Model Reference Adaptive Control of a Flexible Structure

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In this paper, the model reference adaptive control (MRAC) of a flexible structure is investigated. Any mechanically flexible structure is inherently distributed parameter in nature, so that its dynamics are described by a partial, rather than ordinary, differential equation. The MRAC problem is formulated as an initial value problem of coupled partial and ordinary differential equations in weak form. The well-posedness of the initial value problem is proved. The control law is derived by using the Lyapunov redesign method on an infinite dimensional Hilbert space. Uniform asymptotic stability of the closed loop system is established, and asymptotic tracking, i. e., convergence of the state-error to zero, is obtained. With an additional persistence of excitation condition for the reference model, parameter-error convergence to zero is also shown. Numerical simulations are provided.

Key Words : Model Reference Adaptive Control, Distributed Parameter System, Uniform Asymptotic Stability, Weak Form, Persistence of Excitation.

1. Introduction

A large space structure, or any mechanically flexible structure, is inherently a distributed parameter system (DPS) whose dynamics is described by a partial, rather than ordinary, differential equation (PDE). In many cases such a detailed description, i. e., a PDE model, may not be necessary for the successful operation of the system, and a lumped parameter (ordinary differential equation) model may be satisfactory. Nevertheless, a large number of current and newly proposed systems are so thoroughly distributed parameter in nature that it is impossible to ignore this fact in modeling and control. A great deal of research on DPS control have appeared in the literature because of the construction and operation in orbit of large flexible spacecraft and satellites in outer space.

Such a DPS is described by an operator equation on an infinite-dimensional Hilbert (or Banach) space. The analysis of a DPS makes use of the theory of semigroup on an infinite-dimensional state space. The infinite-dimensional approach will yield results that can be used effectively in large-scale finite-dimensional systems as well. One very important consideration in large-scale or distributed parameter systems is to avoid dependence on precise knowledge of the total system dimension and the full system parameters, especially those residual parameters that are not used in the synthesis of the controller. The infinite-dimensional approach can eliminate the uncertainty about the system dimension and spillover problems on residual data.

In flexible systems, not only the geometry of the structure but also physical properties such as the density, stiffness, Poisson ratio and damping coefficients may change with time. Indeed, many of these parameters are unknown even if the material itself is homogeneous and the structure is uniform. Thus, the control problem of flexible

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structures provides challenging issues including parameter estimation, uncertainty quantification, and robustness.

Compared to finite dimensional adaptive control (Astrom and Wittenmark, 1995; Sastry and Bodson, 1989), the adaptive control of infinite dimensional systems is not well developed and has only recently been studied. In (Balas, 1983; Balas, 1998) some of the possible directions of investigation and the main areas of difficulty for infinite dimensional adaptive control were surveyed. Wen and Balas (1985, 1989) proposed a direct model reference adaptive control (MRAC) in an infinite dimensional Hilbert space and analyzed the Lagrange stability of the closed loop system. Finite dimensional adaptive controllers including stability analysis were investigated in (Kobayashi, 1988) for spectral systems and in (Miyasato, 1990) for parabolic systems. An indirect adaptive control algorithm for a class of infinite dimensional stochastic evolution equations has been developed by Duncan et al. (1994). Hong and Bentsman (1994) considered the MRAC of a linear parabolic partial differential equation and established the exponential stability of the closed loop system by applying averaging. The MRAC for a time-varying parabolic system was also investigated in Hong et al. (2000).

The objective of a MRAC scheme is to determine a feedback control law which forces the state of the plant to asymptotically track the state of a given reference model. At the same time, the unknown parameters in the plant model are estimated and used to update the control law. Typically, the resulting closed loop system consisting of the plant, the reference model, and the estimator, will be nonlinear. This is true even if the underlying plant, reference models, and the estimator are linear. The nonlinearity arises due to the coupling of the state and parameter estimation errors. Consequently, the scheme requires a careful stability analysis to ensure that all signals (both input and output) remain, in some sense, bounded. It is also desirable, although not necessarily essential, that some sort of parameter convergence be achieved.

In this paper, the MRAC of a flexible structure which is described by a linear hyperbolic partial differential equation is investigated. Our effort here is closely related to the earlier treatment of adaptive control for distributed parameter systems in (Hong and Bentsman, 1994; Bohm et al., 1998). The present paper makes the following contributions: To the best of the authors' knowledge this paper is the first treatment of a flexible structure in the frame of MRAC. The well-posedness of the closed loop system is established. Using an appropriate Lyapunov function, asymptotic tracking error convergence to zero is established. With the additional condition of persistence of excitation, the convergence of parameter estimation errors to zero is established as well.

