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Two Sequential Wilcoxon Tests for Scale Alternatives

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Abstract

Two truncated sequential tests are developed for the two-sample scale problem based on the usual Wilcoxon rank-sum statistic for two different dispersion indices - absolute median deviations, when the medians of the two populations X and Y are equal or known and sums of squared mean deviations, when the medians are either unknown or unequal. The first test is briefly called SWAMD test and the second SWSMD test. For the SWAMD test, the percentile points for both the one-sided and two-sided alternatives, z_N^α and $|z|_N^\alpha$, have been found by Wiener approximation and their values computed for a range of values of α and N; analytical expression for the power function has been derived through Wiener process and its performance studied for various sequential designs for exponential distribution. This test has been illustrated by a numerical example. All the results of the SWAMD test, being directly applicable to the SWSMD test, are not dealt with separately. Both the tests are compared and their suitable applications indicated.

Keywords: sequential, non-parametric, Wilcoxon rank-sum statistic, scale alternative, Lehmann alternative, percentile point, truncation point, power function.

1. INTRODUCTION

In the past four decades considerable work has been done on the sequential rank tests for location/symmetry. Among the notable are by Wilcoxon, Rhodes and Bradley (1963); Wilcoxon and Bradley (1964); Bradley, Martin and Wilcoxon (1965); Bradley, Merchant and Wilcoxon (1966); Savage and Sethuraman (1966); Sethuraman (1970); Miller (1970,1972); Weed and Bradley (1971,1973); Weed, Bradley and Govindarajulu (1974); Sen and Ghosh (1974a); Lai (1975); Phatarfod and Sudbury (1988); Sen and Mishra (1994); Mishra (1999) and Mishra and

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Sahoo (1999). Most of these tests are based on Lehmann alternatives devised specially for determining the power of the rank tests by Lehmann (1953). As established by several authors, use of such alternatives for measuring location change is not misleading for all practical purposes. In fact, this has facilitated development of sequential rank tests for location problems. By contrast, very little is known about sequential tests for scale alternatives. The purpose of the present article is to investigate this problem and develop some sequential tests based on the well known Wilcoxon rank-sum statistic to test the variability between two independent populations.

In the behavioural and social sciences there is often interest in testing for differences in dispersion. For example, determining that a particular group is more homogenous than another could be of value in developing special instructional materials for that group. In fixed-sample non-parametric set up, the test developed by Siegel and Tukey (1960) is appropriate for comparing differences in scale or variability under the assumption that the medians of the two populations are the same or known. To have a sequential test in this situation, we note that each absolute value of the deviations of observations from the common median is in itself a measure of variability. Therefore, the Wilcoxon rank-sum statistic for the absolute values of such deviations can be used as a consistent test statistic against the scale alternative. Here observations are taken sequentially in pairs from X and Y populations and at each stage n, $n = 1, 2, \cdots, N$ (truncation point), the Wilcoxon statistic W_n is computed for the $2n$ absolute deviations of the observations from the common median M. The logic of the Wilcoxon test for location is then applied in developing the sequential test based on W_n for the scale alternatives in analogy with Miller's (1972) linear barrier test in one-sample case. The test to be called Sequential Wilcoxon Test for Absolute Median Deviations (SWAMD test) has been described in Section 2.

There are many situations in which the medians are either unknown or cannot be assumed equal. The Moses (1963) rank-like test is useful in such cases in the fixed-sample size set up. To compute the Moses statistic it is necessary to divide the observations from the two populations into subsets of equal size, discarding the left over observations from the analysis. Thus the data are not fully utilised. Apart from other advantages, this problem can be avoided if we can develop a sequential rank-like test based on the Wilcoxon rank-sum statistic for some dispersion indices using the same logic of the Wilcoxon test for location for testing the hypothesis of equal variability of the two populations against the one-sided or two-sided alternatives. In Section 3, we have developed such a test, referred to as Sequential Wilcoxon Test for Sums of squared Mean Deviations (SWSMD test). Here random samples of equal size m are drawn sequentially from the two populations and at each stage n, $n = 1, 2, \cdots, N$, the Wilcoxon statistic W_n^* is computed on the basis of the $2n$ sums of squares of the deviations of the observations from the respective sample means. The sequential methods and other results of the SWAMD test are directly applicable to the SWSMD test once W_n is replaced by W_n^* and therefore not dealt with separately.

