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ML-Based Angle-of-arrival Estimation of a Parametric Source

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Abstract

In angle of arrival estimation, the direction of a signal is usually assumed to be a point. If the direction of a signal is distributed due to some reasons in real surroundings, however, angle of arrival estimation techniques based on the point source assumption may result in poor performance. In this paper, we consider angle of arrival estimation when the signal sources are distributed. A parametric source model is proposed, and the estimation techniques based on the well-known maximum likelihood technique is considered under the model. In addition, Various statistical properties of the estimation errors were obtained.

Keywords: *Estimation, Angle-of-arrival, Parametric distributed source, Maximum likelihood*

I. Introduction

In the field of array signal processing, a class of angle-of-arrival (AOA) estimation techniques has been developed based upon the eigen-structure of the array output covariance matrix.

One well-known AOA estimation technique, the multiple signal classification (MUSIC) technique, is proposed in [10] and its variations can also be found in[11-13]. In[9], the statistical properties of MUSIC are analyzed. In[12], an alternative approach based upon the subspace rotation invariance principle is considered.

Other estimation techniques utilize the maximum likelihood (ML) estimate of the covariance matrix. These ML-based AOA estimation techniques can be categorized as depending upon assumptions associated with the signal amplitude[8].

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In[9], the relationship about two group, i.e., between the conventional MUSIC techniques and the conventional ML techniques has been introduced in detail: the MUSIC technique is a large sample realization of an ML technique and the n-dimensional search problem by ML is decoupled into the n one-dimensional search problems by MUSIC.

All of the AOA estimation techniques mentioned above are based on the assumption that the signal sources are point sources: i.e., if the AOA of a source is θ_p, then there is no other source at $\theta_p+\varepsilon$ for a sufficiently small value of ε.

Under this assumption, the AOA estimation technique utilizes a statistic constructed from a weighted sum of sensor outputs where the sensor outputs are modeled by plane waves emanating from a small number of discrete far-field point sources with an additive spatially and temporally uncorrelated Gaussian noise vector.

In real surroundings, the signals received at an array include not only a direct path signal (which can be regarded as a point source), but also angularly spread signals that are coherent, phase-delayed, and amplitude-weighted replicas of the direct path signal: the signals observed from an array can then be regarded as a superposition of plane waves originating from a continuum of directions.

Typical examples are the angularly spread effects created from the local scattering on the lower layers in a multibeam echo sounder and spurious phenomenon due to clutter in radar. In[1,2], a more detailed discussion can be found. In such cases, the signal source direction is spread around θ_p, the signal's direct path, with angularly spread signals existing in some interval$[\theta_p-\varepsilon, \theta_p+\varepsilon]$ on a single frequency for some non-negligible value of ε. We call such a signal source a parametric distributed source. When the direction of signal source is distributed, i.e., angularly spread, the array output is not a weighted sum of the finite number of steering vectors. In addition, although the AOA estimation techniques for point sources may be applied to the AOA estimation for distributed sources, we do not have confidence that the techniques would provide us with good estimates of the AOA's.

In[1-2], the conventional MUSIC technique with slight modification can be applied to obtain the AOA's of the parametric distributed sources in this paper has been considered. The estimate obtained from this MUSIC-based technique should be a true value only under a large samples. If the number of sample is not large, the other estimation technique, e.g., the ML-based technique, must be considered.

In this paper, when the signal sources are distributed in angle due to some reasons in real surroundings, we address the maximum likelihood estimation of the AOA's for parametric distributed sources.

This paper is organized as follows. Parametric source model is considered in Section 2, followed by ML-based AOA estimation techniques for the parametric source in Section 3. Performance analysis is investigated in Section 4. Numerical examples are considered in Section 5, and concluding remarks are given in Section 6.

