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LCC Optimization for Reinforced Concrete Structures under Seismic Hazards

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Abstract

A simple expected damage cost model is developed and a systematic approach to evaluate the economic effects of seismic hazards to reinforced concrete structures is presented. An expected damage cost function during a specific lifetime is modeled by a Poisson's process with uniform continuous cash flow assumption. It is possible that the proposed method can decouple the damage cost effect from random earthquake events. Thus, expected damage cost function can be formulated as a combination of three independent terms; a present worth factor of Poisson's process, a damage cost interpolation function and a mean occurrence rate of earthquake intensity. The validity of the proposed method is demonstrated by a comparative study of LCC evaluations with the previous study.

Keywords: damage cost function, LCC, reliability, seismic hazard, damage function.

1. Expected Life Cycle Cost

Evaluation of economic effects of seismic hazard to reinforced concrete structures can be implemented by a proper integration of a seismic hazard analysis, a damage assessment and a life cycle cost evaluation process. In this study, an expected damage cost model is developed and applied to evaluate the economic effects of seismic hazard.

A Life Cycle Cost (LCC) function can be selected as an objective function to make a cost effective choice for a design of building structures. LCC for a building is expressed as the sum of all the expenditures associated with all items during its entire service life. Cost items of a building at a seismic zone can be classified into an initial cost and a damage cost due to earthquake hazards. Because of the uncertain characteristics of earthquakes, the damage cost of a building at a seismic zone can be expressed by an expected value. The expected life cycle cost (ELCC) function is defined as[(1)] ;

$$E[C_t] = C_i + E[C_d] \tag{1}$$

where $E[C_t]$ is the expected life cycle cost (ELCC), C_i is the initial cost, and $E[C_d]$ is the expected damage cost (EDC) due to an earthquake. Generally the initial cost required for the construction of buildings can be classified into a direct cost and an indirect cost. The direct cost consists of the cost for the structure itself and the cost for the other non-structural components. As the cost of non-structure is nearly unaffected by a level of an earthquake intensity underlying its design, it is taken as a non-variable cost item in this study. However, the indirect cost is assumed as proportionally changed with the direct cost. Under these considerations, the initial cost of a building for different design schemes can be expressed as ;

$$C_i = C_s^0(\omega) + C_{ns}^0 \tag{2}$$

where C_s^0 is the basic cost of structural items, which is a function of a design parameter vector ω. C_{ns}^0 is the basic cost of non-structural items. Basic costs are defined as the costs of the building designed in accordance with the requirement of the current building code. Basic costs include indirect expenditures during construction.

2. Expected Damage Cost Function

The EDC due to seismic occurrences for the life cycle of a structure could be obtained by integrating the damage costs from all possible earthquake intensities. As the damage cost is due to future events, it has to be expressed as a present value on a common basis with initial cost. Thus, the present value of the EDC can be expressed as ;

$$E[C_d]=\int_{a_{min}}^{a_{max}}[\int_0^{\infty}C_d(x)f_{X|A}(x|a)dx](P|F,q,t_a)f_A(a)da \quad (3)$$

where X is a random variable of damage levels, A is a random variable of earthquake intensities at a given location, and $(P|F,q,t_a)$ is the present worth factor of future costs (PWF), in which q is the annual discount rate, which is used to measure the effect of a compound interest rate, and t_a is the occurrence time of an earthquake intensity.

In real problems, it is usually difficult to get an exact solution of the first integral in Eq. (3) which is formulated to obtain the EDC at a given earthquake intensity a. Therefore, the first order approximation method by Tailor series about the mean or median of random variables can be used. Thus, the EDC at a given earthquake intensity a is approximated as a function of global mean damage indices or median damage indices of the structures. [1]

$$\int_0^{\infty}C_d(x)f_{X|A}(x|a)dx \cong C_d(\tilde{x}_a) \quad (4)$$

where $\tilde{x}_a$ is the median global damage index at a given earthquake intensity a.