For the SWAMD test, with pre-assigned probability of type-I error α and truncation point N, analytical expressions for the critical constants for both the one-sided and two-sided tests, z_N^α and $|z|_N^\alpha$, have been found by Wiener approximation and their values computed for a range of values of α and N in Section 4. In Section 5, analytical expression for the power function of the test has been derived through Wiener process and its performance studied for different sequential designs in terms of the power for exponential distributions. This test has been illustrated in Section 6 through a numerical example. In the concluding Section 7, both the tests have been contrasted and their suitable applications briefly discussed.

2. THE SWAMD TEST

First consider the case when the two populations have the same median M or, if the medians are known, they can be subtracted from each sample observation to render the 'adjusted' medians equal (zero). In this case each absolute value of the deviations of observations from the common median is in itself a measure of variability and therefore we consider using this criterion in developing the desired test. Suppose $(X_1, Y_1), (X_2, Y_2), \cdots$ are observations drawn sequentially in pairs from two independent populations X and Y having continuous cumulative distribution functions (cdf's) F and G respectively. Suppose the $X-$ and $Y-$ populations differ only in scale. The logical model for this situation would be

$$H_s : G(x) = F(\theta x) \quad \text{for all } x \text{ and some } \theta > 0, \theta \neq 1 \tag{1}$$

where $\theta = \sigma_x/\sigma_y; \sigma_x$ and σ_y are standard deviations of $X-$ and $Y-$populations respectively. This is appropriately called *scale alternative* because the cdf of the $Y-$ population is the same as that of the $X-$ population but with a compressed or enlarged scale according as $\theta > 1$ or $\theta < 1$ respectively.

The objective of this investigation is to develop a sequential test for testing the null hypothesis $H_0 : \theta = 1$ against the one-sided scale alternative $H_1 : \theta > 1$ (of course, $\theta < 1$ could be an alternative) or against the two-sided scale alternative $H_{1'} : \theta \neq 1$ based on the Wilcoxon rank-sum statistic defined for the absolute deviations of the observations from M.

Let $X_{i'} = |X_i - M|, Y_{i'} = |Y_i - M|, \quad i = 1, 2, \cdots, n$

$Z_i = 1$, if i-th observation in the combined ordered sample of $2n$ absolute deviations $X_{i'}$ and $Y_{i'}$ is an $X_{i'}$.

$= 0$, otherwise $\quad i = 1, 2, \cdots, n$

$D_{ij} = 1$, if $Y_{j'} < X_{i'}$

$= 0$, if $Y_{j'} > X_{i'} \quad i, j = 1, 2, \cdots, n.$

Then the Wilcoxon rank-sum statistic W_n and the Wilcoxon-Mann-Whitney statistic U_n for the transformed variables $X_{i'}$ and $Y_{i'}$ at the n-th stage are given by

$$W_n = \sum_{i=1}^{2n} Z_i, U_n = \sum_{i=1}^{n} \sum_{j=1}^{n} D_{ij} \tag{2}$$

and $U_n = W_n - n(n+1)/2, n = 1, 2, ..., N$.

Under H_0,

$$E(W_n) = n(2n+1)/2 \text{ and } Var(W_n) = n^2(2n+1)/12 \tag{3}$$

Therefore, in analogy with Miller's linear barrier test for the one-sample case, the SWAMD test for testing H_0 against the one-sided scale alternative H_1 is stated as follows: At the n-th stage observe (X_n, Y_n), compute W_n from (2) and take one of the decisions:

(a) Continue sampling as long as $n < N$ and

$$W_n - n(2n+1)/2 \leq z_N^\alpha n \tag{4}$$

(b) Reject H_0 (accept H_1) if for some $n \leq N$

$$W_n - n(2n+1)/2 > z_N^\alpha n \tag{5}$$

(c) Accept H_0 if n reaches N without the inequality (4) being violated.

The truncation point N and the probability of type-I error α are chosen by the statistician that determines the critical constant z_N^α.