2. Problem Formulation

Regarding the reference model in the MRAC of a flexible structure, the model itself can be chosen to have the same structure as the actual plant, but using different mass, stiffness, and damping parameters. Specifically, if a plant is given in operator form, either a linear or a nonlinear distributed parameter system, as

$$M(q_1)u^n+D(q_2)u'+K(q_3)u=f$$

then, the reference model can be chosen as

$$M(q_1^*)v''+D(q_2^*)v'+K(q_3^*)v=g$$

where q_1, q_2, and q_3 are the plant mass, damping, and stiffness parameters, respectively, while q_1^*, q_2^*, and q_3^* are the reference model mass, damping, and stiffness parameters, respectively. The parameters q_1^*, q_2^*, and q_3^* are chosen so that the response $\{v, v'\}$ can have the desired characteristics.

In this paper, the transverse vibrations of an Euler-Bernoulli beam with Kelvin-Voigt damping, as shown schematically in Fig. 1, is considered. In many cases this simple model retains the essential features of more complicated large flexible structures.

The equations of motion for one dimensional Euler-Bernoulli beam, with both ends fixed and

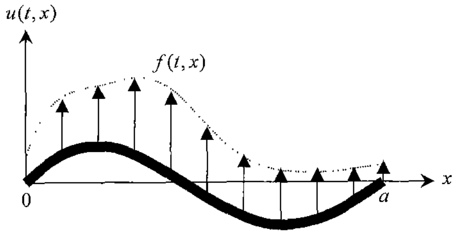

Fig. 1 An Euler-Bernoulli beam deflected due to distributed loadings

Kelvin-Voight damping, is given by (Clough and Penzien, 1993)

$$\rho u_{tt}(t,x)+\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 u(t,x)}{\partial x^2}+c_D I\frac{\partial^3 u(t,x)}{\partial x^2\partial t}\right)$$
$$=f(t,x),\ 0<x<a,\ t>0, \tag{1}$$
$$u(t,0)=\frac{\partial u(t,0)}{\partial x}=u(t,a)=\frac{\partial u(t,a)}{\partial x}=0,$$
$$u(0,x)=u_0(x),\ u_t(0,x)=u_{t0}(x),$$

where $u(t,x)$ is the transverse-displacement (and is also the observed distributed state), ρ is the mass density, E is the Young's modulus or the modulus of elasticity, I is the second moment of inertia of cross section about the centroid axis, c_D is the coefficient of damping, $f(t,x)$ is the control input force, $u_0(x)$ and $u_{t0}(x)$ are the initial conditions, $u_t=\partial u/\partial t$, and $u_{tt}=\partial^2 u/\partial t^2$. The boundary conditions indicate that both ends are fixed.

A typical problem regarding system (1) would be to estimate the mass density (ρ), Kelvin-Voigt damping coefficient ($c_D I$), and stiffness coefficient (EI), which may depend on the time and/or spatial coordinates. If the shape of a beam is uniform, the coefficients can be assumed as constants because the material properties (ρ, E, and c_D) change slowly with respect to time. Then (1) can be rewritten as

$$u_{tt}(t,x)+q_1\frac{\partial^4 u(t,x)}{\partial x^4}+q_2\frac{\partial^5 u(t,x)}{\partial x^4\partial t}=q_3 f(t,x) \tag{2}$$

where q_1, q_2, and q_3 are unknown constant parameters.

To provide a framework that facilitates rigorous analysis, approximation, and implementation, it is advantageous to consider a weak form of (2) (see Banks et al., 1997). To convert (2) into weak form, both sides of (2) are multiplied by a sufficiently smooth test function φ, and are integrated by parts in the usual manner. Assuming that φ satisfies the boundary conditions $\varphi(x)=\partial\varphi(x)/\partial x=0$ at $x=0$ and $x=a$, the weak form of (2) is defined as

$$\int_\Gamma u_{tt}(t,x)\varphi(x)\,dx+\int_\Gamma q_1 D^2 u(t,x)D^2\varphi(x)\,dx$$
$$+\int_\Gamma q_2 D^2 u_t(t,x)D^2\varphi(x)\,dx=\int_\Gamma q_3 f(t,x)\varphi(x)\,dx \tag{3}$$

where $\Gamma=[0,a]$, $D=\partial/\partial x$, and the coefficients q_i, $i=1,2,3$, are unknown. It is pointed out that in the weak form the derivatives have been transferred from the beam moments onto the test function. This eliminates the problem of having to approximate the derivatives of the characteristic function and the Dirac delta function, as is the case with the strong form of the equation. In this paper it is assumed that the system state $u(t,x)$ can be measured at all points of $x\in\Gamma$ and $t\geq 0$.

