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In-situ Determination of Absorption Coefficients in a Room

Jin Sung Suh*

*Department of Mechanical Engineering, Korea Polytechnic University
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Abstract

The possibility is investigated of determining the diffuse absorption coefficients of the wall surfaces in a real room by minimizing the errors between the measured energy impulse response of a real room and the predicted energy impulse responses obtained from the ray tracing simulation of the room. In other words, this can possibly serve as a basis for "acoustical system identification" in attempting to determine the "best fit" of modelled absorption coefficients to measured energy response data. Algorithms for attempting this were investigated. The algorithms developed for this purpose proved to be rigorous and efficient. Instead of using the ray tracing model to determine the absorption coefficients, the phase image model was used in order to determine the acoustic impedances of wall surfaces. However, the numerical algorithms could not find the correct impedance values, primarily due to the wide range of the acoustic impedance values of any single acoustic material and very long computation time.

Keywords: *Absorption coefficient, Impulse response, Ray tracing, Impedance*

I. Introduction

The feasibility is examined of determining the diffuse absorption coefficients of the wall surfaces in a room from the comparison between the measured energy response function of a real room and the predicted energy response functions obtained from the ray tracing model[1-3]. For this to be feasible, two conditions have to be satisfied. Firstly, the ray tracing model should be able to represent the actual physical phenomenon of sound propagation in a room environment. Secondly, the numerical methods employed must be able to find the optimum set of the absorption coefficients of the wall surfaces which minimizes the error between the measured and simulated energy impulse responses.

Corresponding author: Jin Sung Suh (jssuh@kpu.ac.kr)
Korea Polytechnic University, Shihung-city 429-793, Korea

This work discusses only the numerical aspects of the work. It is shown that a combination of two numerical methods, i.e. the "downhill simplex method"[4,5] and the "simulated annealing method"[6,7], could find the correct set of the absorption coefficients which minimizes the error globally. The in-situ determination of the absorption coefficients of wall surfaces in a real room requires that the ray tracing model should be improved further since it cannot represent all the physical aspects of sound propagation in a real room environment.

An attempt was made to use the phase image model [8-11] to identify the acoustic impedances of wall surfaces. At the present stage, however, this attempt was not successful since problems like the heavy computational

burden and very long computation time could not be resolved.

II. Finding Absorption Coefficients by Numerical Techniques

It is first assumed that the result of one ray tracing simulation, when given a particular set of absorption coefficients, is an actual measurement of the energy impulse response function of a room. It needs to be established that this set of absorption coefficients can be found numerically by minimizing the errors between this assumed measurement and the results of the ray tracing simulation obtained from many sets of absoprtion coefficients. When given a large number of the sets of absorption coefficients, each set will produce a different energy impulse response. Among the many sets of absorption coefficients, only one set will give the same energy response function that was assumed to be the actual measurement. So the task in this work will be to establish that the initially given set of absorption coefficients could be found numerically. The procedures will be described below in detail.

Table 1. Coordinates of the source and receiver positions. The units are in metres.

	[illegible]	[illegible]
Case 1	(9.82, 5.35, 1.70)	(5.82, 5.35, 1.70)
Case 2	(9.82, 6.35, 2.70)	(5.82, 4.35, 1.70)
Case 3	(10.82, 3.35, 1.50)	(4.82, 6.35, 2.00)

2.1. Definition of an Error Function

If a room has n wall surfaces whose absosrption coefficients are α_1, α_2, α_3, ..., α_n, then an n-dimensional state space, (α_1, α_2, α_3, ... , α_n), can be defined. If the wall surfaces are covered with only two acoustic materials, a two dimensional (α_1, α_2) state space can be defined. The configuration of a modelled room to be used in this work is shown in Figure 1. It is assumed that the wall surfaces are covered with only two acoustic materials. The absorption coefficients of the floor, and two ceilings were given the value of 0.65. Those of the four side wall surfaces were given the value of 0.35. Three pairs of the positions of monopole source and receiver were used to establish that the same absorption coefficients, i. e. 0.65 and 0.35, could be obtained for each pair. To distinguish between them, "Case 1", "Case 2", "Case 3" were used as specified in Table 1. Each case will give a different energy impulse response. The number of ray particles released from the monopole source is about 18,000. For each of the three cases the ray tracing model with the set of two absorption coefficients (0.65, 0.35) produced the energy impulse response functions as presented in Figure 2. Let us assume these to be the real measurements. Among the many sets of (α_1, α_2), only (0.65, 0.35) will produce the same energy impulse response functions as those shown in Figure 2. The problem at hand will be to investigate whether these two values of (0.65, 0.35) can

