◆ Research Paper

Warranties for Products with Varying Usage Intensity

Kim, Jae Soong*
Kim, Ji Sung**

Abstract

Most warranty studies assume that the usage intensity is the same for all buyers. However, in real life the usage intensity varies across the population of buyers. In the general case, one can divide the population into *k* categories. This has implications for manufacturers of products. Should a manufacturer produce one product and offer different warranties for the *k* groups or produce different products (one for each group) and offer the same warranty. A warranty cost analysis is needed to choose between these options. The analysis complicated by factors such as adverse selection , buyers attitude to risk and the price structure. In this paper we develop models to study the expected warranty cost for products with free replacement warranty with varying usage intensity. Numerical examples are presented.

Keywords: Warranty, Varying Usage Rate, Reliability and Stochastic Model.

1. Introduction

A warranty is a contractual agreement offered by the manufacturer at the point of sale of a product. It requires the manufacturer to rectify all failures occurring within the warranty period or compensate the buyer. In this context, warranty serves as a device to protect the buyer from defective products. Offering warranty results in additional costs to manufacturers from the servicing of the warranty. However, warranties also serve as signals to inform customers about the quality of product. In this context, better warranty terms imply a higher product quality and manufacturers have tended to use warranty as a promotional tool to increase sales and revenue. Hence, from a manufacturers perspective, offering a warranty is worthwhile only if the benefits (in terms of greater sales and/or revenue) exceed the additional costs associated with the servicing of warranty. Many different types of warranty policies for both new and second-hand products have been proposed and analysed. A taxonomy for warranty policies for new products was proposed by Blischke and Murthy (1992). For new products, the cost analysis of various one- and two-dimensional warranty policies can be found in Blischke and Murthy (1994).

The cost analysis of one-dimensional policies is based on the assumption that buyers are homogenous with respect to the usage intensity (or rate). This implies the usage rate is the same for all buyers. In contrast, two-dimensional warranty policies are characterised by a two-dimensional region with one axis representing time (or age) and the other usage. The product usage intensity across the buyer population can vary and is modelled as a random variable in the cost analysis. Two different approaches have been proposed and the details can be found in Blischke and Murthy (1994 and 1996), Kim and Murthy (1999).

* Dept. of Business Administration, Sangji University
** Dept. of Industrial & Systems Engineering, Kyungil University

In this paper we focus our attention on warranty cost analysis for products sold with one-dimensional free replacement warranty and the usage intensity varying across the buyer population. A typical example that reflects this is the following. The usage intensity (in terms of load and frequency of usage per week) of a domestic washing machine varies depending on the size of the family. This is also true for many other domestic and industrial products. The product degradation and failure depends on the usage intensity and this in turn has an impact on the expected warranty cost. This needs to be taken into account in determining the sale price and reliability decisions at the design stage.

The outline of the paper is as follows. In Section 2 we give the details of the model formulation and we consider two different model formulations for modelling the usage intensity. In Sections 3 we carry out the analysis to determine the expected warranty cost with the usage intensity modelled as a continuous random variable. We also deals with special cases that are analytically tractable and are illustrated with numerical examples. Section 4 deals with the case where the usage intensity is modelled as a discrete random variable. Finally, we conclude with a brief discussion of topics for future research in Section 5.

Notations:

$EWC(W)$: Expected warranty cost.

$F(t;\theta)$: Failure distribution function with scale parameter θ.

$\overline{F}(t;\theta)$: Survival function associated with $F(t;\theta)$.

$f(t;\theta)$: Failure density function associated with $F(t;\theta)$.

$r(t;\theta)$: Failure rate function.

$\theta(u)$: Scale parameter of the Weibull distribution as function of usage rate (u).

β: Shape parameter of $F(t;\theta)$.

α: Parameter of Gamma distribution.

δ, ϕ: Load parameters.