II. Parametric Source Model

When M parametric distributed sources are impinging on L array sensors, the output of the array becomes[1]

$$\begin{aligned} \underline{y}(t) &= \sum_{i=1}^{M} \frac{x_i(t)}{2\pi} \int_0^{2\pi} \underline{a}(\theta) \sum_{m=0}^{\infty} c_m(\phi_i) e^{jm\theta} d\theta + \underline{n}(t) \\ &= \sum_{i=1}^{M} x_i(t) \underline{b}(\phi_i) + \underline{n}(t) \end{aligned} \quad (1)$$

where $\underline{y}(t) \in C^{L\times 1}$ is the vector of the array output, $x_i(t)$ represents the ith point source, $\underline{a}(\theta) \in C^{L\times 1}$ is the steering vector, $\phi_i = (\theta_i, \rho_i)$ is the set of unknown parameters with the AOA θ_i(representing the center direction) and distribution parameter ρ_i(representing the extent), $c_m(\phi)$ is a weighting function of the unknown parameters, and

$$\underline{b}(\phi) \triangleq \frac{1}{2\pi} \int_0^{2\pi} \underline{a}(\theta) \sum_{m=0}^{\infty} c_m(\phi) e^{jm\theta} d\theta \quad (2)$$

is the steering vector under the parametric model. It is assumed that the zero-mean white complex normal noise vector $\underline{n}(t) \in C^{L\times 1}$ is stationary with covariance matrices $E[\underline{n}(t)\ \underline{n}^H(s)] = \sigma I \delta_{ts}$ and $E[\underline{n}(t)\ \underline{n}^T(s)] = 0$.

Here $C^{L\times 1}$ denotes the space of $L\times 1$ complex-valued vectors. If we define the point source vector $\underline{x}(\theta, t) = [x_1(\theta, t), x_2(\theta, t), \cdots, x_M(\theta, t)]^T$, $B(\underline{\phi}) = [\underline{b}(\phi_1), \underline{b}(\phi_2), \cdots, \underline{b}(\phi_M)]$ with the vector of unknown parameters $\underline{\phi} = [\phi_1, \phi_2, \cdots, \phi_M]^T$, (1) can be rewritten as

$$\underline{y}(t) = B(\underline{\phi})\underline{x}(t) + \underline{n}(t) \quad (3)$$

and the covariance matrix of $\underline{y}(t)$ is

$$R_0 = B(\underline{\phi}) R_x B^H(\underline{\phi}) \underline{x}(t) + \sigma I \quad (4)$$

III. ML-Based AOA Estimation

Let us consider a ML-based technique to obtain $\underline{\phi}$ from the observations. Here, as in[6], we consider the random process $\underline{x}(t)$ as a conditioning parameter: then the distributional results $\{\underline{y}(t)\}_{t=1}^{N}$ should be interpreted as being conditioned on $\{x(\theta,t)\}_{t=1}^{N}$, since $x(\theta,t)$ is a function of $x(t)$. The conditional log-likelihood function of the observed data can be derived to be

$$\ln L(Y \mid x(t)) = -N \ln \sigma$$
$$-\frac{1}{N\sigma}\sum_{t=1}^{N}[y(t)-B(\phi)x(t)]^{H}[y(t)-B(\phi)x(t)] \quad (5)$$

where $Y=[y(1),y(2),\cdots,y(N)]\in C^{L\times M}$ is the observation matrix. Let us first maximize (5) with respect to $x(t)$ for given ϕ and σ.

Then we obtain

$$x(t)=(B^{H}(\phi)B(\phi))^{-1}B^{H}(\phi)y(t) \quad (6)$$

Let $S_{\phi} \triangleq B(\phi)(B^{H}(\phi)B(\phi))^{-1}B^{H}(\phi)$ be the projection operator onto the space spanned by the columns of $B(\phi)$, $N_{\phi} \triangleq I-S_{\phi}$ be the orthogonal operator of S_{ϕ}, and $\hat{R}_0=\frac{1}{N}\sum_{t=1}^{N}y(t)\,y^{H}(t)$ be the sample covariance matrix of $y(t)$.