A PWF function for a Poisson's process of earthquake intensity a during a specific time interval L was proposed by Ang and et al. [2] and modified by Park[1] as following.

$$(P|F,q,L_a)$$

$$=\frac{1}{\nu L}\sum_{n=1}^{\infty}\sum_{k=1}^{n}\frac{\Gamma(k,\alpha L)}{\Gamma(k,\nu L)}\left(\frac{\nu}{\alpha}\right)^k\frac{(\nu L)^n}{n!}e^{-\nu L} \quad (5)$$

where $\alpha=\nu+\ln(1+q)$, in which ν is the mean arrival rate of earthquake. It was illustrated that the numerical values of PWF from Eq. (5) are independent from the variable of annual arrival rate ν. [1]

Since an earthquake is occurred arbitrarily in the time interval, it can be approximated as a PWF of an uniform continuous cash flow assumption for the time interval L. In this consideration, Eq. (5) can be simplified as ;

$$(P|F,q,L_1)=\int_0^L\frac{1}{L}e^{-qt}dt \quad (6)$$

where L_1 implies the time interval L in which only one earthquake is occurred arbitrarily. In Fig. 1 results from Eq. (5) and (6) are compared for 50 year time interval. This fig. shows the difference between these equations is negligible. Similarly, when earthquakes are occurred arbitrarily n times in the time interval L, Eq. (6) can be expressed as;

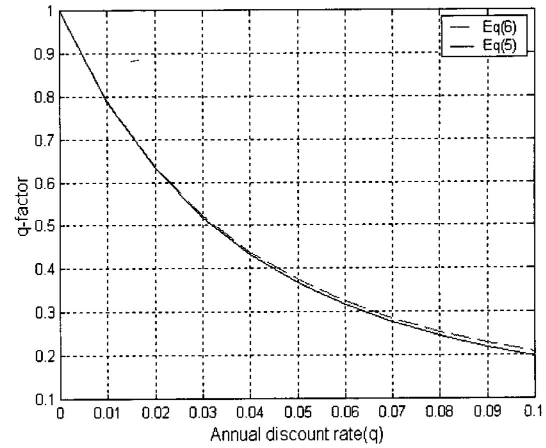


Fig. 1 Present worth factor

$$(P|F,q,L_n)=\int_0^L\frac{n}{L}e^{-qt}dt \quad (7)$$

If the occurrence of a discrete earthquake intensity a_i is modeled by a Poisson's process and PWF of damage costs is assumed as an uniform continuous cash flow model for the time interval L, Eq. (3) can be expressed as ;

$$E[C_d]=\sum_{a_{min}}^{a_{max}}C_d(\tilde{x}_{a_i})\sum_{n=1}^{\infty}[\int_0^L e^{-qt}\frac{n}{L}dt]\frac{(\nu_{a_i}L)^n}{n!}e^{-\nu_{a_i}L}$$

$$=\sum_{a_{min}}^{a_{max}}C_d(\tilde{x}_{a_i})\frac{1}{L}[\int_0^L e^{-qt}dt]\sum_{n=1}^{\infty}n\frac{(\nu_{a_i}L)^n}{n!}e^{-\nu_{a_i}L}$$

$$=\sum_{a_{min}}^{a_{max}}C_d(\tilde{x}_{a_i})\frac{e^{qL}-1}{qLe^{qL}}\nu_{a_i}L \quad (8)$$

where ν_{a_i} is the mean occurrence rate of discrete earthquake intensity a_i. By an assumption of earthquake occurrence events as an extreme type II distribution, ν_{a_i} of certain discrete interval Δa_i can be expresses as ;

$$\nu_{a_i}=\frac{\partial}{\partial a}[1-\exp\{-(\frac{a}{\sigma})^{-\gamma}\}]\Delta a_i \quad (9)$$

A damage cost function under a certain earthquake intensity is formulated by combining a damage cost interpolation function with a maximum damage cost at the completely collapsed states.