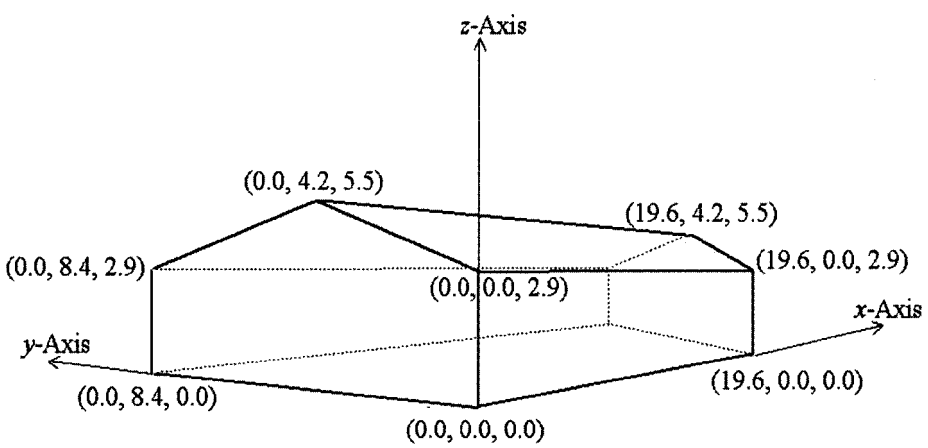

Figure 1. The configuration of the modelled room. All dimensions are in metres.

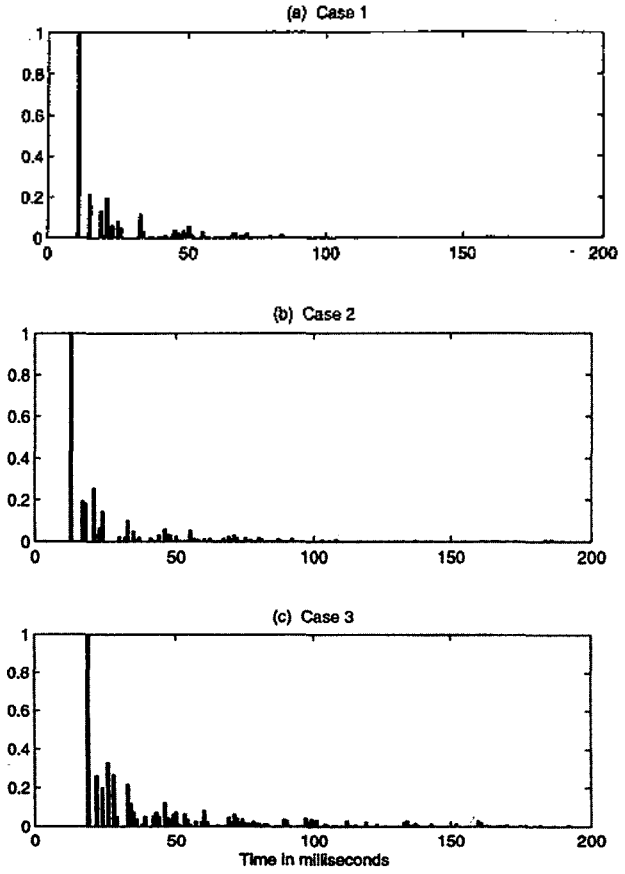

Figure 2. Comparison of the energy impulse response functions in 1 msec resolution.