An analogous test for the two-sided alternative $H_{1'} : \theta \neq 1$ would be to continue sampling as long as $|W_n - n(2n+1)/2| \leq |z|_N^\alpha n$ and $n < N$. If for some $n \leq N$, $|W_n - n(2n+1)/2| > |z|_N^\alpha n$, then reject H_0 (accept $H_{1'}$); otherwise accept H_0. $|z|_N^\alpha$ is the upper α-percentile point for the two-sided test, also determined by α and N. Both z_N^α and $|z|_N^\alpha$ have been estimated by Wiener approximation in Section 4.

3. THE SWSMD TEST

Let us now consider the situation when the two population medians are either unknown or can not be assumed equal. Let groups of observations are taken sequentially, with each group consisting of m independent observations from the $X-$ population and m independent observations from the $Y-$ population, where X and Y have the cdf's as defined in section 2. Then at n-th stage of the sequential process, we have the observations $X_{n1}, X_{n2}, \cdots, X_{nm}$ and $Y_{n1}, Y_{n2}, \cdots, Y_{nm}, n = 1, 2, \cdots, N$ where N is the upper bound on the sequential sampling. We calculate the dispersion indices for the sample of m $X-$observations and m $Y-$observations obtained at each stage n by

$$D(X_n) = \sum_{i=1}^{m}(X_{ni} - \bar{X}_n)^2, D(Y_n) = \sum_{i=1}^{m}(Y_{ni} - \bar{Y}_n)^2, n = 1, 2, \cdots, N$$

where $\bar{X}_n = (1/m)\sum_{i=1}^{m} X_{ni}, \bar{Y}_n = (1/m)\sum_{i=1}^{m} Y_{ni}$ are the sample means of $X-$ and $Y-$ observations at stage n.

Now, if the null hypothesis of equal variability is true, we would expect that the values of $D(X_n)$ and $D(Y_n)$ should be well mixed in that the dispersion measures obtained at different stages should be similar. On the other hand, if the alternative hypothesis that variability of X is more than the variability of Y is true, then we would expect that the values $D(X_n)$ would generally tend to be larger than the $D(Y_n)$. Hence the logic of Wilcoxon test for location can be applied to test the hypothesis of equal dispersion against the one- or two-sided alternatives on the basis of Wilcoxon rank-sum statistic W_n^* defined for the dispersion indices $D(X_n)$ and $D(Y_n)$ as follows:

$$\begin{aligned} W_n^* &= \text{Sum of the ranks of the } D(X_n) \text{ in the combined ordered sample of} \\ &\quad\ 2n \text{ indices } D(X_n) \text{ and } D(Y_n) \\ &= \sum_{i=1}^{2n} iZ_i^* \end{aligned}$$

where $Z_i^* = 1$, if the i-th item in the combined ordered array of $2n$ indices $D(X_n)$ and $D(Y_n)$ is a $D(X_n)$

$= 0$, otherwise , $\quad i = 1, 2, \cdots, n$

Then the mean, variance and all other properties of W_n^* are same as that for W_n . Consequently, the sequential procedures and other results on critical points, power, etc., of the SWAMD test are directly applicable to the test based on W_n^* , called the SWSMD test, once W_n is replaced by W_n^* . Therefore in the following sections we will restrict our study to the SWAMD test only.

4. CRITICAL CONSTANTS

Let $V_n = [W_n - n(2n+1)/2]/n$

$Z_N = \max\{V_1, V_2, \cdots, V_N\}; \quad |Z|_N = \{|V_1|, |V_2|, ..., |V_N|\}$

In order for the one-sided SWAMD test to have size α , the critical constant z_N^α must be the upper $\alpha-$ percentile point of the distribution of Z_N, i.e., $P\{Z_N > z_N^\alpha\} = \alpha$. Similarly $|z|_N^\alpha$ is to be determined by $P\{|Z|_N > |z|_N^\alpha\} = \alpha$. Since the exact distribution of $V_1, V_2, ..., V_N$ is a complicated discrete multivariate distribution, both Z_N and $|Z|_N$ have discrete distributions and are therefore intractable. However, the values of z_N^α and $|z|_N^\alpha$ can be approximated by a Wiener process. Under H_0, we have from (3),