$$C_d(\tilde{x}_{a_i}) = c_0 h(\tilde{x}_{a_i}) \tag{10}$$

where c_0 is the maximum damage cost and $h(\tilde{x}_{a_i})$ is the damage cost interpolation function of the median damage level $\tilde{x}$ at a discrete earthquake intensity a_i. The damage cost interpolation function relates a damage level to a associated damage cost. The patterns of building damage cost can be classified into three different groups. Those are serviceability or acceleration sensitive, moderate sensitive and collapse sensitive costs. Damage cost patterns for those characteristics can be effectively expressed by the two parameters' exponential function of the median global damage index [1] as ;

$$h(\tilde{x}_{a_i}) = 1 - e^{\alpha \tilde{x}_{a_i}^{\gamma}} \tag{11}$$

The values of parameter α, γ of the damage cost interpolation function can be determined from the statistical data which can show the relation between real earthquake damages and damage cost. But such kind of real data are hardly available. In this study, parameters are determined by curve fitting method applying to the data from ATC-13. [3] Fig. 2 shows the typical shapes of interpolation functions. From Eq. (3), (4), (8) and (11) the EDC function can be formulated as a combination of three independent functions; a present worth factor of Poisson's process, a damage cost interpolation function and a mean occurrence rate of earthquake intensity. Thus EDC can be simply expressed as ;

$$E[C_d] = c_0 \frac{e^{qL} - 1}{qLe^{qL}} \sum_{a_{min}}^{a_{max}} (1 - e^{\alpha \tilde{x}_{a_i}^{\gamma}}) \nu_{a_i} L \tag{12}$$

3. Comparison with Previous Study

3.1 LCC Optimization Process

To obtain an optimal solution of the LCC for reinforced concrete structures, it requires integration of a seismic hazard analysis, a damage assessment of structural system under earthquake and an estimation of life cycle cost for the numerous design schemes of structures. In order to make it possible to include such a complicate process effectively, a discrete optimization technique of a coordinate descendant method can be applied. [4] In this study, nominal strengths of the structures and load effects under earthquake events are considered as discrete design variables for LCC optimization.

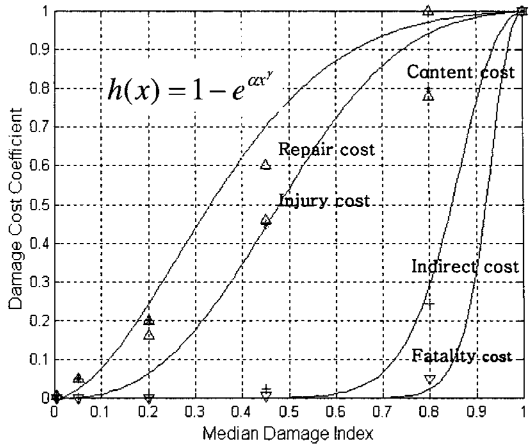


Fig. 2 Damage cost interpolation function

The discrete optimization problem can be formulated as ;

$$\begin{aligned} &Min\ \ E[C_t(\omega_{R_i})] = C_i^0(\omega_{R_i}) + E[C_d(\omega_{R_i})] \\ &Subject\ to\ \ g_1(\omega_{R_i}, \omega_S) = 0\ (i = 1, \cdot\cdot, n) \\ &g_2(\tilde{x}, \omega_{R_i}, \omega_S, a) = 0 \end{aligned} \tag{13}$$

where the first constraint function is for the requirement of the strength capacity of the structure and the second is for the damage level evaluation of the structure at a given earthquake intensity. Among above variables, ω_{R_i} is a vector of nominal strength capacities of the structures and ω_s is a vector of load effects. The Optimal LCC satisfying constraint conditions can be determined by testing the ELCC of building candidates with different levels of nominal strength. The systematic process for the LCC optimization is illustrated in Fig. 3.

The validity of the proposed approach is tested for the same building of 7-story reinforced concrete framed structure as studied in previous papers[1] (Fig. 4). Member properties and strength or deformation capacities of members for each design scheme are shown in Table 1. The seismic hazard analysis for the site is performed on the basis of historical earthquake records. An extreme type II distribution is assumed for the event of earthquake and characteristic values of the distribution are obtained such as; $\sigma = 1.128e-05$ and $\gamma = 0.58317$ (Fig. 5). Mean occurrence rate ν_{a_i} of certain discrete interval a_i are obtained from Eq. (9). The ground motion due to earthquake is simulated by stochastic ground motion. [5] The statistics of parameters for the ground motion simulation and the damage analysis of the example structure are shown in Table 2.

