smooth vector fields, is input-state linearizable if, and only if, there exists a region Ω such that the following conditions hold :

(i) the vector fields $\{ g, ad_f g, \cdots, ad_f^{r-1} g \}$ are linearly independent in Ω.

(ii) the distribution

$$\triangle(x) = span \{ g, ad_f g, \cdots, ad_f^{r-2} g \}$$

is involutive in Ω. Where Ω is an open and connected subset of R^n which includes the origin.

Theorem 2.2 A nontrivial real-valued function $h(x)$ exists whose differential is a annihilator of the distribution $\triangle(x)$ defined in theorem 2.1, i.e.,

$$dh(x) \neq 0 \text{ and } dh(x) \cdot \triangle(x) = 0$$

if and only if system (3) satisfies the theorem 2.1. We can see that theorem 2.2 is equivalent to the fact that the function $h(x)$, when interpreted as the output of the system (3), produces a relative degree r. From the above theorem 2.1 and 2.2, we can construct the diffeomorphism $\Phi(x) = [h(x),$ $L_f h(x), \cdots, L_f^{r-1} h(x), \psi_{r+1}(x), \cdots, \psi_n(x)]^T$ which has the property $\Phi(0) = 0$, and transforms the system (3) to the following cannonical form

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \vdots \\ \dot{z}_{r-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{r-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} z_r \quad (4)$$

and

$$\dot{z}_r = a(z, \Psi) + b(z, \Psi) u \quad (5)$$

$$\begin{bmatrix} \dot{\psi}_{r+1} \\ \vdots \\ \dot{\psi}_n \end{bmatrix} = \begin{bmatrix} w_{r+1}(z, \Psi) \\ \vdots \\ w_n(z, \Psi) \end{bmatrix} \quad (6)$$

where $\dot{\Psi} = w(0, \Psi)$ is zero dynamics and in this paper we assume that it is stable(or minimum phase system).

$$z = [z_1, \cdots, z_r]^T, \quad \Psi = [\psi_{r+1}, \cdots, \psi_n]^T$$

$$z_i = h(x) = L_f^{i-1} h(x) \quad \text{for } i = 1, \cdots, r$$

$$a(z, \Psi) = L_f^r h(x)|_{x=\Phi^{-1}(z)},$$

$$b(z, \Psi) = L_g L_f^{r-1} h(x)|_{x=\Phi^{-1}(z)}$$

Ⅲ. VSS Controller Design

In this section, we propose a control law which guarantees that closed-loop system has the uniformly ultimate bounded stability with a tolerable tracking error. Because the states of zero dynamics are not accessible, we consider these states as bounded disturbances under the assumption that zero dynamics is stable and also we have no priori knowledge concerning the magnitude of these disturbances. The following assumptions are needed for the development of a controller.

Assumption 1 Assume that zero dynamics is stable(or nonlinear minimum phase). $a(z, \Psi)$ and $b(z, \Psi)$ can be approximated as follows

$$a(z, \Psi) \cong a(z, \Psi_o) + \frac{\partial a}{\partial \Psi}(z, \Psi_o) \delta \Psi$$

$$b(z, \Psi) \cong a(z, \Psi_o) + \frac{\partial b}{\partial \Psi}(z, \Psi_o) \delta \Psi.$$

Assumption 2 There exists some positive constant vector $\rho_{1\nu}, \rho_{2\nu}$ such that

$$\| \frac{\partial a}{\partial \Psi}(z, \Psi_o) \delta \Psi \| \leq \rho_{1\nu}^T \mu_\nu(t, z)$$

$$\| \frac{\partial b}{\partial \Psi}(z, \Psi_o) \delta \Psi \| \leq \rho_{2\nu}^T \mu_\nu(t, z)$$

where

$$\rho_{1\nu}^T = (\rho_{11} \ \rho_{12} \ \rho_{13})$$

$$\rho_{2\nu}^T = (\rho_{21} \ \rho_{22} \ \rho_{23})$$

are unknown parameter vectors and

$$\mu_\nu(t, z) = (1 \ \|z\| \ \|z\|^2)^T.$$

From the above assumptions, the time derivative of z_r can be rewritten as

$$\dot{z}_r = a(z, \Psi_o) + b(z, \Psi_o) u + \eta(z, u, t) \quad (7)$$

where

$$\eta(z, u, t) = \frac{\partial a}{\partial \Psi}(z, \Psi_o) \delta \Psi + \frac{\partial b}{\partial \Psi}(z, \Psi_o) \delta \Psi u$$