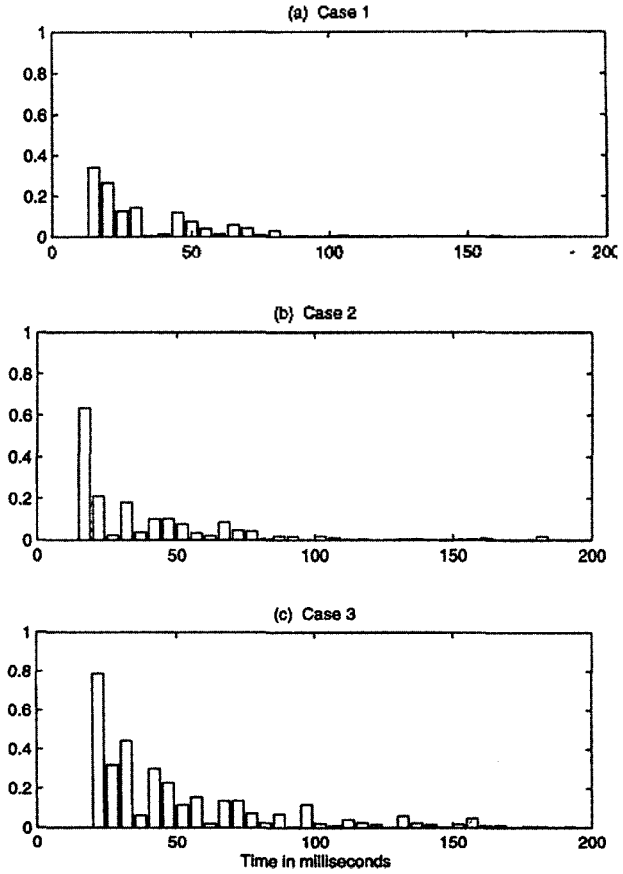

Figure 3. The energy impulse response functions in 5 msec resolution from the detection of the first reflected sound. The arrival times of the first reflected sound are 15 msec for Case 1, 17 msec for Case 2, and 22 msec for Case 3, respectively.

be found among infinitely many sets of (α_1, α_2) by minimizing the errors between the assumed real measurement and the results of the ray tracing simulation tried with many sets of absorption coefficiens.

An error function should thus be defined first and it will be a function of (α_1, α_2). There can be an infinite number of ways of defining an error function depending on the characteristics of each problem at hand. In this work, the error function was defined as follows. From the three assumed measurements shown in Figure 2, the detection of direct sound occurred at t = 11, 13, 19 msec, respectively. Since the direct sound does not contain any information about the absorptive properties of wall surfaces at all, it would be better not to include this in the error function. From the time when the first reflected sound is detected, each 5 consecutive values were all added together, i.e. in a 5 msec resolution. The energy impulse response functions in a 5 msec resolution are shown in Figure 3 for each of the three cases. Likewise this procedure can be applied to the simulated energy impulse response functions obtained from the ray tracing model tried with many sets of absorption coefficients. Let us denote M(i), at the i-th interval at 5 msec time resolution, as the value of the energy impulse response function from one of the assumed real measurements, and R(i) as the value of an energy impulse response function obtained from the ray tracing model with an arbitrary set of absorption coefficients (α_1, α_2). An error function E(α_1, α_2) can be defined by

$$E(\alpha_1,\alpha_2) = \sum_{i=1} \{M(i) - R(i)\}^2 \qquad (1)$$

where i means the i-th interval at 5 msec time resolution.