3.2 Damage Evaluation

In this study, the damage level of reinforced concrete members is defined by the damage index proposed.[6]

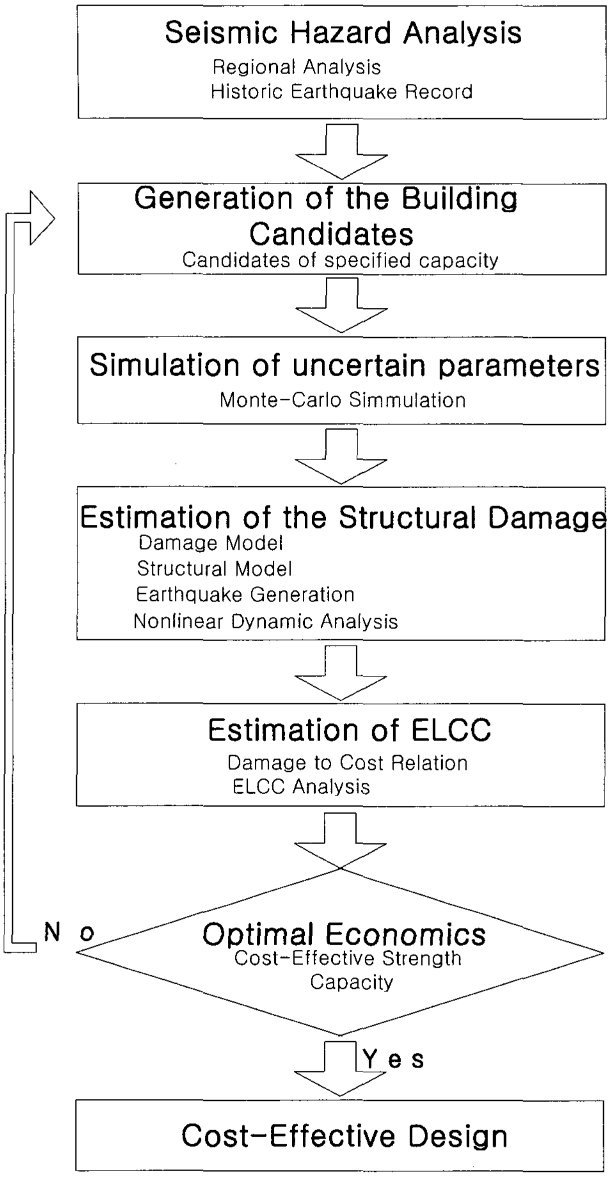


Fig. 3 LCC optimization process

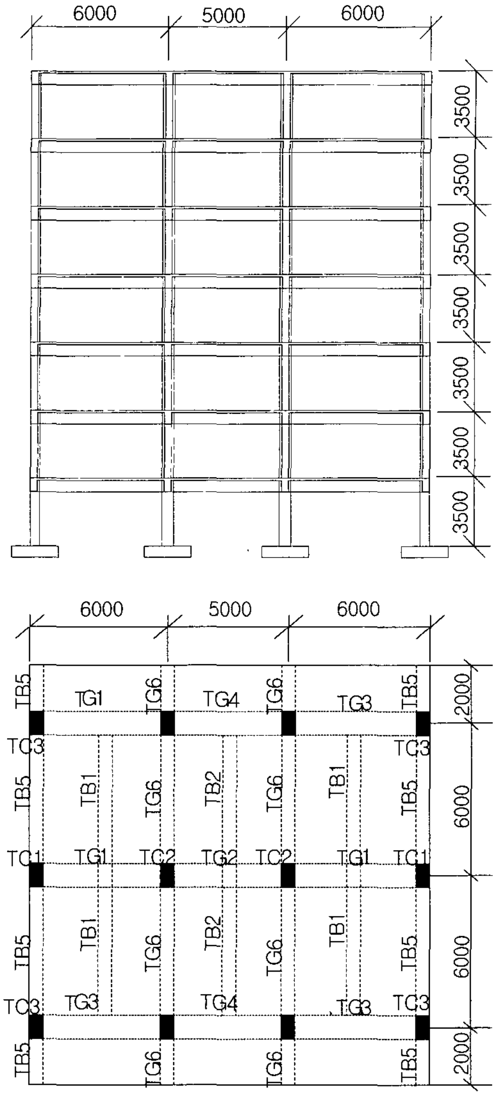


Fig. 4 Illustrated Building (unit : mm)