Because control input $u(t)$ must be bounded, the norm of $\eta(z, u, t)$ can satisfy the following inequality.

$$\| \eta(z, u, t) \| \leq \sigma_\nu^T \varphi_\nu(t, z)$$

where $\sigma_\nu^T = (\gamma_1 \ \gamma_2 \ \gamma_3)$ is unknown parameter vector and $\varphi_\nu(t, z)$ can be any positive vector function. In this paper, we set $\varphi_\nu(t, z)$ to be the same as $\mu_\nu(t, z)$ as follows

$$\varphi_\nu(t, z) = (1 \ \|z\| \ \|z\|^2)^T.$$

Throughout this paper, the norm $\|\cdot\|$ is assumed to be the Euclidean vector norm. Now we utilize the VSS concept to derive a control law. First let us define a sliding surface as follows

$$s(z_1, \cdots, z_r) = \alpha_1 z_1 + \cdots + \alpha_{r-1} z_{r-1} + z_r. \quad (8)$$

where α_i, $i = 1, \cdots, r-1$, are chosen so that the following polynominals $p(s)$ are Hurwitz

$$p(s) = s^r + \alpha_{r-1} s^{r-1} + \cdots + \alpha_1. \quad (9)$$

Now, we consider a following VSS-like type control law

$$u = u_{eq} + u_\Delta, \quad (10)$$

where u_{eq} is the equivalent control input of the nominal system, u_Δ is the control input overcoming the uncertainties (or disturbance which represent the term concerning zero dynamics). We derive u_{eq} from the fact that the derivative of $\frac{1}{2}s^2$ along the trajectory of the closed-loop system should be equal to zero and u_Δ is found such that

$$s[\dot{s}] < -\beta \|s\|, \quad \beta > 0. \quad (11)$$

This inequality implies that the trajectory reaches the sliding surface in a finite time and stays on the sliding surface thereafter. Now we discuss how to derive the u_{eq} and u_Δ which satisfy the above conditions. The time derivative $\dot{s}$ can be expressed as

$$\begin{aligned} \dot{s}(z_1 \cdots z_r) &= \alpha_1 \dot{z}_1 + \alpha_2 \dot{z}_2 + \cdots + \dot{z}_r = 0 \\ &= \alpha_1 z_2 + \cdots + \alpha_{r-1} z_r + a(\mathbf{z}, \Psi_o) \\ &\quad + b(\mathbf{z}, \Psi_o) u + \eta(\mathbf{z}, \mathrm{u}, t). \end{aligned}$$

If we choose u_{eq} as follows

$$u_{eq} = \frac{1}{b(\mathbf{z}, \Psi_o)} \{ -\alpha_1 z_2 - \cdots - \alpha_{r-1} z_r - a(\mathbf{z}, \Psi_o) \} \quad (12)$$

then

$$\dot{s}(z_1 \cdots z_r) = b(\mathbf{z}, \Psi_o) u_\Delta + \eta(\mathbf{z}, \mathrm{u}, t). \quad (13)$$

Now u_Δ is chosen by

$$u_\Delta = -\frac{1}{b(\mathbf{z}, \Psi_o)} \{ \widehat{\sigma_\nu}^T \varphi_\nu(t, \mathbf{z}) + k \} \cdot sgn(s) \quad (14)$$

where $\widehat{\sigma_\nu}$ is estimate of σ_ν. We can summarized the controller structure as follows

$$\begin{aligned} u = u_{eq} &+ \frac{1}{b(z, \Psi_o)} \{ -k \\ &- \widehat{\sigma_\nu}^T \varphi_\nu(t, \mathbf{z}) \} \, sgn(s). \end{aligned} \quad (15)$$

