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Time-Varying Subspace Tracking Algorithm for Nonstationary DOA Estimation in Passive Sensor Array

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Abstract

In this paper we propose a new subspace tracking algorithm based on the PASTd (Projection Approximation Subspace Tracking with deflation). The algorithm is obtained via introducing the variable forgetting factor which adapts itself to the time-varying subspace environments. The tracking capability of the proposed algorithm is demonstrated by computer simulations in an abruptly changing DOA scenario. The estimation results of the variable forgetting factor PASTd (VFF-PASTd) outperform those of the PASTd in the nonstationary case as well as in the stationary case.

Keywords *: Sonar signal processing, Array processing, Subspace tracking, Variable forgetting factor, PASTd*

I. Introduction

High-resolution subspace methods have been studied by many researchers for DOA (Direction of Arrival) estimation with sensor array processing in sonar and radar field. Many situations of the DOA applications need to estimate the DOA of the fast time varying objects, e.g. fast moving targets or fast moving jammers. In these days, mobile antenna researchers also take interest in the problem for mobile users. Thus DOA in nonstationary environments has drawn attention in high-resolution array processing area. In the nonstationary environments it is necessary to recursively update the subspace. Many efficient updating algorithms categorized as subspace tracking, have been proposed and evaluated in [1].

One of the novel subspace tracking algorithms is the projection approximation subspace tracking with deflation (PASTd)[2]. The basic idea of the PASTd is that a projection-like unconstrained criterion is approximated using an RLS-like algorithm for tracking the signal subspace. However the PASTd algorithm doesn't work well in fast time-varying environments or time varying and stationary mixed environments because it uses a fixed forgetting factor for subspace tracking.

This paper proposes a new algorithm for fast subspace tracking based on the PASTd. For the fast tracking this paper introduces the variable forgetting factor to the PASTd. Since the variable forgetting factor can effectively adjust itself to the varying nonstationarity of subspace, it provides the performance improvement to the PASTd.

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II. Problem Formulation

Consider an array of M sensors on which K ($M>K$) incident narrowband point sources impinge. These source signals are assumed to be emitted by targets moving with arbitrary trajectories. At time t, the sample vector $x(t) \in C^{M\times 1}$ of the sensor outputs can be written according to the model,

$$x(t) = A(\theta(t))s(t) + n(t), \tag{1}$$

where $s(t) \in C^{K\times 1}$ is the vector of the complex envelopes of the target signals, $n(t) \in C^{M\times 1}$ is an additive noise, $A(\theta(t)) = [a(\theta_1(t)), \cdots, a(\theta_K(t))] \in C^{M\times K}$ is the matrix of the steering vectors $a(\theta_k(t))$, and $\theta_k(t)$, $k=1,\cdots,K$ is the DOA of target k measured with respect to the normal of the array [3].

In the subspace-based method, the DOA's are determined as the minimizing arguments of the cost function,

$$f(\theta) = a^H(\theta)\tilde{\Pi}a(\theta), \tag{2}$$

where $\tilde{\Pi}$ is the orthogonal projector onto the noise subspace. Classically, $\tilde{\Pi}$ is obtained via an off-line eigendecomposition of the covariance matrix of the received data. The $\tilde{\Pi}$ can also be recursively obtained via the PASTd algorithm. The PASTd algorithm tracks time-varying subspace with a single fixed forgetting factor. However, the tracking ability of the PASTd with the single fixed forgetting factor is not satisfactory for its practical use, because the fixed forgetting factor limits the traceable range of the nonstationarity of signal and it may increase the estimation variance even in stationary interval. In addition to that, the PASTd needs *a priori* knowledge of the signal nonstationarity to set the forgetting factor. In order to improve the tracking capability of the PASTd, this paper derives a method that estimates the subspace by adaptively adjusting the forgetting factor without *a priori* knowledge of the nonstationarity.

III. Projection Approximation Subspace Tracking (PAST) and Projection Approximation Subspace Tracking with deflation (PASTd)

In the following, two subspace tracking algorithms are discussed. They are based on the minimum property of the unconstrained cost function,

$$\begin{aligned} J(W(t)) &= \sum_{i=1}^{t} \beta^{t-i} \| x(i) - W(t)W^H(t)x(i) \|^2 \\ &= tr[C(t)] - 2tr[W^H(t)C(t)W(t)] \\ &\quad + tr[W^H(t)C(t)W(t)W^H(t)W(t)] \end{aligned} \tag{3}$$

Here $C(t)$ is the sample correlation matrix and $W(t)$ is the $N\times r$ matrix. It has been proven that $J(W(t))$ has a global minimum as $W(t) = V_s(t)Q$ and no other local minima [2,4]. $V_s(t)$ is the matrix containing the r signal eigenvectors of $C(t)$ and Q is an arbitrary unitary matix.