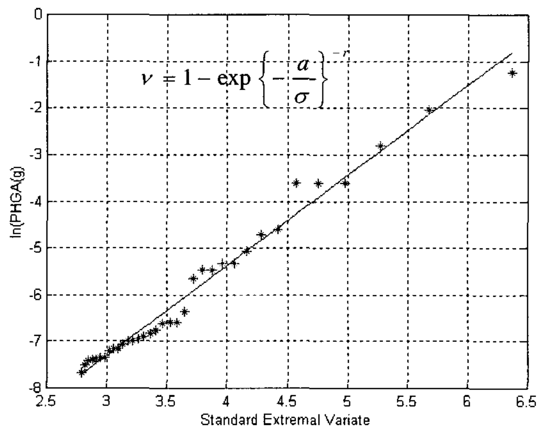


Fig. 5 Analysis for historical earthquake records

Table 1. Properties of design schemes

Nominal Strength	Member	B (cm)	H (cm)		As_p (cm²)	As_n (cm²)	Av (cm²)	Sv (cm)	Ls (cm)	c (cm)	y (cm)	u (cm)	Mu (tm)
0.0En	Column	40	40	0.48	3.2	3.2	1.42	20	350	0.35	1.06	5.48	11.7
	Beam	15	60	1.2	7.8	3.6	1.42	20	600	0.35	5.59	21.6	12.3
0.5En	Column	40	40	0.48	12	12	1.42	20	350	0.57	1.33	5.83	19.4
	Beam	15	60	1.2	11.7	3.6	1.42	20	600	0.35	5.47	18.2	18.3
1.0En	Column	40	40	0.48	20.8	20.8	1.42	20	350	0.6	1.35	5.89	27.7
	Beam	20	60	0.98	15.6	4.8	1.42	15	600	0.35	5.49	17.5	23.4
2.0En	Column	45	45	0.48	30.4	30.4	1.42	15	350	0.46	1.65	5.94	49
	Beam	30	60	0.61	22.6	7.2	1.42	15	600	0.35	5.49	16.7	35.5
3.0En	Column	50	50	0.48	31.3	31.3	2.54	15	350	0.36	1.78	5.83	64
	Beam	40	60	0.48	32.2	32.2	1.42	15	600	0.35	5.42	15.5	5.23

B : beam width, H : beam depth, ρ : reinforcing ratio
As_p : tension steel area, As_v : compression steel area,
A_v : shear steel area, S_v: stirrup spacing, Ls: member length,
δc: crack displacement, δy: yield displacement,
δu: ultimate displacement, Mu : ultimate moment,
En : nominal strength of the structures for code specified earthquake load

Table 2. Statstics of parameters

Parameter	Mean	C.O.V	Distribution
δu	1.0 $\bar{\delta u}$	0.3	Normal
β	1.0 $\bar{\beta}$	0.3	Normal
Mu	1.0 $\bar{Mu}$	0.2	Normal
m	1.0 $\bar{m}$	0.1	Log-Normal
ξ	0.05	0.6	Log-Normal
fy	1.0 $\bar{fy}$	0.1	Log-Normal
f'c	1.0 $\bar{f'c}$	0.3	Log-Normal
Ec	1.0 $\bar{Ec}$	0.3	Log-Normal
ω_g	16.9	0.4	Log-Normal
ζ_g	0.94	0.4	Log-Normal
ω_f	0.7	0	.
ζ_f	0.6	0	.
T(sec)	5.5	0.7	Log-Normal

$\delta\mu$: ultimate displacement, β : dissipated energy parameter, Mu : ultimate moment, m : mass damping coe fficient, ξ : damping coefficient fy : yield strength of rebar, f'c : compressive srength of concrete, Ec : modulus of elasiticity of concrete, ϖ_g , ζ_g , ω_f , ζ_f : filter parameters representing the ground condition.