U: Usage rate (random variable).

u_l: Usage rate of the light user.

u_h: Usage rate of the heavy user.

$G(u)$: Usage distribution function.

$g(u)$: Usage intensity function.

$E[N(t)]$:Expected value of $N(t)$.

C_r: Cost of each repair.

C_m: Cost of each replacement(including costs of parts, labor and transport).

2. Model Formulation

2.1 Item Failures

Let $F(t;\theta)$ be the failure distribution function for the product. θ is the scale parameter and is a function of the usage intensity as will be discussed later in the section. The product is sold with a free replacement warranty period that requires the manufacturer to rectify all failures over the warranty period at no cost to the buyer.

The first failure is a random variable with distribution function $F(t;\theta)$. Subsequent failures depend on the type of rectification action used. We consider two cases; Firstly, for non-repairable products, a failed item under warranty needs to be replaced with a new one. If the claims are exercised immediately and the time to replace is small relative to the mean time between failures, then it can be ignored. As a result, failures over the warranty period occur according to a renewal process associated with $F(t;\theta)$(see, Ross (1972)). Secondly, for repairable product, we assume that the failures are minimally repaired (see, Barlow and Hunter (1960)) and the time to repair is negligible so that it can be ignored. This implies that failures over the warranty period occur according to a non-homogeneous Poisson process with intensity function given by the failure rate function $r(t;\theta)$ associated with $F(t;\theta)$. This failure rate function is given by

$$r(t;\theta) = \frac{f(t;\theta)}{1-F(t;\theta)} \tag{1}$$

2.2 Usage Intensity

Let U denote the usage rate. This is a random variable and characterises the different usages across the buying population. We can model this in two different approaches; the usage either as a continuous variable or as a discrete variable.

Approach 1 [Continuous Variable]: The usage rate is modelled a continuous random variable distributed over an interval $u_{\min}$ and $u_{\max}$ according to a distribution function $G(u)$ with density function $g(u)$. These two limits denote the minimum and maximum usage rates. The form of distributions that will consider later in the paper is given by a Gamma distribution with parameter α, thus $g(u) = \frac{1}{\Gamma(\alpha)} u^{\alpha-1} e^{-u}$, $u \geq 0$. Note that in the case $u_{\min} = 0$ and $u_{\max} = \infty$.

Conditional on the usage rate $U = u$, θ is given by

$$\theta(u) = \delta(u)\,\theta_d \tag{2}$$

where θ_d is design parameter and $\delta(u)$ defines the effect of the usage rate(or load on the item) and is modelled as

$$\delta(u) = \sum_{j=1}^{i}\left(\frac{u}{u_{j-1}}\right)^{j-1} \qquad u_{i-1} \leq u \leq u_i, \quad i = 1,\ 2,\ \cdots\ k \tag{3}$$

with u_i as the additional design parameters. The design parameters depend on the design decisions and are under the control of the manufacturer. Higher values for θ_d and u_0 are resulted from better design.

Approach 2 [Discrete Variable]: The usage population can be clustered into several groups. The general case is to cluster them into k groups. Let p_i denote the probability that the buyer is i group's user. Then conditional on the usage rate $U = u$ θ is given by

$$\theta(u) = \phi^{i-1}\theta_d, \qquad i = 1, 2, \cdots k, \ \phi \geq 1 \tag{4}$$

2.3 Expected Warranty Cost Per Unit

Let $N(W)$ denote the number of failures over the warranty period for an item sold with warranty period W. We first consider the case where the item is non-repairable so that a failed item is replaced with a new one. Let C_m denote the cost of each replacement. Then, the expected warranty cost per unit is given by

$$EWC(W) = C_m E[N(W)] \tag{5}$$

For repairable item, the failed item is repaired minimally. Let C_r denote the cost of each minimal repair. Then, the expected warranty cost per unit is given by

$$EWC(W) = C_r E[N(W)] \tag{6}$$

3. Continuous Model Analysis

In this section we obtain expressions for the expected warranty cost per unit with usage rate being modelled as a continuous random variable.