$$E(V_n) = 0 \text{ and } Var(V_n) = (2n+1)/12 \cong n/6 \tag{6}$$

Also, using the backward martingale property of U_n , we have for $m < n$,

$$E(U_m/U_n) = (m/n)^2 U_n; E(U_m.U_n) = (m/n)^2 E(U_n^2)$$

Hence,

$$Cov(V_m, V_n) = Cov(U_m, U_n)/mn = m^2(2n+1)/12mn \cong m/6 \tag{7}$$

The approximate normality of V_n with the mean, variance and covariance given by (6) and (7) suggests that the $V_n, n = 1, 2, \cdots, N$ are behaving like $Z(n)/\sqrt{6}$, where $Z(n)$ is a standard Wiener process with zero mean and variance n. This approximation gives

$$\begin{aligned} P\{Z_N > c\} \cong P\{\max_{0 \le n \le N} Z(n)/6 > c\} &= 2P\{Z(N) > c\sqrt{6}\} \\ &= 2[1 - \Phi(c\sqrt{6}/\sqrt{N}] \end{aligned} \tag{8}$$

where Φ is the standard normal cdf. The approximation (8) implies that $z_N^\alpha \cong \sqrt{N} g^{\alpha/2}/\sqrt{6}$, where $g^{\alpha/2}$ is the upper $\alpha/2$- percentile point of Φ. If the probability of crossing both the positive and negative boundaries by time N is negligible, then $|z|_N^\alpha \cong \sqrt{N} g^{\alpha/4}/\sqrt{6}$. The values of z_N^α and $|z|_N^\alpha$ for $N = 10(5)30(10)50$ and $\alpha = 0.01, 0.05, 0.1$ are computed and displayed in Tables 1a,1b.

Miller (1972) has shown that values of the percentile points for his linear barrier test obtained by Wiener process approximation are in close agreement with the Monte Carlo estimates for the above range of α and N. Further, Miller and Sen (1972) have established an invariance principle for $U-$statistics which gives an asymptotic justification of the Wiener process approximation for the critical constants. Therefore, it is felt that the estimates of z_N^α and $|z|_N^\alpha$ by Wiener approximation in the given range are quite justified and there is no need to venture into simulation process for the purpose.

Table 1a: Values of z_N^α by Wiener Approximation

α	N						
	10	15	20	25	30	40	50
0.01	3.32	4.07	4.70	5.26	5.76	6.65	7.44
0.05	2.53	3.10	3.58	4.00	4.38	5.06	5.66
0.10	2.12	2.60	3.01	3.36	3.68	4.25	4.75

Table 1b: Values of $|z|_N^\alpha$ by Wiener Approximation

α	N						
	10	15	20	25	30	40	50
0.01	3.62	4.44	5.13	5.73	6.28	7.25	8.10
0.05	2.89	3.54	4.09	4.58	5.01	5.79	6.47
0.10	2.53	3.10	3.58	4.00	4.38	5.06	5.66

Table 2a: Wiener Power of the Tests (one-sided), $N = 20$

α	θ				
	1	2	3	4	6
0.01	0.01	0.21	0.64	0.82	0.98
0.05	0.05	0.56	0.82	0.96	1.00

Table 2b: Wiener Power of the Tests (one-sided), $N = 50$

α	θ				
	1	1.5	2	2.5	3
0.01	0.01	0.19	0.66	0.90	0.98
0.05	0.05	0.49	0.84	0.98	1.00

5. POWER OF THE TESTS

We know that the asymptotic normality of U_n holds even in the nonnull case, where the mean and variance of U_n depend on the parameters $p, p1$ and $p2$ given by

$$p = P(Y_j < X_i) = \int_{-\infty}^{\infty} G(x)dF(x) \tag{9}$$

$$p_1 = P(Y_j < X_i \cap Y_k < X_i) = \int_{-\infty}^{\infty} [G(x)]^2 dF(x) \tag{10}$$

$$p_2 = P(Y_j < X_i \cap Y_j < X_h) = \int_{-\infty}^{\infty} [1 - F(x)]^2 dG(x) \tag{11}$$