The above results states that one can seek the signal subsapce of $C(t)$ by minimizing $J(W(t))$ via iterative methods. Since there are no local minima, a global convergence is guaranteed. B. Yang is interested in adaptive algorithms which compute $W(t)$ from $W(t-1)$ and the new sensor output vector $X(t)$ recursively. For this purpose, $W^H(t)X(t)$ in eqn.3 is approximated as the unknown projection of $X(t)$ onto the columns of $W(t)$ by the expresion $y(i) = W^H(i-1)x(i)$ which can be calulated for $1 < i < t$ at the time instant t. This results in a modified cost function,

$$J'(W(t)) = \sum_{i=1}^{t} \beta^{t-i} \| X(i) - W(t)y(i) \|^2, \tag{4}$$

which is now quadratic in the elements of $W(t)$.

Note that the above projection approximation changes the error performance surface of $J(W(t))$. For stationary or slowly varying signals, however, the difference between $W^H(t)x(i)$ and $W^H(i-1)x(i)$ is small, in particular when i is close to t. This differnce may be larger in the distance past with $i \ll t$. But the contribution of the past data to the cost function is decreasing for growing t. Therefore it is expected $J'(W(t))$ to be a good approximation for $J(W(t))$ and the matrix $W(t)$ minimizing $J'(W(t))$ to be a good estimate for the signal subspace of $C(t)$.

This projection approximation is the key issue of PAST and PASTd for subspace tracking. Its main advantage is the exponentially weighted least squares criterion ,eqn.4 which is well studied in adaptive filtering. $J'(W(t))$ is minimized if

$$
\begin{aligned}
W(t) &= C_{xy} C_{yy}^{-1}(t), \\
C_{xy}(t) &= \sum_{i=1}^{t} \beta^{t-i} x(i) y^H(i) = \beta C_{xy}(t-1) + x(t) y^H(t), \qquad (5) \\
C_{yy}(t) &= \sum_{i=1}^{t} \beta^{t-i} y(i) y^H(i) = \beta C_{yy}(t-1) + y(t) y^H(t).
\end{aligned}
$$

A recursive computation of the $N \times r$ matrix $C_{xy}(t)$ and the $r \times r$ matrix $C_{yy}(t)$ requires $O(Nr)$ and $O(r^2)$ operations. The computation of $W(t)$ from $C_{xy}(t)$ and $C_{yy}(t)$ demands additional $O(Nr)+O(r^3)$ operations. A more efficient and numerically more robust way is to apply the matrix inversion lemma to compute the inverse of $C_{yy}(t)$ recursively.

Table 1 summarizes the resulting algorithm. B. Yang called it projection approximation subspace tracking(PAST). The operator Tri{ } indicates that only the upper (or lower) triangular part of $P(t) = C_{yy}^{-1}(t)$ is calculated and its Hermitian transposed vesion is copied to the another lower (or upper) triangular part. The PAST algorithm requires only $3Nr+O(r^2)$ operations every update.

In the above table, the columns of $W(t)$ are not perfectly orthonormal because it minimizes the modified cost function $J'(W(t))$ instead of the original one $J(W(t))$.

In the following, a second subspace tracking algorithm is introduced in the same approach. It is based on the deflation technique and is referred to as the PASTd algorithm. The basic idea of deflation is the sequential estimation. First the most dominant eigenvector is updated by applying the PAST algorithm with r=1. Then the projection of the current data vector x(t) onto this eigenvector is removed from x(t) itself. Because the second dominant eigenvector becomes the most dominant one in the updated data, it can be extracted in the same way as before. Applying this procedure repeatedly, all desired eigenvectors are estimated sequentially.