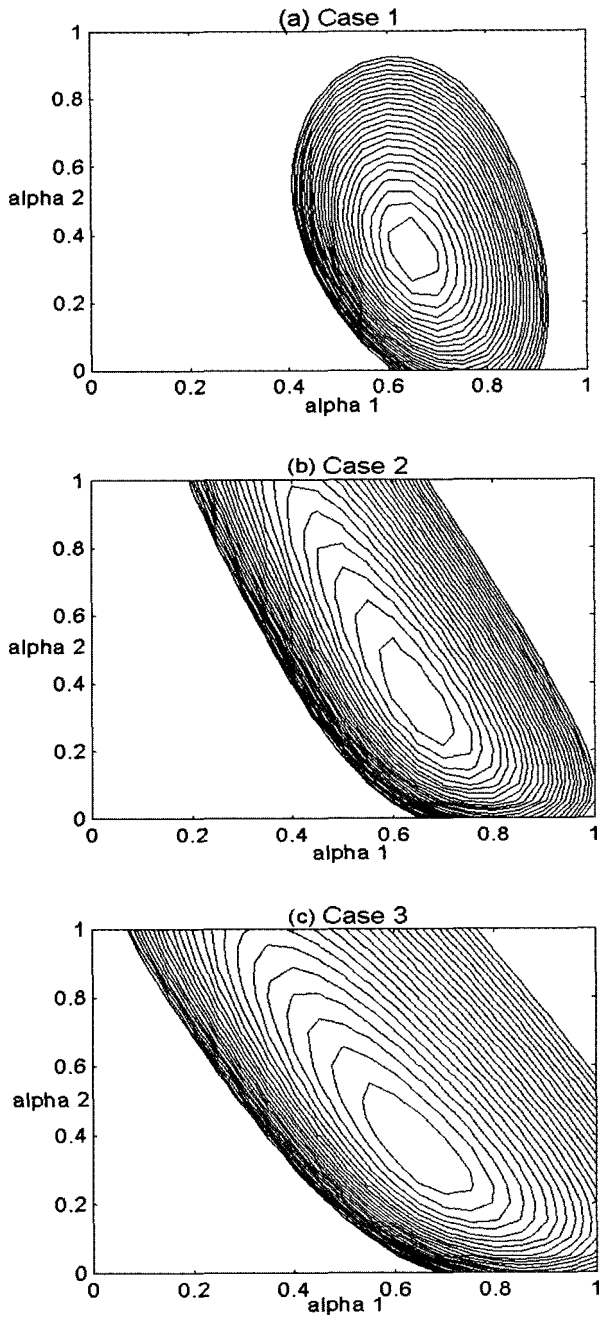

Figure 4. The error contour plots for the three cases which are listed in Table 1. In each of the three cases there exists only one global minimum at (0.65, 0.35).

Among the infinite number of the set (α_1, α_2), only one set (0.65, 0.35) will give a global or true minimum of E(α_1, α_2), which is zero in this problem. Thus the objective is to find the (α_1, α_2) which minimizes E(α_1, α_2) globally by using numerical methods. Before delving into the numerical method the error function E(α_1, α_2) was computed at each grid of the coordinate (0.05i, 0.05j)

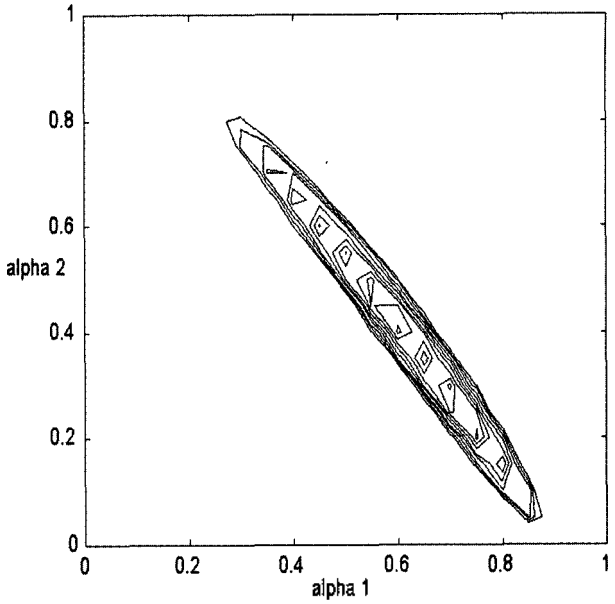

Figure 5. The error contour plot for Case 1 with an ill-defined error function. There are many local minima as well as the true global minimum at (0.65, 0.35).

such that i, j = 0, 1, 2, ..., 20 in order to visualize the form of the error function. So E(α_1, α_2) was computed on 441 points in (α_1, α_2) space. A contour plot which has some lowest levels of E(α_1, α_2) are shown in Figure 4, for each of the three cases.

An error function can be defined in a different way. For Case 1, if the direct sound is included and a time resolution of 50 msec is chosen, a new error function can be similarly defined by equation(1), but in this case "i" in equation(1) means the i-th interval at 50 msec time resolution. This will produce an error contour plot shown in Figure 5. Some local minima as well as the true global minimum can be easily seen. This error function can be considered as ill-defined. Even in this case the numerical methods should be capable of finding the true global minimum with the presence of many other local minima.