The global damage index of framed structures is a function of the local damages of their members such as columns and girders. The levels of global damage are defined as ;

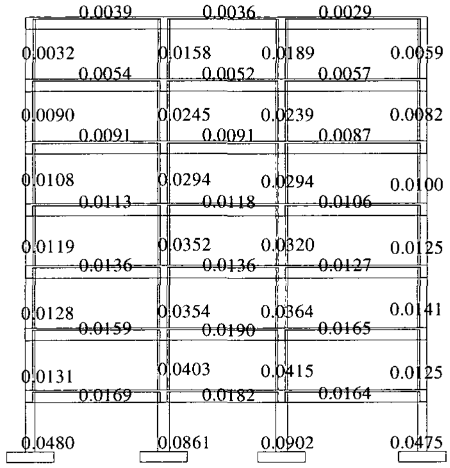
$$D_g = \sum_{i=1}^{n} w_i D_i \tag{14}$$

where D_g is a global damage index, D_i is damage of critical component i and w_i is weighted average factor. In Eq. (14), D_i may be computed as a combination of the damage to several components, such as a story damage that is computed as a weighted average of the damage to columns and beams in the story.

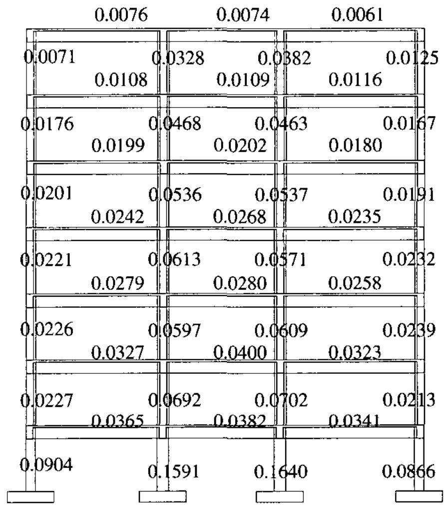
The median local damage indices of structural elements are obtained from the damage analysis for the 200 randomly simulated sample buildings and results are shown in Fig. 6. Median global damage index of design schemes under the different seismic intensities are shown in Fig. 7. From the damage evaluation, it is found that median local damage indices of columns at the first story are much larger than those of the other members, which imply that all simulated buildings are to be collapsed by column mechanism. Magnitude of median global damage index increases with the load effects from earthquake and decreases with nominal strength. Under the moderate seismic intensity of PGA less than 0.2g, median global damage indices are less than 0.4 and the failure probabilities of most of the buildings are known to be negligible.

3.3. Results of LCC Optimization

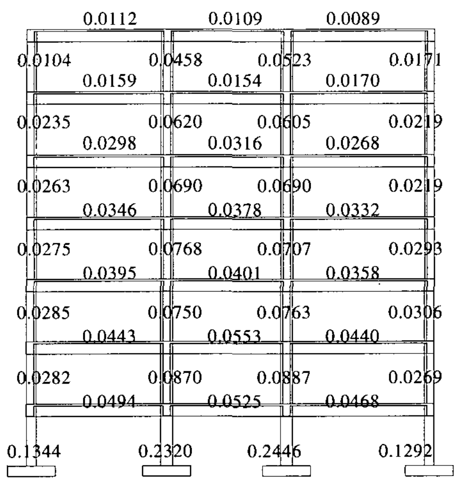
The ELCC evaluated by the proposed model is compared with those of the previous study.[1] The relation between the ratio of strength capacity and the normalized ELCC are shown in Fig. 8. It is found that error of the proposed model is less than 1.8%. The cost effective levels of design schemes obtained by the proposed model are compared with previous study and summarized in table 3. It is also found