Table 2 summarizes the PASTd algorithm. Actually, the main body within the second For loop corresponds to the PAST algorithm in the one-vector case r=1. The quantity $\lambda_i(t)$ palys the same role as the $r \times r$ matrix $C_{yy}(t) = P^{-1}(t)$ in Table 6 and the gain vector g(t) becomes a scalar gain $y_i(t)/\lambda_i(t)$ in deflation case. The last equation in Table 1 describes the deflation step. It subtracts the component of $x_i(t)$ along the direction of the i-th eigenvector $w_i(t)$ from $x_i(t)$. The PASTd algorithm requires $4Nr+O(r)$ operations per update.

Table 1. Summary of PAST.

Choose P(0) and $\underline{W}(0)$ suitably

For t=1,2,⋯ Do

$y(t) = W^H(t-1)x(t)$	(T1.1)
$h(t) = P(t-1)y(t)$	(T1.2)
$g(t) = h(t)/[\beta + y^H(t)h(t)]$	(T1.3)
$P(t) = \frac{1}{\beta} Tri\{P(t-1) - g(t)h^H(t)\}$	(T1.4)
$e(t) = x(t) - W(t-1)y(t)$	(T1.5)
$W(t) = W(t-1) + e(t)g^H(t)$	(T1.6)

END

Table 2. Summary of PASTd.

Choose $\lambda_i(0)$ and $W_i(0)$ suitably

For t=1,2,⋯ Do

$x_1(t) = x(t)$	(T2.1)
For i=1,⋯,r Do	
$y_i(t) = W_i^H(t-1)x_i(t)$	(T2.2)
$\lambda_i(t) = \beta\lambda_i(t-1) + \lvert y_i(t)\rvert^2$	(T2.3)
$\varepsilon_i(t) = x_i(t) + w_i(t-1)y_i(t)$	(T2.4)
$w_i(t) = w_i(t-1) + \varepsilon_i(t)\frac{y_i(t)}{\lambda_i(t)}$	(T2.5)
$x_{i+1}(t) = x_i(t) + w_i(t)\, y_i(t)$	(T2.6)
END	

END

IV. Variable Forgetting Factor PASTd Algorithm

The PASTd algorithm handles the nonstationarity with a forgetting factor, β . Practical DOA's estimation situation is apt to be stationary / nonstationary mixed or unpredictable nonstationary environments so that the single fixed forgetting factor cannot handle such various cases effectively. To cope with the nonstationary DOA's environments, this paper introduces the variable forgetting factor to the PASTd algorithm. The variable forgetting factor properly adjusts itself to the nonstationarity of DOA. The variable forgetting factor is determined as the minimizing arguments of cost function, which is modified from eqn.4, eqn.T2.4 and eqn.T2.3.

$$J_i=\frac{1}{2}\sum_{j=1}^{t}\omega_i(t,j)\,\varepsilon_i^H(j)\varepsilon_i(j), \tag{6}$$

where $\omega_i(t,j)=\begin{cases}\prod_{k=j+1}^{t}\beta_i(k), & 1\le j\le t-1,\\ 1, & j\ge t\end{cases}$

$0\le\beta_i(k)<1$, $\omega_i(k,k)=1$, $\omega_i(k-1,k)=\beta_i(k)$ and $\varepsilon_i(j)=x_i(j)-w_i(j)y_i(j)$. The steepest descent method can be applied to minimize eqn.6 and the variable forgetting factor migrates as follows [5].

$$\omega_i(t+1,t)=\omega_i(t,t-1)-\frac{1}{2}\alpha\frac{\partial E_i}{\partial\beta} \tag{7}$$

or $\beta_i(t+1)=\beta_i(t)-\frac{1}{2}\alpha\frac{\partial E_i}{\partial\beta}$ from definition of $\omega_i(t,j)$ in eqn. 6. (8)

$$\beta_i(t+1)=\beta_i(t)-\alpha Re[\varepsilon_i^H(t)\varepsilon_i'(t)], \tag{9}$$

where $E_i=\varepsilon_i^H(t)\varepsilon_i(t)$, $\varepsilon_i'(t)=\frac{\partial\varepsilon_i(t)}{\partial\beta_i}$ and α is a control factor. Since eqn.9 contains the sample derivative, $\varepsilon_i^H(t)\varepsilon_i'(t)$, which is noise sensitive, it should be replaced by the smoothed version as eqn.10 [6].

$$\tilde{J}_i=\sum_{j=1}^{t}\omega_i(t,j)\,\varepsilon_i^H(j)\varepsilon_i'(j). \tag{10}$$