Even under the more specific scale alternative (1), these integrals depend on both θ and F. Therefore, evaluating even approximations to power requires that the basic parent population be specified. From Gibbons (1985, p. 142), we have, under H_s

$$E(V_n) = n\mu(\theta); Var(V_n) = n\sigma^2(\theta) + a(\theta) \cong n\sigma^2(\theta) \tag{12}$$

where $\mu(\theta)$ and $\sigma^2(\theta)$ depend on F, but not on n. Now, using the asymptotic normality of V_n, the behaviour of $V_n, n = 1, 2, ..., N$ can be approximated by a Wiener process $Z(n)$ with drift μn and variance $\sigma^2 n$ and, noting from Miller (1972, p.104), the Wiener approximation to power function of the one-sided SWAMD test for specified F is given by

$$\begin{aligned} P_F(\theta; N, \alpha) &= P\{\max_{0 \le n \le N} Z(n) > z_N^\alpha\} \\ &= exp(2\mu z_N^\alpha/\sigma^2).\Phi(-\mu\sqrt{N}/\sigma - z_N^\alpha/\sigma\sqrt{N}) + \Phi(\mu\sqrt{N}/\sigma - z_N^\alpha/\sigma\sqrt{N}) \end{aligned} \tag{13}$$

The power of this test has been investigated for the exponential distribution F, which represents the scale alternative (1).

Let

$$f(x) = e^{-x}, x \ge 0 \text{ and } g(x) = \theta e^{-\theta x}, x \ge 0, \theta > 0, \theta \ne 1 \tag{14}$$

Then

$$F(x) = 1 - e^{-x}, x \ge 0 \text{ and } G(x) = 1 - e^{-\theta x}, x \ge 0, \theta > 0, \theta \ne 1 \tag{15}$$

so that $G(x) = F(\theta x)$ for all x and some $\theta > 0, \theta \ne 1$, which is same as the alternative (1). Now, using the probability functions (14) and (15) in (9), (10) and (11), we easily find that

$$p = \theta/(\theta + 1), \quad p_1 = 2\theta^2/(\theta + 1)(2\theta + 1), \quad p_2 = \theta/(\theta + 2)$$

$$E(V_n) = \mu n; \quad Var(V_n) = \sigma^2 n + O(1) \cong \sigma^2 n$$

where

$$\mu = \theta/(\theta+1) - 0.5; \quad \sigma^2 = \theta(\theta^2 + 4\theta + 1)/(\theta+1)^2(\theta+2)(2\theta+1) \tag{16}$$

Substituting the values of μ and σ obtained from (16) for various true values of θ and the specified α and N in (13), we get the Wiener power of the SWAMD test (or the SWSMD test) against the alternative $H_1 : \theta > 1$. For $N = 20$, the performance of this test has been studied at $\theta = 1, 2, 3, 4, 6$; for $N = 50$, at $\theta = 1, 1.5, 2, 2.5, 3$. The error probability α is taken to be either 0.05 or 0.01. The results are displayed in Tables 2a,2b, which indicate that the test is having very satisfactory power in the entire range of the alternative considered. Wiener approximation to the power of the two-sided test can be found in a similar way.

It is interesting to note that the results in Tables 2a,2b are in close agreement with the Wiener powers of Miller's one-sample sequential test with linear barriers, as given in Tables Vb, VIb of Miller (1972) for different values of the location parameter under the double exponential distribution, but with the same set of values of α and N.

6. AN ILLUSTRATIVE EXAMPLE

An institute of microbiology is interested in purchasing microscope slides of uniform thickness and needs to choose between two different Suppliers. Both have the same specifications for median thickness, but may differ in variability. It is desired to test the variability in stages with a minimum of total experimentation, but subject to maximum of 10 pairs of slides. It is also desirable to conduct sequential experiments with known small risks of type-I error, say $\alpha = 0.05$ or 0.01. Accordingly, the institute selects randomly a pair of slides from the two Suppliers at each stage, gauges the thickness of the slides using a micrometer and reports the data as deviations of the measurements from the specified median M. The data so reported in the form of X-M and Y-M, where X and Y refer to the observations from the 1st and 2nd Suppliers respectively, are given below. Which Supplier makes slides with a smaller variability in thickness ?

X-M : $0.028, 0.029, 0.011, -0.030, 0.017, -0.012, -0.027, -0.018, 0.022, -0.023$

Y-M : $-0.002, 0.016, 0.005, -0.001, 0.000, 0.008, -0.005, -0.009, 0.001, -0.019$

The one-sided SWAMD test is the most appropriate for this problem. Here we have to test the null hypothesis $H_0 : \theta = 1 (\sigma_x = \sigma_y)$ against the alternative $H_1 : \theta > 1 (\sigma_x > \sigma_y)$ by sequentially examining the statistic $W_n - n(2n+1)/2$ and comparing it with the boundary $n.\ z_N^\alpha$ for $\alpha = 0.05$ or $0.01, N = 10$ and

$n = 1, 2, 3, \cdots$ to take a decision regarding the variability in thickness of slides supplied by the two Suppliers. The sequential procedure is summarised in Table 4. It reveals that the SWAMD test rejects the null hypothesis of equal variability in favour of the alternative that the 2nd Supplier has the smaller variability in thickness of the slides ($\sigma_x > \sigma_y$) at 6−th stage of the sequential experimentation at 5 % level of significance. The same conclusion is also drawn at 1 % level, but at the cost of two more pairs of observations.