(6-a) PGA= 0.05g

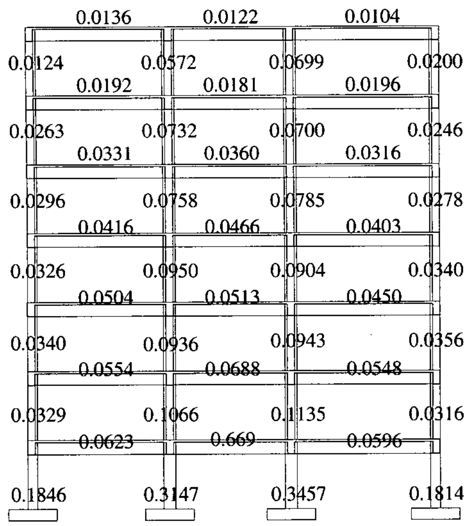


(6-b) PGA= 0.10g

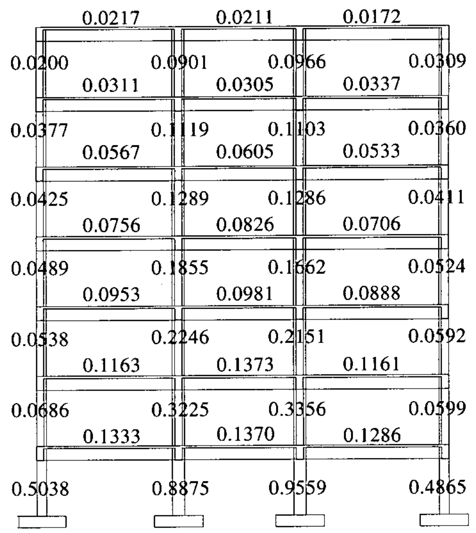


(6-c) PGA= 0.15g

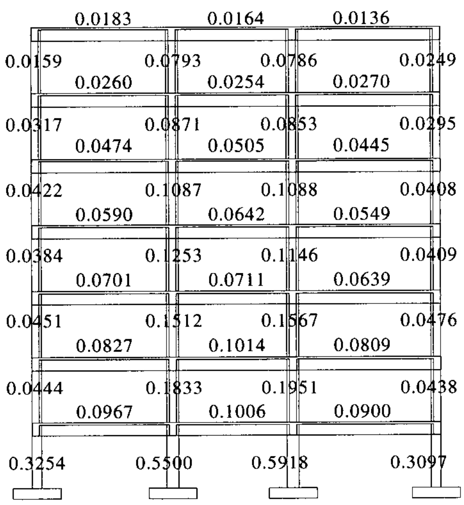
Fig. 6 Local damage index(continued)



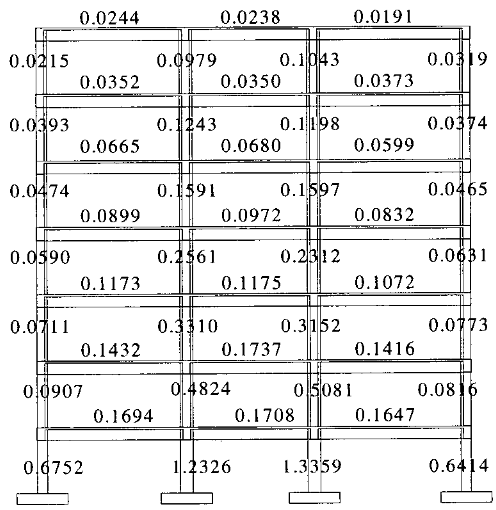
(6-d) PGA= 0.20g



(6-f) PGA= 0.40g



(6-e) PGA= 0.3g



(6-g) PGA= 0.50g

Fig. 6 Local damage index (Nominal srength En =1.0)