2.2. Numerical Techniques

Since the calculation of the gradients of the error function in (α_1, α_2, ..., α_n) space is very difficult to obtain in this particular problem, a numerical method which does not require gradient data are considered. However, this method also has the possibility of being trapped in local minima, thus failing to find the true global

minimum. Therefore they should be tried repeatedly with different initial positions in (α_1, α_2, ..., α_n) space to confirm that the final positions the numerical technique has reached are identical. In higher dimensional space of (α_1, α_2, ..., α_n), in particular, each try can possibly end up with a different position, which means that the algorithm fails to find the true global minimum. This is true of numerical methods which require gradient data. However, the simulated annealing method can have a much better chance of finding the true global minimum since it can escape from local minima according to some probabilistic criterion based on the properties of the system being investigated. In this work, the simulated annealing method combined with the downhill simplex method is used.

2.2.1. Downhill Simplex Method

This method was devised by Nelder and Mead[4]. This method needs only functional evaluations, not gradient. It is not a very efficient method in respect of the number of calculations. However it is quite a good method in a relatively low dimensional state space (α_1, α_2, ..., α_n) of absorption coefficients. The basic principles of this method are well explained by Jacoby et. al.[5]. This is based on the comparison of the error function values at the (n + 1) vertices of a general simplex in (α_1, α_2, ..., α_n) coordinate space and gradually transforming this simplex towards the minimum point. In a two dimensional space a simplex is a triangle, and in a three dimensional space it is a tetrahedron. Initially, e.g. in a two dimensional space, three vertices of a triangle are arbitrarily chosen as a starting simplex. This is shown in Figure 6. At each of the three vertices, the error function values are computed. x^h is the highest point of the error function, which means that the error evaluated at x^h gives the highest value. x^s is the second highest point. x^l is the lowest point. This method moves x^h to a lower point by the three basic operations. x^o is the centroid of all x^i except i = h, given by (Refer to Figure 6)

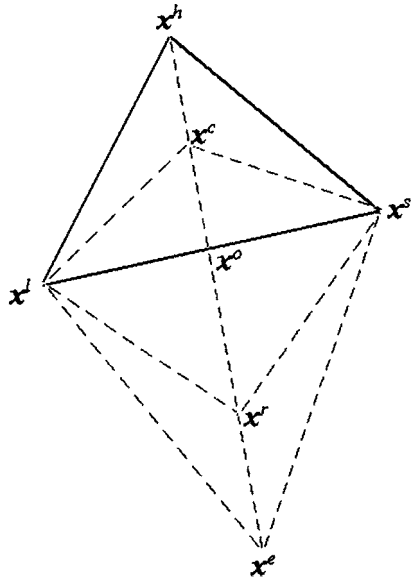

Figure 6. The vertices of the starting simplex are located at x^l, x^s, x^h. This figure represents the reflection, expansion, and contraction operations in the downhill simplex method in a two dimensional space (α_1, α_2).

$$x^o = \frac{1}{n}\sum_{\substack{i=1 \\ i\neq h}}^{n+1} x^i \tag{2}$$

Firstly, a reflection operation is tried, which can be represented by

$$x^r = x^o + \alpha(x^o - x^h),\ 0 < \alpha \leq 1 \tag{3}$$

where the usual choice of is 1.0. If the value of the error function at x^r is smaller than that at x^h, then x^r is selected as a vertex of a newly transformed simplex. For this case the vertices of the new simplex are located at x^l, x^s, x^r. Otherwise an expansion operation is tried. x^r in equation (3) is expanded in the direction along which a further improvement of the error may be expected. This can be represented by

$$x^e = x^o + \gamma(x^o - x^h),\ \gamma > 1 \tag{4}$$

where the usual choice of γ is 2.0. If the value of the error function at x_e is smaller than that at x_h, then x_e is selected as a vertex of a newly transformed simplex. Otherwise a contraction operation is tried, which can be represented by

$$x^c = x^o + \beta(x^h - x^o),\ 0 < \beta < 1 \tag{5}$$

where the usual choice of β is 0.5.

