研究論文

수리 가능한 시스템에서의 최적 예방 보전 정책

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Optimal Preventive Maintenance Policy for a Repairable System

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Abstract

In this paper, a preventive maintenance(PM) policy for a repairable system is considered. The failure rate model proposed by Park et al.(2000) is generalized by assuming that after each PM not only the PM slows down the degradation process of the system but also reduces down the system failure rate by a certain fixed amount. Long-run expected cost rate of the PM policy is derived and the properties of joint solution of the optimal PM period and optimal number of PM which minimizes the expected cost rate are obtained. Numerical examples for the case of a Weibull-type failure rate are given.

1. Introduction

At an early stage of developing the replacement model, it was generally assumed that a failure system after repair will yield a functioning system which is "as good as new". However when a complex system ceases to function by the failure of one of its constituent, replacing only the failed component can return the system to the functioning state. In this situation, as the great majority of components have not been replaced, the remaining life distribution and the failure rate of the system are not changed. This leads to the well-known minimal repair model. Thus by a minimal repair the system is returned to the "as bad as old" condition. For a precise definition of minimal repair, see Nakagawa(1983). Since Barlow and Hunter(1960) have investigated systems subject to minimal repair and replacement, extensive research has been conducted. The surveys of Pierskalla and

Voelker(1976) and Valdez-Flores and Feldman(1989) are excellent references for this research area.

In many cases, since system deteriorates as the operating time of the system increases, preventive maintenance actions, such as oil change, cleaning, greasing and replacing some worn components, are performed. Obviously such maintenance action improves the condition of the system but does not return the system to "as good as new". Many authors have addressed the effect of PM and studies on various maintenance types have been considered(See, for example, Canfield(1986), Chan and Downs(1978), Lie and Chun(1986), Malik(1979), Nakagawa(1979, 1980, 1987, 1988), Jayabalan and Chaudhuri(1992a)).

More recently, researches on the system which is subject to three kinds of maintenance action - minimal repair, preventive maintenance(or overhaul) and replacement - have been conducted. Researches of Nakagawa(1986), Jayabalan and Chaudhuri(1992b), Liu et al.(1995), Park et al.(2000), Zhang and Jardin(1998) and Usher et al.(1998) are recent works dealing with such a model.

The work proposed in this paper is most closely related to Canfield(1986), Park et al.(2000) and Zhang and Jardin(1998). Canflied(1986) discusses a periodic PM model for which the PM slows down the degradation process of the system, while the failure rate keeps the shape of monotone increase. In his(her) model, it was only considered that complete replacement was done on each system failure. Park et al.(2000) considered the same failure rate model as Canflied(1986)'s but they adopted the minimal repair action on each failure occurring during a renewal cycle. In the model the system is preventively maintained at periodic times kx and is replaced by a new system at the Nth PM, where $k=1,2,\cdots,N$. According to the system failure rate assumed in Park et al.(2000) the system improvement by a PM is limited by the state of the system just prior to the PM, i.e. the reliability of the system right after a PM can not be better than that just prior to the PM. Zhang and Jardin(1998) consider a system improvement model by assuming that each PM makes the system failure rate between "as bad as old" and "as good as previous PM period". In this paper we generalize the failure rate model proposed by Park et al.(2000) by assuming that after each PM not only the PM slows down the degradation process of the system but also reduces down the system failure rate by a certain fixed amount. Also, in many cases, for most deteriorating systems the PM cost becomes higher and higher as the system operating time increases because of its cumulative wear and ageing. Hence it is assumed that the cost of PM is proportional to the failure rate just prior to the PM with a proportionality constant γ_{pm}.

In Section 2, the notations and the failure rate model, which will be treated in this paper, are described. The property of joint solution of the optimal PM period and the optimal number of PM is obtained in Section 3. Numerical examples for the case of a Weibull-type failure rate are also given in Section 4.

2. Preventive Maintenance Model

In this section a preventive maintenance model is proposed. The notations which will be used in this paper are given as follows.