Now the objective of control is to drive the parameter update law which guarantee that $\mathbf{z}(t)$ converge to zero vector as time goes to infinity. Therefore, we suggest the parameter update law as follows

$$\begin{aligned} \dot{\hat{\gamma}}_1 &= s \cdot sgn(s) \\ \dot{\hat{\gamma}}_2 &= s \|z\| \, sgn(s) \\ \dot{\hat{\gamma}}_3 &= s \|z\|^2 \, sgn(s) \end{aligned} \quad (16)$$

where

$$\begin{aligned} \tilde{\gamma}_1 &= \hat{\gamma}_1 - \gamma_1 \\ \tilde{\gamma}_2 &= \hat{\gamma}_2 - \gamma_2 \\ \tilde{\gamma}_3 &= \hat{\gamma}_3 - \gamma_3. \end{aligned}$$

The stability of the proposed control law is analyzed by the following theorem.

Theorem 3.3 Under the assumption [1]-[2], the uncertain dynamical system (7) with a robust control law (15) and parameter update law (16), is globally uniformly ultimately bounded.

Proof: The proof is based on the Lyapunov-like function

$$V = \frac{1}{2} (\tilde{\gamma}_1^2 + \tilde{\gamma}_2^2 + \tilde{\gamma}_3^2 + s^2). \quad (17)$$

Taking the time derivative of V along the trajectory of (13) yields

$$\begin{aligned} \dot{V} \le\ & \tilde{\gamma}_1 \dot{\hat{\gamma}}_1 + \tilde{\gamma}_2 \dot{\hat{\gamma}}_2 + \tilde{\gamma}_3 \dot{\hat{\gamma}}_3 + s\, b(\mathbf{z}, \Psi_o)\, u_\Delta \\ & + |s| \sigma_\nu^T \varphi_\nu(t, \mathbf{z}) \\ \le\ & \tilde{\gamma}_1 \dot{\hat{\gamma}}_1 + \tilde{\gamma}_2 \dot{\hat{\gamma}}_2 + \tilde{\gamma}_3 \dot{\hat{\gamma}}_3 \\ & - s(\widehat{\sigma_\nu}^T \varphi_\nu(t, \mathbf{z}) + k) \cdot sgn(s) \\ & + |s| \sigma_\nu^T \varphi_\nu(t, \mathbf{z}) \\ \le\ & \tilde{\gamma}_1 \dot{\hat{\gamma}}_1 + \tilde{\gamma}_2 \dot{\hat{\gamma}}_2 + \tilde{\gamma}_3 \dot{\hat{\gamma}}_3 \end{aligned}$$

$$- s\, \tilde{\sigma}_\nu^{\,T}\, \varphi_\nu(t,\, z)\, sgn(s) - s\, k\, sgn(s). \quad \blacksquare$$

From (17), $\dot{V}$ can be expressed as

$$\dot{V} \le -k\, s\, sgn(s) < 0.$$

Therefore $s \to 0$ and $z \to 0$ as $t \to \infty$.

IV. Computer Simulation

In this section, computer simulations are conducted to verify the feasibility and effectiveness of the proposed control scheme. The following fourth-order nonlinear dynamical system is used for a nonlinear plant.

$$\dot{x} = \begin{bmatrix} x_2 - x_3^2 \\ x_3 + 2x_1^2 x_3 + 2x_3 x_4 \\ x_1^2 + x_4 \\ x_1 + x_3^2 - x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 2x_3 \\ 1 \\ 0 \end{bmatrix} u. \qquad (18)$$

From the feedback linearization, we can derive the following equations

$$\begin{aligned} y &= h(x) = x_1 = z_1 \\ \dot{y}^{(1)} &= \dot{x}_1 = x_2 - x_3^2 = z_2 \\ \dot{y}^{(2)} &= \ddot{x}_1 = x_3 = z_3 \\ \dot{y}^{(3)} &= \dot{x}_3 = x_1^2 + x_4 + u. \end{aligned} \qquad (19)$$