With the smoothed sample derivative, the eqn.9 becomes eqn.11,

$$\beta_i(t+1)=\beta_i(t)-\alpha Re[\tilde{J}_i(t)] \tag{11}$$

and the smoothed sample derivative $\tilde{J}_i$ can be expressed recursively.

$$\tilde{J}_i(t)=\omega_1(t,t-1)\tilde{J}_i(t-1)+\varepsilon_i^H(t)\varepsilon_i'(t) \tag{12}$$

or $\tilde{J}_i(t)=\beta_i(t)\tilde{J}_i(t-1)+\varepsilon_i^H(t)\varepsilon_i'(t)$ from definition of $\omega_i(t,j)$. (13)

Table 3. Summary of VFF-PASTd.

Choose $\lambda_i(0)$ and $W_i(0)$ suitably

$$X_1(t)=X(t) \tag{T3.1}$$

For $i=1,\cdots,r$ Do

$$y_i(t)=W_i^H(t-1)X_i(t) \tag{T3.2}$$

$$\lambda_i(t)=\beta_i(t)\lambda_i(t-1)+|y_i(t)|^2 \tag{T3.3}$$

$$\varepsilon_i(t)=X_i(t)-W_i(t-1)y_i(t) \tag{T3.4}$$

$$\varepsilon_i'(t)=-\Psi_i(t-1)W_i^H(t-1)X_i(t)-W_i(t-1)\Psi_i^H(t-1)X_i(t) \tag{T3.5}$$

$$\tilde{J}_i(t)=\beta_i(t)\tilde{J}_i(t-1)+\varepsilon_i^H(t)\varepsilon_i'(t) \tag{T3.6}$$

$$\beta_i(t+1)=\beta_i(t)-\alpha Re[\tilde{J}_i(t)] \tag{T3.7}$$

$$y_i'(t)=\Psi_i^H(t-1)X_i(t) \tag{T3.8}$$

$$S_i(t)=d_i(t-1)-\beta_i(t)S_i(t-1)+2Re[\Psi_i^H(t-1)X_i(t)X_i^H(t)W_i(t-1)] \tag{T3.9}$$

$$\Psi_i(t)=\Psi_i(t-1)+\varepsilon_i'(t)[y_i^*(t)/d_i(t)]+\varepsilon_i(t)[y_i'^*(t)d_i(t)-y_i^*(t)S_i(t)]/d_i^2(t) \tag{T3.10}$$

$$w_i(t)=w_i(t-1)+\varepsilon_i(t)\frac{y_i^*(t)}{\lambda_i(t)} \tag{T3.11}$$

$$X_{i+1}(t)=X_i(t)-W_i(t)y_i(t) \tag{T3.12}$$

END

where $y_i'(t)=\frac{\partial y_i(t)}{\partial\beta_i}$, $\Psi_i(t-1)=\frac{\partial W_i(t-1)}{\partial\beta_i}$ and $S_i(t)=\frac{\partial\lambda_i(t)}{\partial\beta_i}$

The $\varepsilon_i'(t)$ can also be expressed recursively as eqn.14.

$$\varepsilon_i'(t) = -\frac{\partial W_i(t-1)}{\partial \beta_i} W_i^H(t-1)X_i(t) - W_i(t-1)\frac{\partial W_i^H(t-1)}{\partial \beta_i} X_i(t) \quad (14)$$

For the partial derivative of $\omega_i(t-1)$, three recursive equations for $\frac{\partial W_i(t-1)}{\partial \beta_i}$, $\frac{\partial y_i(t)}{\partial \beta_i}$ and $\frac{\partial \lambda_i(t)}{\partial \beta_i}$ are derived additionally from eqn.T2.5, eqn.T2.2 and eqn.T2.3 respectively.