It is also found that this test terminates with acceptance of the two-sided alternative that the two variances are not equal at 7−th and 8−th stages for $\alpha = 0.05$ and 0.01 respectively (since, $7.|z|_N^\alpha = 20.23$ for $\alpha = 0.05$ and $8.|z|_N^\alpha = 28.96$ for $a = 0.01$).

The data used for this illustration have been adapted from Example 15.8.1 of Gibbons (1985, p.314), where same decision is taken on the basis of fixed sample of 10 pairs of slides by applying the Siegel-Tukey test. Thus, in this particular instance, there has been 40 percent saving in the number of observations by applying the sequential procedure. Of course, this is not true in general.

Table 4: Summarisation of sequential computation of the test statistic and rejection boundaries of the SWAMD test for the data on thickness of microscope slides from two Suppliers

n	$\|X-M\|$ (x)	$\|Y-M\|$ (y)	Configurations of ordered x and y upto stage n	W_n	$W_n - \frac{n(n+1)}{2}$	nz_N^α $\alpha = .05$ $N = 10$	nz_N^α $\alpha = .01$ $N = 10$
1	0.028	0.002	yx	2	0.5	2.53	3.32
2	0.029	0.016	yyxx	7	2.0	5.06	6.64
3	0.011	0.05	yyxyxx	14	3.5	7.59	9.96
4	0.030	0.001	yyyxyxxx	25	7.0	10.12	13.28
5	0.017	0.000	yyyyxyxxxx	39	11.5	12.65	16.60
6	0.012	0.008	yyyyyxxyxxxx	55	**16.0**	**15.18**	19.92
7	0.027	0.005	yyyyyyxxyxxxxx	75	22.5	17.71	23.24
8	0.018	0.009	yyyyyyyxxyxxxxxx	98	**30.0**	20.24	**26.56**
9	0.022	0.001	yyyyyyyyxxyxxxxxxx	124	38.5	22.77	29.88
10	0.023	0.019	yyyyyyyyxxyxxyxxxxxx	149	44.0	25.30	33.20

7. COMPARISON AND APPLICATION OF THE TESTS

A modest attempt has thus been made in this paper to develop two sequential Wilcoxon tests for testing the scale differences between two independent populations under two different situations. When the population medians are equal or known, the SWAMD test is most appropriate because of considerable savings in total number of observations. When medians are unknown or unequal, the SWSMD test is applied. But in this case some additional computations are required. Due to grouped sequential sampling, the number of observations required for coming to a decision will inevitably rise, but it cannot go beyond $2mN, N$ being the upper bound on the amount of sampling. Further, the efficiency of this test is a function of the size of the sample at each stage. The efficiency increases with increase in sample size, but there is a trade-off, since increasing the sample size leads to rapid increase in total number of observations and make the test uneconomical. With pre-assigned truncation point N to terminate the test, rigorous study of the expected sample size through simulation is of little practical value and hence not carried out for any of the tests. Unlike the SWAMD test, the SWSMD test requires that the observations are measured on at least an interval scale.

Although ties in the original data are no problem in the application of the tests, adjustment for ties must be made in the usual way applied to Wilcoxon test, if there are ties in the dispersion indices. As the power of the tests is indirectly determined by α and N, these have to be specified with due care.

Both the tests are straightforward to apply and easy to explain to nonstatisticians. They are most suitable in medical applications, where selection of an alternative hypothesis and its associated power is often difficult or extremely arbitrary and a bound on the amount of sampling is usually easier to determine due to limitations of resources like time, money, manpower, patients, etc. Many other applications for these tests can also be found in social, psychological and educational studies where difference in variability between two independent groups needs to be tested with controlled type I error and with a minimum of total experimentation.

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