To pose the MRAC problem, two function spaces are first introduced. Let H and V be the Hilbert spaces given by $H\triangleq L^2(\Gamma)$ and $V\triangleq H_0^2(\Gamma)$, which are defined as follows:

$$L^2(\Gamma)=\left\{\eta:[0,a]\to R\ \middle|\ \int_\Gamma \eta^2 dx<\infty\right\} \tag{4}$$
$$H_0^2(\Gamma)=\{\eta\in L^2(\Gamma)\,|\,D\eta,\ D^2\eta\in L^2(\Gamma),$$
$$\text{and } \eta(x)=D\eta(x)=0 \text{ at } x=0,\ a\}$$

The inner products in and are defined respectively as

$$\langle\phi(x),\psi(x)\rangle_H=\int_\Gamma \phi(x)\psi(x)\,dx,$$
$$\langle\phi(x),\psi(x)\rangle_V=\int_\Gamma D^2\phi(x)D^2\psi(x)\,dx,$$

and the corresponding induced norms are denoted by $|\cdot|_H$ and $|\cdot|_V$, respectively. Since V is densely and continuously embedded in H, the Hilbert spaces H and V form a Gelfand triple (Showalter, 1977)

$$V\mapsto H\cong H^*\mapsto V^* \tag{5}$$

where the symbol $\mapsto$ denotes embedding, H^* and V^* denote the continuous duals of H and V, respectively. All of the embeddings in (5) are dense and continuous. The following is also satisfied:

$$|\varphi|_H\leq K|\varphi|_V,\ \varphi\in V \tag{6}$$

for some positive (embedding) constant K.

For ϕ, ψ in V, the sesquilinear forms $\sigma_i(q_i\,;\cdot,\cdot): V\times V\rightarrow C$, $i=1, 2$ are defined as

$$\sigma_i(q_i\,;\phi,\psi)=\int_\Gamma q_i D^2\phi D^2\psi dx,$$

for $q_i\in R$ and $i=1,2$ (7)

where σ_i, $i=1,2$, satisfy various continuity, symmetry, coercivity, and linearity conditions, i. e., for each $q_i=R^+$, σ_i satisfies

(A1) $|\sigma_i(q_i\,;\phi,\psi)|\leq q_i|\phi|_V|\psi|_V$, (boundedness)

(A2) $\text{Re}\sigma_i(q_i\,;\phi,\phi)\geq\alpha_i|\phi|^2_V$, for some $\alpha_i>0$, (-coercivity)

(A3) $\sigma_i(q_i\,;\phi,\psi)=\sigma_i(q_i\,;\psi,\phi)$ (symmetricity)

(A4) The map $q_i\rightarrow\sigma_i(q_i\,;\phi,\psi)$ from R into C is linear, (R-linearity)

for $i=1,2$ and all $\phi,\psi\in V$. The boundedness results from the Schwarz's Inequality for inner products (Luenberger, 1968, p. 47), while the V-coercivity follows from the fact that there exists a $\alpha_i>0$ such that

$$\text{Re}\sigma_i(q_i\,;\phi,\phi)=\int_\Gamma q_i(D^2\phi)^2dx\geq\alpha_i\int_\Gamma(D^2\phi)^2dx=\alpha_i|\phi|^2_V,$$

for $i=1,2$ and all $\phi\in V$.

Now, two product spaces $\mathbf{H}=V\times H$ and $\mathbf{V}=V\times V$ are introduced. Note that $\mathbf{H}$ and $\mathbf{V}$ are Hilbert spaces with the standard inner products given by

$$\left\langle\begin{pmatrix}\phi_1\\\phi_2\end{pmatrix},\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}\right\rangle_{\mathbf{H}}=\sigma_1(q_1^*\,;\phi_1,\psi_1)+\langle\phi_2,\psi_2\rangle_H,\ \begin{pmatrix}\phi_1\\\phi_2\end{pmatrix},\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}\in\mathbf{H}\quad(8)$$