$$\frac{\partial W_i(t)}{\partial \beta_i} = \frac{\partial W_i(t-1)}{\partial \beta_i} + \varepsilon_i'(t)\frac{y_i^*(t)}{\lambda_i(t)} + \varepsilon_i(t)\left[\frac{\partial y_i^*(t)}{\partial \beta_i}\lambda_i(t) - y_i^*(t)\frac{\partial \lambda_i(t)}{\partial \beta_i}\right]/(\lambda_i(t))^2. \quad (15)$$

$$\frac{\partial y_i(t)}{\partial \beta_i} = \frac{\partial W_i^H(t-1)}{\partial \beta_i} X_i(t). \quad (16)$$

$$\frac{\partial \lambda_i(t)}{\partial \beta_i} = \lambda_i(t-1) + \beta_i(t)\frac{\partial \lambda_i(t-1)}{\partial \beta_i} + \frac{\partial W_i^H(t-1)}{\partial \beta_i} X_i(t) X_i^H(t) W_i(t-1) + W_i^H(t-1) X_i(t) X_i^H(t)\frac{\partial W_i(t-1)}{\partial \beta_i}. \quad (17)$$

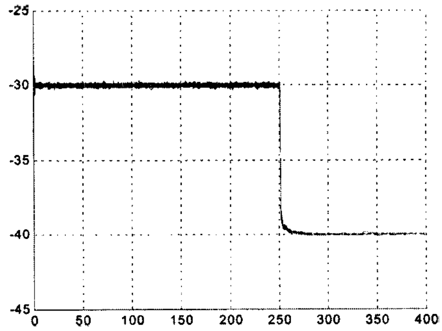
With eqn.15~eqn.17, we modify the PASTd to effectively track the time-varying DOA's. Table 3 gives a summary of the proposed algorithm.

V. Simulation Results

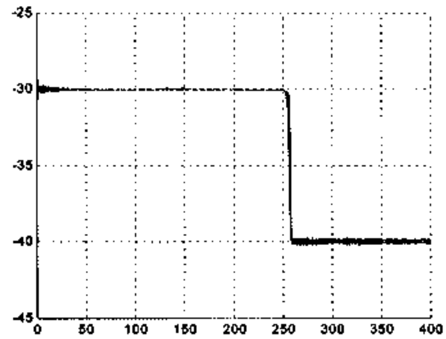
In this section, the performance of PASTd and VFF-PASTd are demonstrated. All algorithms find the DOA estimates using the ESPRIT algorithm. Although other approaches for finding DOA may be considered, the ESPRIT approach has an advantage that the columns of $W(t)$ in eqn. 3 are not required to be orthogonal.

For the performance demonstration, two different simulations are performed. The first simulation tests the effect of α factor in eqn.7 on the variation speed of $\beta_i(t)$ and the other shows the tracking capability.

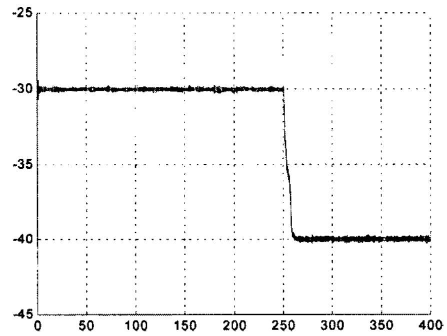
Simulation 1. Selection of α in VFF-PASTd : The factor of α is necessary to recursively calculate the variable



(a) α=1



(b) α=0.1

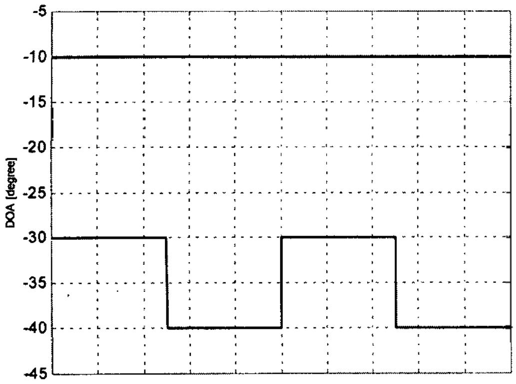


(c) α=0.01

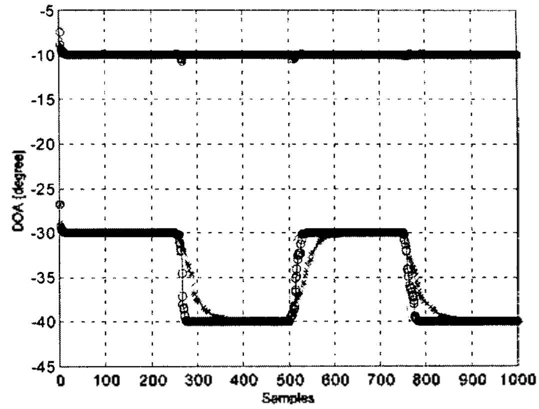
Fig. 1. α factor effect test for tracking performance of the proposed algorithm.

forgetting factor. To select the proper α, we perform test trials for 3 different values. Fig.1 (a), (b) and (c) show the results for the α's of 1, 0.1 and 0.01 respectively. In each test, 50 independent trials have been performed. The results show that the factor of α does not make great difference in quick responses although the larger α makes the faster response. In the following experiment, α of 1 has been used.