3.1 Non-repairable Product

For non-repairable product, each failure over warranty is replaced with a new one. Since replacements are instantaneous, failures over the warranty period occur according to a renewal process. We use the conditional approach to obtain an expression for the expected warranty cost per unit.

Let $N(W;\theta(u))$ denotes the number of failures over warranty conditional on $U=u$. The expected value of $N(W;\theta(u))$ is given by

$$E[N(W;\theta(u))] = M(W;\theta(u)) \tag{7}$$

where $M(W;\theta(u))$ is the conditional renewal function associated with the distribution function $F(W;\theta(u))$ and is given by

$$M(W;\theta(u)) = F(W;\theta(u)) + \int_0^W M(W;\theta(u))\, dF(W;\theta(u)) \tag{8}$$

using the formula for conditional expectation, the expected number of failures over warranty, $E[N(W)]$, is given by

$$E[N(W)] = \int_{u_{min}}^{u_{max}} M(W;\theta(u))dG(u) \tag{9}$$

The expected warranty cost per unit sale is obtained from (5) using (9).

3.2 Repairable Product

Item failure over the warranty period, conditional on $U=u$, occur according to a non-homogeneous Poisson process with intensity function given by $r(t;\theta(u))$. As a result, the conditional expected value of the number of failures is given by

$$E[N(W;\theta(u))] = \int_0^W r(t;\theta(u))dt \tag{10}$$

Using the formula for conditional expectation argument, the expected number of failures over the warranty period is given by

$$E[N(W)] = \int_{u_{min}}^{u_{max}} \int_0^W r(t;\theta(u))dtdG(u) \tag{11}$$

The expected warranty cost per unit is obtained from (6) using (11).

In general, it is not possible to derive analytical expressions for the expected warranty costs. In this case, one needs to use some computational schemes to obtain cost estimates. In the next section, we discuss some special cases for which it is possible to derive analytical expressions.

3.3 Numerical Examples

The usage population can be clustered into several groups. The general case is to cluster them into k groups. In this example, the simplest case is to cluster them into 3 groups, i. e., the light users, medium users and heavy users.

Example 1: Non-Repairable Product with Gamma Usage Rate Distributions

Theusage rate is given by a Gamma distribution with parameter α, therefore $g(x) = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x}$, $x \geq 0$. Also if $F(t;\theta)$ is an exponential with parameter $\theta(u)$, then from (9) we have

$$E[N(W)] = W\theta_d \sum_{i=1}^{k} \prod_{j=1}^{i} \left(\frac{1}{u_{j-1}}\right)^{j-1} \frac{\Gamma(\alpha + i(i-1)/2)}{\Gamma(\alpha)} \cdot \left[G\left(u_i;\alpha+\frac{i(i-1)}{2}\right) - G\left(u_{i-1};\alpha+\frac{i(i-1)}{2}\right)\right] \tag{12}$$

Let $\theta_d = 0.1$(mean time to failure is 10 years) and $\beta = 1$. W is in years and the unit for usage rate is 10^4km/*year*. Table 1 shows $E[N(W)]/W$ for different combinations of u_1, u_2 and α.

[Table 1] $E[N(W)]/W$ for different combinations of u_1, u_2 and α.