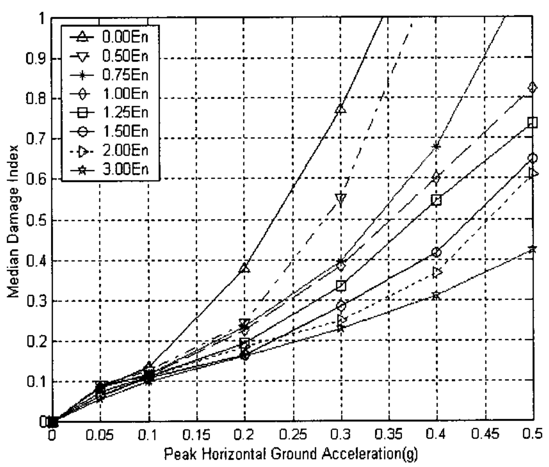


Fig. 7 Median global damage index

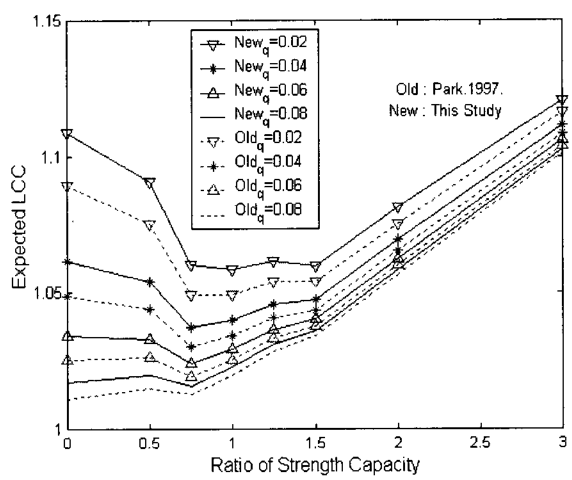


Fig. 8 ELCC curves

Table 3 Optimal ELCC comparison

Discount Rate		q=0.02			q=0.04			q=0.06			q=0.08		
R.S.C	Initial Cost	$ELCC^{(1)}$	$ELCC^{(2)}$	ERROR	$ELCC^{(1)}$	$ELCC^{(2)}$	ERROR	$ELCC^{(1)}$	$ELCC^{(2)}$	ERROR	$ELCC^{(1)}$	$ELCC^{(2)}$	ERROR
0.000	0.9590	1.0891	1.1088	1.1088	0.0181	1.0488	1.0614	0.0120	1.0255	1.0341	1.0111	1.0172	0.0060
0.500	0.9750	1.0751	1.0906	1.0906	0.0144	1.0441	1.0541	0.0096	1.0262	1.0329	1.0151	1.0199	0.0047
0.750	0.9880	1.0494	1.0601	1.0601	0.0102	1.0304	1.0373	0.0067	1.0194	1.0241	1.0126	1.0160	0.0034
1.000	1.0000	1.0494	1.0586	1.0586	0.0088	1.0341	1.0401	0.0058	1.0253	1.0294	1.0198	1.0228	0.0029
1.250	1.0120	1.0539	1.0614	1.0614	0.0071	1.0409	1.0458	0.0047	1.0334	1.0367	1.0288	1.0312	0.0023
1.500	1.0210	1.0539	1.0598	1.0598	0.0056	1.0437	1.0476	0.0037	1.0378	1.0405	1.0342	1.0361	0.0018
2.000	1.0440	1.0754	1.0812	1.0812	0.0054	1.0657	1.0695	0.0036	1.0601	1.0626	1.0566	1.0584	0.0017
3.000	1.0910	1.1160	1.1206	1.1206	0.0041	1.1083	1.1113	0.0027	1.1038	1.1058	1.1010	1.1025	0.0014

R.S.C.: The Ratio of Strength Capacity. $ELCC^{(1)}$: Park.1997. $ELCC^{(2)}$: this study.

that the same cost effective level of design schemes are obtained from proposed model. The result indicates the validity of the EDC model of Eq. (12).

4. Conclusion

By an assumption of continuous cash flow model during a specific lifetime L, the PWF of a Poisson's process of earthquake events can be independently decoupled from damage evaluation.

A more simple and effective expected damage cost function can be formulated as a combination of three independent functions; a PWF of Poisson's process, a damage cost interpolation function and a mean occurrence rate of earthquake intensity.

A systematic approach of a discrete optimization to evaluate the economic effect of seismic hazard is proposed and it is also demonstrated that the cost effective design level can be reasonably obtained by the proposed method.

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