Next, let us maximize (5) with respect to σ. Then we get,

$$\hat{\sigma}=\frac{1}{L-M}tr[N_{\phi}\hat{R}_0] \quad (7)$$

Thus, the ML cost function is, from (6) and (7),

$$V(\phi)=tr[N_{\phi}\hat{R}_0] \quad (8)$$

Therefore, the estimate $\hat{\phi}$ of ϕ can be obtained from

$$\hat{\phi}=\arg\max_{\phi} tr[N_{\phi}\hat{R}_0] \quad (9)$$

Eq. (9) can be solved with, for example, the Newton algorithm[7].

IV. Performance Analysis

Let us consider the asymptotic statistical properties of the estimates. Let us define the AOA parameter vector $\theta(=[\theta_1,\theta_2,\cdots,\theta_M]^T)$ and the distribution parameter vector $\rho(=[\rho_1,\rho_2,\cdots,\rho_M]^T)$. Then $B(\phi)$ can be rewritten as $B(\theta,\rho)$. The estimates of ρ and θ are denoted by $\hat{\rho}$ and $\hat{\theta}$, respectively. For notational convenience, let us define

$$h_{\theta\theta} \triangleq \left[\frac{\partial}{\partial\theta}B^{H}(\theta,\rho)\right]N_{\phi}\left[\frac{\partial}{\partial\theta^{T}}B(\theta,\rho)\right],$$
$$h_{\theta\rho} \triangleq \left[\frac{\partial}{\partial\theta}B^{H}(\theta,\rho)\right]N_{\phi}\left[\frac{\partial}{\partial\rho^{T}}B(\theta,\rho)\right],$$
$$h_{\rho\theta} \triangleq \left[\frac{\partial}{\partial\rho}B^{H}(\theta,\rho)\right]N_{\phi}\left[\frac{\partial}{\partial\theta^{T}}B(\theta,\rho)\right],$$
$$h_{\rho\rho} \triangleq \left[\frac{\partial}{\partial\rho}B^{H}(\theta,\rho)\right]N_{\phi}\left[\frac{\partial}{\partial\rho^{T}}B(\theta,\rho)\right],$$

and $[A\circ B]_{ij} \triangleq [A]_{ij}[B]_{ij}$.

Under the assumption that $(\hat{\theta},\hat{\rho})$ is sufficiently close to (θ,ρ), the estimation error vector is obtained from $V'(\hat{\theta},\hat{\rho})=0$, or

$$\begin{bmatrix}\hat{\theta}-\theta\\ \hat{\rho}-\rho\end{bmatrix}\cong -H(\theta,\rho)^{-1}V'(\theta,\rho) \quad (10)$$

where

$$V'(\theta,\rho)=\begin{bmatrix}\frac{\partial}{\partial\theta}V(\theta,\rho)\\ \frac{\partial}{\partial\rho}V(\theta,\rho)\end{bmatrix} \quad (11)$$

and

$$H(\theta,\rho)=\begin{bmatrix}\frac{\partial}{\partial\theta}\left(\frac{\partial}{\partial\theta}V(\theta,\rho)\right)^{T} & \frac{\partial}{\partial\theta}\left(\frac{\partial}{\partial\rho}V(\theta,\rho)\right)^{T}\\ \frac{\partial}{\partial\rho}\left(\frac{\partial}{\partial\theta}V(\theta,\rho)\right)^{T} & \frac{\partial}{\partial\rho}\left(\frac{\partial}{\partial\rho}V(\theta,\rho)\right)^{T}\end{bmatrix} \quad (12)$$

From (10) and by use of the statistical results of[8], the estimation error vector $[(\hat{\theta}-\theta)^{T}\ \ (\hat{\rho}-\rho)^{T}]^{T}$ is easily shown to be zero-mean normal with covariance matrix

$$C=E\left[\begin{bmatrix}\hat{\theta}-\theta\\ \hat{\rho}-\rho\end{bmatrix}[\hat{\theta}-\theta\ \ \hat{\rho}-\rho]\right]=\overline{H}^{-1}\,\overline{C}\,\overline{H}^{-1} \quad (13)$$