		$u_1=0.5$	$u_1=1.0$	$u_1=1.5$	$u_1=2.0$	$u_1=2.5$
$u_2=1.0$	$\alpha=1$	0.9333	-	-	-	-
	$\alpha=2$	2.7035	-	-	-	-
$u_2=1.5$	$\alpha=1$	0.4667	0.2595	-	-	-
	$\alpha=2$	1.2337	0.6256	-	-	-
$u_2=2.0$	$\alpha=1$	0.3177	0.1850	0.1470	-	-
	$\alpha=2$	0.7408	0.3791	0.2646	-	-
$u_2=2.5$	$\alpha=1$	0.2584	0.1553	0.1272	0.1160	-
	$\alpha=2$	0.5319	0.2747	0.1949	0.1592	-
$u_2=3.0$	$\alpha=1$	0.2324	0.1424	0.1186	0.1095	0.1055
	$\alpha=2$	0.4339	0.2257	0.1623	0.1347	0.1209

Note that as α increases $\theta(u)$ also increases. As a result, the expected number of failures for the same warranty period increases. $\theta(u)$ decreases u also increases and as a result the expected number of failures decreases.

Example 2: Repairable Product with Gamma Usage Rate Distributions

$F(t;\theta)$ is a Weibull distribution with parameter $\theta(u)$ and β, thus $F(t;\theta)=1-e^{-(\theta(u)t)^{\beta}}$. Then from (11) we have the expected number of failures over warranty is given by

$$E[N(W)]=(W\theta_d)^{\beta}\sum_{i=1}^{k}\prod_{j=1}^{i}\left(\frac{1}{u_{j-1}}\right)^{(j-1)\beta}\frac{\Gamma(\alpha+i(i-1)\beta/2)}{\Gamma(\alpha)}\cdot$$

$$\left[G\left(u_i;\alpha+\frac{i(i-1)\beta}{2}\right)-G\left(u_{i-1};\alpha+\frac{i(i-1)\beta}{2}\right)\right] \quad (13)$$

Let $\beta=2$ and the remaining parameters are as in Example 1. Table 2 shows $E[N(W)]/W^{\beta}$ for different combinations of u_1, u_2 and α.

[Table 2] $E[N(W)]/W^{\beta}$ for different combinations of u_1, u_2 and α.

		$u_1=0.5$	$u_1=1.0$	$u_1=1.5$	$u_1=2.0$	$u_1=2.5$
$u_2=1.0$	$\alpha=1$	6.8559	-	-	-	-
	$\alpha=2$	26.8907	-	-	-	-
$u_2=1.5$	$\alpha=1$	1.3657	0.3454	-	-	-
	$\alpha=2$	5.3206	1.3315	-	-	-
$u_2=2.0$	$\alpha=1$	0.4486	0.1162	0.0556	-	-
	$\alpha=2$	1.7014	0.4267	0.1917	-	-
$u_2=2.5$	$\alpha=1$	0.2042	0.0551	0.0284	0.0196	-
	$\alpha=2$	0.7251	0.1827	0.0832	0.0491	-
$u_2=3.0$	$\alpha=1$	0.1216	0.0344	0.0193	0.0145	0.0125
	$\alpha=2$	0.3875	0.0982	0.0457	0.0280	0.0203

As a result, note that as β increases the expected number of failures for the same warranty period increases.

4. Discrete Model Analysis

In this section we obtain expressions for the expected warranty cost per unit with usage rate being modeled as a discrete random variable.

4.1 Non-repairable Product

It is easily shown that the expected number of failures over warranty is given by,

$$E[N(W)]=\sum_{i=1}^{k}p_iM(W;\theta_i) \quad (14)$$

where $M(W;\theta_i)$ is the reneal function associated with $F(t;\theta_i)$. When the failure distribution is exponential, $E[N(W)]$ is given by

$$E[N(W)] = W\theta_d \sum_{i=1}^{k} p_i \phi^{i-1} \quad (15)$$

4.2 Repairable Product

It is easily seen that the expected number of failures over warranty is given by

$$E[N(W)] = \sum_{i=1}^{k} p_i \int_0^W r(t;\theta_i)dt \quad (16)$$

When $F(t;\theta_i)$ is a Weibull distribution, the expected number of failures over the warranty period is given by

$$E[N(W)] = (W\theta_d)^{\beta} \sum_{i=1}^{k} p_i \phi^{\beta(i-1)} \quad (17)$$

4.3 Numerical Examples

Example 1: Non-repariable Product

Let $\theta_d = 0.1$(mean time to failure is 10 years), $u_l = 1$, $u_m = 3$ and $u_h = 4$(the mean usage per year the light user is $1(\times 10^4)$, for the medium user is $3(\times 10^4)$ and for the heavy user is $4(\times 10^4)$. Table 3 shows $E[N(W)]/W$ for different combinations of ϕ and p_i.