Notations

x time interval between two consecutive PMs

α the factor which represents the degree of system improvement

γ_{pm} the proportionality constant for PM cost

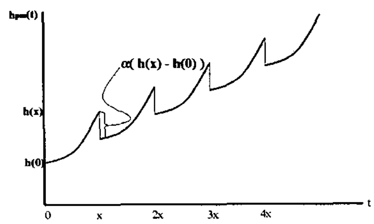
C_{mr} the cost of a minimal repair

C_{re} the cost of a replacement

Now we describe the model under consideration. The system is preventively maintained at periodic times kx and is replaced by a new system at the Nth PM, where $k=1,2,\cdots,N$. For intervening failures only minimal repair is done. Let $h(t)$ be the failure rate function of the system without PM. Then the failure rate of the system under PM, $h_{pm}(t)$, is defined by

$$h_{pm}(t)=\begin{cases} h(t), & \text{for } 0\le t\le x, \\ k\{h(x)-h(0)-\alpha(h(x)-h(0))\}+h(t-kx), & \text{for } kx< t\le (k+1)x, \end{cases} \qquad (1)$$

$k=1,2,\cdots,$ where $0\le\alpha\le 1$, which represents the degree of system improvement. Note that the case when $\alpha=0$ reduces to the case of Park et al.(2000) and when $\alpha=1$, each PM



<Figure 1 > Failure Rate of System under PM

corresponds to a replacement.

3. Optimal Maintenance Policy

Before proceeding to derive the long-run expected cost rate, the assumptions are described as follows.

Assumptions

1. A PM not only slows down the degradation process of the system but also reduces down the system failure rate by a certain fixed amount.
2. A minimal repair does not change the system failure rate.
3. All renewal cycles have the same number of PM dividing each cycle into equal length period.
4. The times for a minimal repair, a PM, or a replacement are negligible.
5. The cost of PM is proportional to system failure rate which is just prior to the PM with a proportionality constant γ_{pm}.

If we define $C_{pm(k)}$ as the cost incurred at kth PM, $C_{pm(k)}$ is given by

$$\begin{aligned} C_{pm(k)} &= \{h(x)+(k-1) \\ &\quad \times(1-\alpha)[h(x)-h(0)]\}\gamma_{pm} \\ &= [kh(x)-(k-1)(h(0) \\ &\quad +\alpha\{h(x)-h(0)\})]\gamma_{pm}. \end{aligned} \tag{2}$$

Note that the cost incurred in a renewal cycle is composed of three parts. One part is incurred by the minimal repairs in a renewal cycle. Another part is related to the PM(s) done in a renewal cycle. The other is the part which is caused by a replacement. Observe that the expected number of minimal repairs in a renewal cycle is

$$\frac{N(N-1)}{2}x(h(x)-h(0)-\alpha\{h(x) - h(0)\})+N\int_0^x h(t)dt \tag{3}$$

and the cost incurred by PM is given by

$$\begin{aligned} \sum_{k=1}^{N-1} C_{pm(k)} = [&\frac{N(N-1)}{2}h(x) \\ &-\frac{(N-1)(N-2)}{2}(h(0) \\ &+\alpha\{h(x)-h(0)\})]\gamma_{pm}, \end{aligned} \tag{4}$$

when $N\geq 2$ and is 0 when $N=1$. Hence by the renewal theory the long-run expected cost rate is given by

$$\begin{aligned} C(x,N) = \frac{1}{Nx}[\{&\frac{N(N-1)}{2}x(h(x) \\ &-h(0)-\alpha\{h(x)-h(0)\}) \\ &+N\int_0^x h(t)dt\}C_{mr}+\{\frac{N(N-1)}{2}h(x) \\ &-\frac{(N-1)(N-2)}{2}(h(0)+\alpha[h(x) \\ &-h(0)])\}\gamma_{pm}+C_{re}], \end{aligned} \tag{5}$$

for $x>0$ and $N\geq 1$. Let x^* and N^* be the values which satisfy

$$C(x^* \cdot N^*) = \min_{x, N} C(x, N). \qquad (6)$$

Then the property of joint solution of $(x^* \cdot N^*)$ is given in the following theorem.

Theorem 3.1. Suppose that $h(t)$ is increasing failure rate(IFR) and $h''(t) > 0$ for all $t > 0$. Then for any fixed N there exists the x_N^* which satisfies

$$C(x_N^*, N) < C(x, N), \quad \forall\, x > 0, \qquad (7)$$

and the x_N^* is the solution of the following equation;

$$N\{xh(x) - \int_0^x h(t)dt + \frac{(N-1)}{2}(1-\alpha)x^2 h'(x)\}C_{mr} + \frac{(N-1)}{2}(N-\alpha(N-2))(xh'(x) - h(x)) \cdot \gamma_{pm} + \frac{(N-1)(N-2)}{2}(1-\alpha)h(0) \cdot \gamma_{pm} - C_{re} = 0. \qquad (8)$$

proof.