where

$$\overline{H}=2\begin{bmatrix}Re(h_{\theta\theta}\circ W^{T}) & Re(h_{\theta\rho}\circ W^{T})\\ Re(h_{\rho\theta}\circ W^{T}) & Re(h_{\rho\rho}\circ W^{T})\end{bmatrix} \quad (14)$$

and

$$C=\frac{\sigma}{2N}\begin{bmatrix}Re(h_{\theta\theta}\circ(W_x)^{T}) & Re(h_{\theta\rho}\circ(W_x)^{T})\\ Re(h_{\rho\theta}\circ(W_x)^{T}) & Re(h_{\rho\rho}\circ(W_x)^{T})\end{bmatrix}^{-1} \quad (15)$$

In (15), $W_x \triangleq \underline{x}(t) W_c \underline{x}(t)$ and

$W_c \triangleq \underline{x}^{-1}(t) + \sigma \underline{x}^{-1}(t)(B^H(\underline{\theta},\underline{\rho})B(\underline{\theta},\underline{\rho}))^{-1}\underline{x}^{-1}(t)$.

Next, let us obtain the Cramer Rao bound CRB of the variance of the estimation error vector. From (5) and by extending the statistical results of[8], the asymptotic CRB is obtained as,

$$CRB = \frac{\sigma}{2N}\begin{bmatrix} Re(h_{\theta\theta}\circ \underline{x}(t)^T) & Re(h_{\theta\rho}\circ \underline{x}(t)^T) \\ Re(h_{\rho\theta}\circ \underline{x}(t)^T) & Re(h_{\rho\rho}\circ \underline{x}(t)^T) \end{bmatrix}^{-1} \quad (16)$$

V. Numerical Examples

In this section let us illustrate the results of previous sections more explicitly. Let us assume that the number L of sensors of a uniform linear array is 5, the number M of signal source is 2, and the number N of snapshots is 100. The signal sources generated from the parametric model are assumed to be uncorrelated.

Example 1:

In this example, we compare the variance of the AOA estimation error in (10) with the CRB. The comparison between the variance (the variance term of the AOA estimation error in (13)) and the CRB (the Cramer Rao Bound term of the AOA estimation error in (16)) is shown in Figure 1 under the condition that one signal source is located at 30° with the distribution parameter 0.99 and the AOA of the other signal source is changed with the distribution parameter fixed at 0.95.

Example 2:

Similar to Example 1, we compare the variance of the distribution parameter estimation error part in (10) with the CRB. The comparison between the variance (the variance term of the distribution parameter estimation error in (13)) and the CRB (the Cramer Rao Bound term of the distribution parameter estimation error in (16)) is shown in Figure 2 under the condition that one signal source is located at 30° with the distribution parameter 0.99 and the distribution parameter of the other signal source is changed with the AOA fixed at 40 degree.

From Examples 1 and 2, we see that the difference between the variance and the CRB of the AOA estimation error and that between the variance and the CRB of the distribution parameter error has been approached zero as the difference between the two AOA's or that between the two distribution parameters is larger. We also observe that the variance and CRB in Figure 1 are larger than those in Figure 2, which implies that the resolvability of the AOA is inferior to that of distribution parameter.

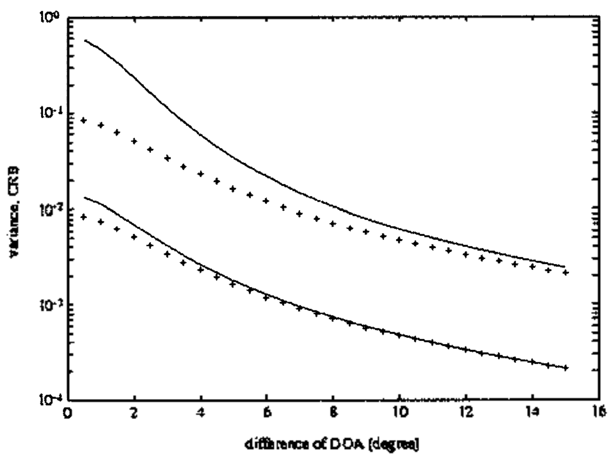

Figure 1. The variance (-) and CRB (+) of AOA estimation errors versus difference of AOA when $\rho_1=0.99$, $\rho_2=0.95$, $L=5$, $N=100$ and SNR=10, 20 (below) dB.