[Table 3] $E[N(W)]/W$ for different combinations of ϕ and p_i.

		$\phi = 1.5$	$\phi = 2.0$	$\phi = 2.5$	$\phi = 3.0$
$p_1 = 0.3$	$p_2 = 0.3$	0.1650	0.2500	0.3550	0.4800
	$p_2 = 0.4$	0.1575	0.2300	0.3800	0.4200
	$p_2 = 0.5$	0.1500	0.2100	0.2800	0.3600
$p_1 = 0.4$	$p_2 = 0.3$	0.1525	0.2200	0.3025	0.4000
	$p_2 = 0.4$	0.1450	0.2000	0.2650	0.3400
	$p_2 = 0.5$	0.1375	0.1800	0.2275	0.2800
$p_1 = 0.5$	$p_2 = 0.3$	0.1400	0.1900	0.2500	0.3200
	$p_2 = 0.4$	0.1325	0.1700	0.2125	0.2600
	$p_2 = 0.5$	0.1250	0.1500	0.1750	0.2000

The result show that as ϕ increases. this increases the number of failure over warranty period. As p_i increases, the expected number of failure decreases as to be expected since the buyer is more likely to be a light user.

Example 2: Repariable Product

Let $\beta = 2$ and remaining parameters are as in Example 5. Table 4 shows $E[N(W)]/W^{\beta}$ for different combinations of ϕ and p_i.

[Table 4] $E[N(W)]/W^{\beta}$ for different combinations of ϕ and p_i.

		$\phi = 1.5$	$\phi = 2.0$	$\phi = 2.5$	$\phi = 3.0$
$p_1 = 0.3$	$p_2 = 0.3$	0.0300	0.0790	0.1778	0.3540
	$p_2 = 0.4$	0.0272	0.0670	0.1452	0.2820
	$p_2 = 0.5$	0.0244	0.0550	0.1124	0.2100
$p_1 = 0.4$	$p_2 = 0.3$	0.0259	0.0640	0.1399	0.2740
	$p_2 = 0.4$	0.0231	0.0520	0.1071	0.2020
	$p_2 = 0.5$	0.0203	0.0400	0.0743	0.1300
$p_1 = 0.5$	$p_2 = 0.3$	0.0219	0.0490	0.1019	0.1940
	$p_2 = 0.4$	0.0191	0.0370	0.0691	0.1220
	$p_2 = 0.5$	0.0163	0.0250	0.0363	0.0500

The result show that as ϕ increases, this increases the number of failure over warranty period. As p_i increases, the expected number of failure decreases as to be expected since the buyer is more likely to be a light user.

5. Conclusions

Warranty cost model for one-dimensional warranties assumes that the usage intensity is the same for all buyers. But in real life the usage intensity varies across the population of buyers. In the paper we deals with models to study the expected warranty cost for products with free replacement warranty with varying usage intensity.

We have confined our discussion to the free replacement warranty policy. New incentive-based policies can help reduce some of the moral hazard issues and also the adverse selection problem. One such policy is a combination policy where the buyer incurs no cost for rectifying failure in the early stages of the warranty and an increasing share of the cost later in the warranty period. Also, the manufacturer might offer extra period of warranty coverage at the expiry of the initial warranty if the buyer makes the right selection. This would discourage high intensity users from claiming to be light intensity users and the extra coverage for light intensity users. This would compensate for the unfairness. Some of these problems are currently under investigation by the authors.

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