After x^h is transformed into a new vertex, the error values at each vertex of the newly transformed simplex are compared in order to determine which are the highest, second highest, and lowest points. The above operations are repeated until the test for convergence is satisfied, which is given by

$$\left\{\sum_{i=1}^{n+1}\frac{\left[E(x^i)-E(x^o)\right]^2}{n+1}\right\}^{\frac{1}{2}} \le \tau \tag{6}$$

where τ is usually a small, preselected tolerance, e.g. 10^{-6}. Another possible convergence test will be to test whether the fractional difference between the function values at the highest and the lowest points is smaller than a tolerance value, which can be stated as

$$\frac{E(x^h)-E(x^l)}{E(x^h)} \le \tau \tag{7}$$

If the convergence test is satisfied, the computation is terminated. This convergence test should be undertaken for every current simplex.

Although this method has some advantages, it also has some disadvantages as well. One of them is that there is a possibility that the final simplex may collapse into a local minimum not finding the true global minimum. It is a good idea to repeat this method by starting from different initial simplexes.

2.2.2. Simulated Annealing Method

This method, in its essence, is deeply related with the fundamentals of statistical physics and was originated by Metropolis et al.[6]. As shown in Figure 7, when a system is in thermal equilibrium with a heat bath which is at a constant temperature T then the probability P_r of finding the system in any one particular microscopic state(or microstate) r of energy E_r is given by[12]

$$P_r = \frac{e^{\frac{-E_r}{kT}}}{\sum_r e^{\frac{-E_r}{kT}}} \tag{8}$$

where the denominator is called a "partition function" and its sum is carried out over all the accessible microstates of the system. The symbol k denotes the Boltzmann constant, and T is the absolute temperature in Kelvin. Equation(8) is known as "Boltzmann distribution". By utilizing Boltzmann distribution, it is possible to get out of a local minimum and to reach the true global minimum. By analogy the error function $E(\alpha_1, \alpha_2)$ can be comparable to E_r in equation(8), and (α_1, α_2) to a microstate. Let us denote "c" as current state and "n" as next state. Then $(\alpha_1, \alpha_2)_c$ is the current state, and $(\alpha_1, \alpha_2)_n$ is the next state. Then (P_n/P_c) can be written, according to the Boltzmann distribution, in the form

$$\frac{P_n}{P_c} = e^{\frac{-(E_n-E_c)}{kT}} \tag{9}$$

where E_c is the current error value at $(\alpha_1, \alpha_2)_c$ and E_n is the error value at $(\alpha_1, \alpha_2)_n$. If En is smaller than

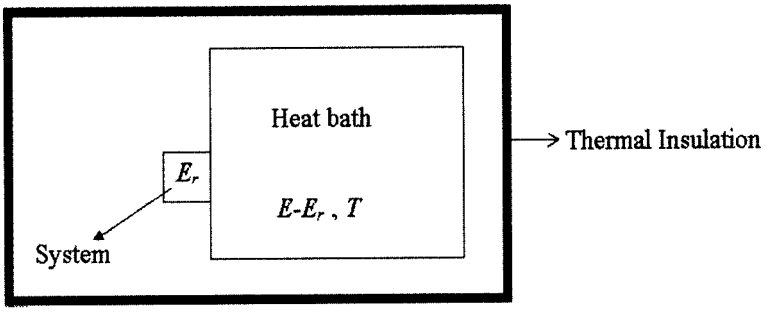

Figure 7. A system in thermal equilibrium with a heat bath. The heat bath is at a constant temperature T. The system and heat bath are thermally interacting with each other but they are isolated from their environment. Thus the total energy E remains constant.