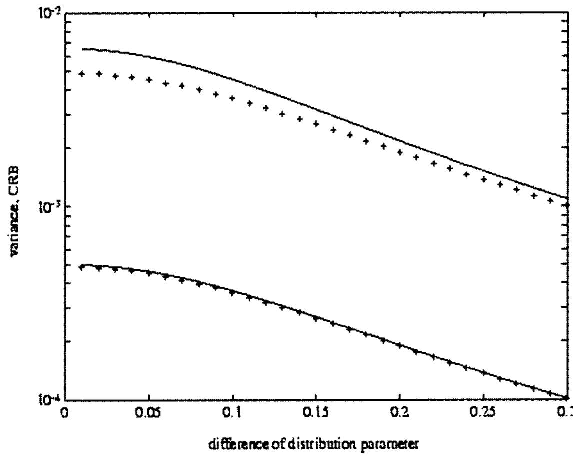

Figure 2. The variance (-) and CRB (+) of distribution parameter estimation errors versus difference of distribution parameter when $\theta_1=30°$, $\theta_2=40°$, $L=5$, $N=100$ and SNR= 10, 20 (below) dB.

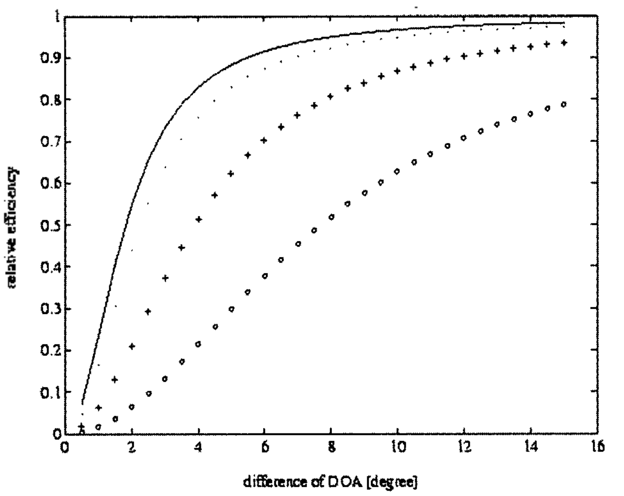

Figure 3. The relative efficiency of AOA estimation errors versus difference of AOA when $\rho_1=\rho_2=1$, 0.9 (.), 0.7 (+), 0.5 (o), $L=5$, SNR=20 dB, $\theta_1=30°$, and θ_2 is changed.

Example 3:

Under the assumption that the two distribution parameters have the same values 0.5, 0.7, 0.9, and 1, the relative efficiency of the AOA estimation error is shown in Figure 3, from which we observe that the relative efficiency of the AOA estimation increases as the distribution parameter gets larger.

From some additional studies, we obtained that the AOA estimation error is more sensitive to the change of the difference of AOA than that of the distribution parameter. Also, we observe that when the SNR is larger than 20 dB, the variance of the AOA estimation error is almost equal to the *CRB*.

VI. Concluding Remarks

When the signal sources are distributed over an area, we considered the ML-based AOA estimation technique in the parametric source model.

It was shown that because of the source distribution, we cannot exactly obtain the AOA's with the conventional ML technique and therefore this difficulty can be overcome by using the parametric source model and the modified ML-based technique.

Various statistical properties of the AOA's and distribution parameter estimation errors were obtained. First, the difference between the variance and *CRB* of the AOA estimation error has been approached at zero as the difference between the two AOA's is larger. We observed that the resolvability of the AOA is inferior to that of distribution parameter, and that the relative efficiency of the AOA estimation increases as the distribution parameter gets larger.

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[Profile]

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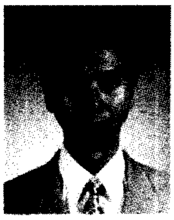

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