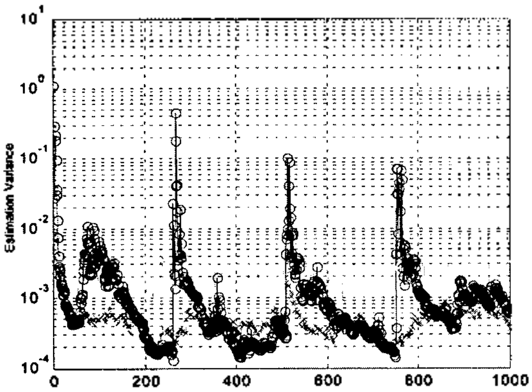
Simulation 2. Sensor Array DOA Tracking : In this scen-



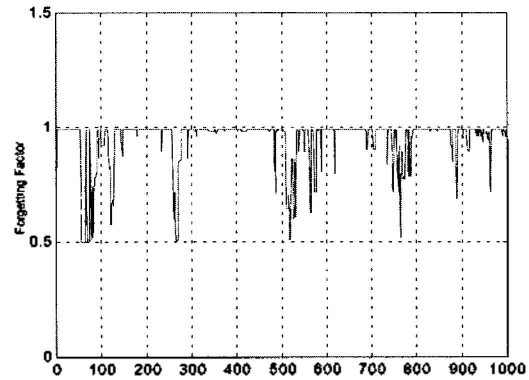
(a) DOA variation scenario



(b) PASTd with the forgetting factor of 0.97 (-*-), PASTd with the forgetting factor of 0.8 (solid line) and the proposed algorithm (-o-)



(c) Variance of the estimated DOA's (PASTd with the forgetting factor of 0.97 (-*-), PASTd with the forgetting factor of 0.8 (solid line) and the proposed algorithm (-o-))



(d) Variable forgetting factor track.

Fig. 2. Tracking performance comparison.

ario, two planar wavefronts arrive at the array. The emitter signals are zero-mean white and Gaussian. A uniform linear array of 8 elements is studied. The signals have individual SNR's of 20 dB.

For comparison, the PASTd algorithm has been also simulated with different forgetting factors of 0.97 and 0.8. The proposed algorithm and the PASTd algorithm have been simulated in 50 independent trials. For the initial value, $\lambda_i(0)$ sets to 1 and $W_i(0)$ sets to {0...0,1,0...0} in which the i-th element is '1' and the others are set to '0'.

Fig.2 (a) displays the DOA change scenario. Fig.2 (b) shows the mean value of DOA's estimated by the proposed algorithm and PATSd algorithm respectively. Fig.2 (c) demonstrates the variance of the estimated DOA's and Fig. 2(d) displays the mean of variable forgetting factor in 50 independent trials.

In Fig.2 (b), the PASTd with small forgetting factor converges faster than that with large one. In Fig.2(c), however, the PASTd with large forgetting factor has the lower estimation variance than that with small one. The proposed algorithm shows that the convergence is compatible to the PASTd with smaller forgetting factor and that the variance is almost the same as that of the PASTd with large forgetting factor.

VI. Conclusion

In this paper, a subspace tracking algorithm was proposed. The presented algorithm was able to track time varying subspaces without *a priori* knowledge of the forgetting factor. To track subspace effectively, we introduced a variable forgetting factor to the PASTd algorithm. To illustrate the tracking capability, we compared the proposed algorithm and the PASTd in scenarios with abruptly time-varying DOA targets. In the comparison, the proposed algorithm showed the improved subspace tracking capability and revealed the compatible or better estimation performance without *a priori* knowledge of forgetting factor.

The improved subspace tracking capability can be applied to many different kinds of areas with time-varying subspace such as target tracking and underwater communication.

Acknowledgements

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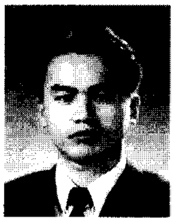

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