E_c, i.e. $0 < P_n/P_c > 1$, then $(\alpha_1, \alpha_2)_n$ is always chosen as the new current state and En always chosen as the new current error value. If En is bigger than E_c, i.e. $0 < P_n/P_c < 1$, $(\alpha_1, \alpha_2)_n$ and E_n could become a new current state and a new current error value according to a certain probabilistic criterion. At first glance this looks strange, but if we are trapped in a local minimum, any next state near a local minimum (current state+incremental change of state) will always give a bigger value than the value at the local minimum. So there is no way to get out of a local minimum if a smaller error value is always to be chosen as a new current error value. It is possible to set a probabilistic criterion which makes it possible for a value bigger than the current error value to become a new current value. By taking the natural logarithm of equation(9), and generating a random number which is between 0 and 1, this criterion can be written by[7]

$$T \times log(random) < -(E_n - E_c) \tag{10}$$

where (random) means a random number between 0 and 1. In equation(10) the Boltzmann constant k need not be included. Here T is a control parameter which is analogous to temperature. At the starting value of T, e.g. 10^6, the process of finding a new minimum should be carried out as many times as possible. T should be lowered sowly to simulate the annealing process. At the next lower value of T, e.g. 0.9×10^6, this searching process should be carried out again. It can be understood that major improvements would be accomplished at high values of T and small refinements would be accomplished at low values of T as T is being lowered slowly. It seems that the simulated annealing method still leaves room for further improvement and development for its detailed application.

III. Numerical Results

The entire process of searching for the global minimum is listed in Table 2 for Case 1 only. The last line corresponds to the true global minimum where the searching process came to an end. The numerical technique combined with the downhill simplex method and simulated annealing method could find the true global minimum, and thus the correct set of absorption coefficients (0.65, 0.35). For Case 2 and Case 3, they also gave the same results. (α_1, α_2) in each line corresponds to the lowest point x^l in each current simplex and $E(\alpha_1, \alpha_2)$ is the error value at x^l. (Refer to Figure 6)

Also for the case of ill-defined error function for Case 1 as shown in Figure 5, the numerical technique used could find the true global minimum with the presence of the many local minima. This numerical technique could find the true global minimum in other situations where

(1) The correct set of absoprtion coefficients (α_1, α_2) are in their extremities such as (0.01, 0.99).
(2) The room shape is very disproportionate.
(3) Combination of (1) and (2).

All the above procedures were carried out for the case

Table 2. Searching process for the global minimum for Case 1. "..." indicates some intermediate searching process which is omitted.

α_1	α_2	$E(\alpha_1, \alpha_2)$
0.010	0.900	448.681
0.025	0.880	423.263
0.020	0.900	430.133
0.010	0.900	448.681
0.020	0.900	430.133
0.018	0.930	432.507
0.003	0.950	458.833
0.031	0.925	409.098
0.049	0.867	382.489
0.111	0.849	293.459
0.384	0.688	63.880
0.977	0.221	69.751
0.402	0.630	53.198
0.977	0.221	69.751
0.241	0.790	157.592
...	...	...
0.653	0.340	0.022
0.662	0.352	0.129
0.653	0.340	0.022
0.653	0.363	0.053
0.650	**0.350**	**0.000**

of three dimensional state space (α_1, α_2, α_3), i.e. when the wall surfaces of a room are covered with three acoustic materials. The numerical technique used could reach the correct set of absorption coefficients (α_1, α_2, α_3).

IV. Conclusion

The simulated annealing method combined with the downhill simplex method could search for the correct set of the absorption coefficients. By using the simulated annealing method it was possible to escape from many local minima and to reach the true global minimum. This technique could search for three unknown absorption coefficients as well as two. The error minimization in multi-dimensional space higher than three dimensional space will require further development together with faster computing power.

The numerical technique was extended to the phase image model in order to investigate whether the correct values of the acoustic impedances could be obtained. However, this attempt was not computationally possible due to the wide range of the acoustic impedance values of any single acoustic material, and very long computation time. An attempt to overcome these difficulties will be left as a future work.

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[Profile]

• Jin Sung Suh

Jin Sung Suh received B.Sc. degree in Mechanical Design & Production Engineering from Seoul National University in 1985, and M.Sc. degree in Noise and Vibration Engineering from the University of Southampton in 1988. From 1989 to 1992 he was with Daewoo Motor Corporation. He received his Ph.D. degree in Noise and Vibration Engineering from the University of Southampton in 1998. Since March in 2000, he has been in the Department of Mechanical Engineering at Korea Polytechnic University.