For a fixed N, differentiate $C(x, N)$ with respect to x then

$$\frac{\partial C(x,N)}{\partial x} = \frac{1}{x^2}\{xh(x) - \int_0^x h(t)dt + \frac{(N-1)}{2}(1-\alpha)x^2 h'(x)\}C_{mr} + \frac{(N-1)}{2Nx^2} \cdot (N-\alpha(N-2))(xh'(x) - h(x)) \cdot \gamma_{pm} + \frac{1}{Nx^2}\left[\frac{(N-1)(N-2)}{2}(1-\alpha)h(0) \cdot \gamma_{pm} - C_{re}\right]. \qquad (9)$$

Setting (9) equal to 0, then we obtain

$$N\{xh(x) - \int_0^x h(t)dt + \frac{(N-1)}{2}(1-\alpha)x^2 h'(x)\}C_{mr} + \frac{(N-1)}{2}(N-\alpha(N-2))(xh'(x) - h(x))\gamma_{pm} + \frac{(N-1)(N-2)}{2}(1-\alpha)h(0)\gamma_{pm} - C_{re} = 0. \qquad (10)$$

Let the left side of (10) be $g(x)$ then since $g(x)$ is strictly increasing and $\lim_{x\to 0} g(x) = -(N-1)h(0)\gamma_{pm} - C_{re} < 0$, $\lim_{x\to\infty} g(x) = \infty$, there exists the unique x_N^* which satisfies (10). This completes the proof.

Now, in order to obtain the optimal solution (x^*, N^*), it is sufficient to find the N^o which satisfies

$$C(x_{N^o}^*, N^o) = \min_N C(x_N^*, N), \quad N = 1, 2, \cdots, \qquad (11)$$

where x_N^* is the solution of (8). Then the obtained $(x_{N^o}^*, N^o)$ is the optimal solution (x^*, N^*) which minimizes $C(x, N)$ in (5).

4. Numerical Examples

In this section we present some numerical examples of the PM model under consideration. In the example, we assume that the failure rate of the system is given by

$h(t) = t^2+5, \ t\geq 0.$

Let X_p be the pth percentile of the system lifetime. Then $X_{0.50}=0.13844$ $X_{0.90}=0.45417$, $X_{0.95}=0.58575$, $X_{0.97}=0.68032$, $X_{0.99}=0.87617$. To obtain the optimal solution, it is impossible to calculate $C(x_N^*, N)$ for all positive integer N and compare them. Thus, the following stopping procedure is adopted;

1. For each N, we can obtain x_N^* which is the unique solution of the equation (8) and can calculate $H(N)\equiv C(x_N^*, N)$.

2. If there exists N^o which satisfies

$$H(1)\geq H(2)\geq\cdots\geq H(N^o)\leq H(N^o+1)\leq \cdots\leq H(N^o+m)$$

then stop the process and choose the $(x_{N^o}^*, N^o)$ as the optimal solution.

In this example, we took $m=5$. Table 4.1 and 4.2 give the optimal solution (x^*, N^*), the corresponding renewal cycle $(N^*\times x^*)$, and $C(x^*, N^*)$ for different choices of C_{re} and α when $C_{mr}=1$, $\gamma_{pm}=0.2$.

Table 4.1. Optimal solutions for $C_{mr}=1$, $\gamma_{pm}=0.2$, and $\alpha=0.4$.

C_{re}	N^*	x^*	$N^*\times x^*$	$C(x^*, N^*)$
5	3	1.0470	3.1410	8.4331
7	5	0.8633	4.3165	8.9535
10	7	0.7800	5.4600	9.5626
15	11	0.6664	7.3304	10.3390
20	15	0.5980	8.9700	10.9585
30	21	0.5410	11.3610	11.9450
50	34	0.4609	15.6706	13.4187

Table 4.2. Optimal solutions for $C_{mr}=1$, $\gamma_{pm}=0.2$, and $\alpha=0.6$.

C_{re}	N^*	x^*	$N^*\times x^*$	$C(x^*, N^*)$
5	4	0.9868	3.9472	8.1428
7	6	0.8725	5.2350	8.5690
10	9	0.7625	6.8625	9.0721
15	12	0.7189	8.6268	9.7185
20	16	0.6558	10.4928	10.2364
30	23	0.5872	13.5056	11.0664
50	37	0.5040	18.6480	12.3099

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