Using these new coordinates, we obtain the following equations

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_1^2 + z_4 + u \\ \dot{\psi}_4(z, \psi) &= w_4(z, \psi) = \dot{z}_4 \\ &= x_1 + x_3^2 - x_4 = z_1 + z_3^2 - z_4 \end{aligned} \qquad (20)$$

where the zero dynamics of the system is $\dot{z}_4 = -z_4$. To derive a robust controller based on VSS concepts, it can be easily verified that $L_g L_f^2 h(x) \neq 0$ and thus relative degree of the system is 3. We select sliding surface as follows

$$s = 2z_1 + 3z_2 + z_3. \qquad (21)$$

Then the equivalent control input can be expressed as follows

$$u_{eq} = -(2z_2 + 3z_3 + z_1^2). \qquad (22)$$

We choose u_Δ as follows

$$u_\Delta = -(\hat{\sigma}_\nu^{\,T}\, \varphi_\nu(t,\, z) + k)\, sgn(s). \qquad (23)$$

and we set k to be 1. Therefore the total control input u and parameter update law can be expressed as

$$\begin{aligned} u = & -2z_2 - 3z_3 - z_1^2 \\ & - \hat{\sigma}_\nu^{\,T}\, \varphi_\nu(t, z)\, sgn(s) - sgn(s) \end{aligned} \qquad (24)$$

$$\begin{aligned} \hat{\gamma}_1 &= s \cdot sgn(s) \\ \hat{\gamma}_2 &= s\|z\|\, sgn(s) \\ \hat{\gamma}_3 &= s\|z\|^2\, sgn(s). \end{aligned} \qquad (25)$$

Fig.1 shows a input of robust VSS controller, and Fig.2 shows z_4 the state of zero dynamics. The regulation of the state trajectory $z(t) = (z_1\ z_2\ z_3)^T$ to zero vector is shown in Fig.2 and corresponding estimate parameter updation γ_1, γ_2, γ_3 are shown in Fig.3. From the results, we can see that the proposed control scheme is very effective.

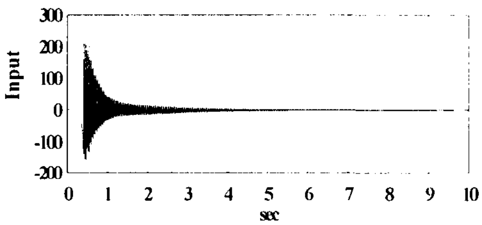

Fig.1. Robust VSS controller input.

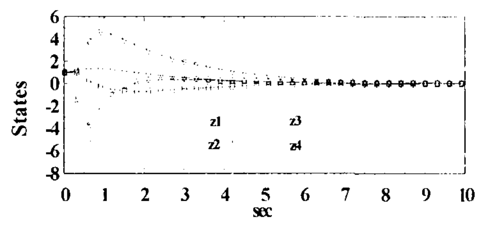

Fig.2. The states of z_1, z_2, z_3, z_4.

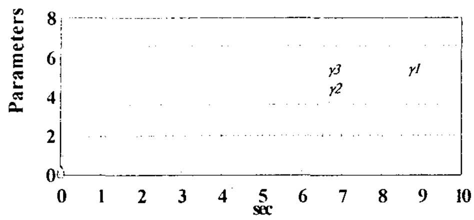

Fig.3. The adaptation parameters of controller.

V. Conclusions

In this paper, a robust control scheme for a class a uncertain nonlinear dynamical systems has been proposed. The presented control scheme is based on the VSS robust control structure with an parameter adaptation law for the uncertainty bounds. The uniformly ultimate boundness of the control scheme is guaranted and has been demonstrated by a simulation. In the future, we will propose that robust controller based on the proposed control scheme for one-link or two-link flexible robot arm with the vibration mode.

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