$$\left\langle\begin{pmatrix}\phi_1\\\phi_2\end{pmatrix},\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}\right\rangle_{\mathbf{V}}=\sigma_1(q_1^*\,;\phi_1,\psi_1)+\sigma_1(q_1^*\,;\phi_2,\psi_2),\ \begin{pmatrix}\phi_1\\\phi_2\end{pmatrix},\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}\in\mathbf{V}\quad(9)$$

for $q_1^*\in R^+$. Instead of (8), a modified inner product in the product space $\mathbf{H}$, from the work of (Bohm et at., 1998), is introduced as follows:

$$\left\langle\begin{pmatrix}\phi_1\\\phi_2\end{pmatrix},\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}\right\rangle_{\mathbf{H}}=\sigma_1(q_1^*\,;\phi_1,\psi_1)+\langle\phi_2,\psi_2\rangle_H+\mu\{\langle\phi_1,\psi_2\rangle_H+\langle\phi_2,\psi_1\rangle+\sigma_2(q_2^*\,;\phi_1,\psi_1)\}\quad(10)$$

where $\begin{pmatrix}\phi_1\\\phi_2\end{pmatrix},\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}\in\mathbf{H}$ and q_i^*, $\mu\in R^+$, $i=1,2$. The symmetry, linearity with respect to the first component, and homogeneity properties of (10), as an inner product, are apparent. To show the positiveness, i. e., $\left\langle\begin{pmatrix}\phi_1\\\phi_2\end{pmatrix},\begin{pmatrix}\phi_1\\\phi_2\end{pmatrix}\right\rangle_{\mathbf{H}}\geq0$, the following inequality is asserted:

$$\sigma_1(q_1^*\,;\phi_1,\phi_1)+\langle\phi_2,\phi_2\rangle_H+\mu\sigma_2(q_2^*\,;\phi_1,\phi_1)\geq2\mu\langle\phi_1,\phi_2\rangle_H\quad(11)$$

The right hand side of (11) satisfies

$$\sigma_1(q_1^*\,;\phi_1,\phi_1)+\langle\phi_2,\phi_2\rangle_H+\mu\sigma_2(q_2^*\,;\phi_1,\phi_1)\geq\alpha_1|\phi_1|^2_V+|\phi_2|^2_H+\mu\alpha_2|\phi_1|^2_V$$

where the V-coercivity of the σ_i's was used. On the other hand, the right hand side of (11) satisfies

$$2\mu\langle\phi_1,\phi_2\rangle_H\leq\mu(|\phi_1|^2_H+|\phi_2|^2_H)\leq\mu(K^2|\phi_1|^2_V+|\phi_2|^2_H),$$

where equation (6) has been utilized in deriving the second inequality. Therefore, if $\mu\leq\min\{\alpha_1/|K^2-\alpha_2|,1\}$ and μ is sufficiently small, then (10) is an inner product in $\mathbf{H}$. The norm induced by (10) is equivalent to the standard norm induced by the inner product (8), i.e. there exist k_1, $k_2>0$ such that

$$k_1\{\sigma_1(q_1^*\,;\phi_1,\phi_1)+\langle\phi_2,\phi_2\rangle_H\}\leq\sigma_1(q_1^*\,;\phi_1,\phi_1)+\langle\phi_2,\phi_2\rangle_H+\mu\{\sigma_2(q_2^*\,;\phi_1,\phi_1)+2\langle\phi_1,\phi_2\rangle_H\}\leq k_2\{\sigma_1(q_1^*\,;\phi_1,\phi_1)+\langle\phi_2,\phi_2\rangle_H\}$$

With these definitions, (3) can be rewritten as

$$\langle\mu_{tt},\varphi\rangle+\sigma_1(q_1\,;u,\varphi)+\sigma_2(q_2\,;u,\varphi)=q_3\langle f,\varphi\rangle\quad(12)$$

for $\varphi\in V$ and $t>0$, where $q_1\in R$, $i=1,2$, are unknown and the control input force f is assumed to satisfy $f\in L_2(0,T\,;V^*)$ for all $T>0$. The MRAC problem, in the presence of unknown parameters q_i's, for plant (12) is now to find a control input f in feedback form which forces the state, u, to track the state of a reference model, v, given by

$$\begin{aligned}&\langle v_{tt},\varphi\rangle+\sigma_1(q_1^*:v,\varphi)+\sigma_2(q_2^*:v_1,\varphi)\\&\quad=q_3^*\langle g,\varphi\rangle,\ 0<x<a,\ t>0\qquad(13)\\&v(t,0)=\frac{\partial v(t,0)}{\partial x}=v(t,a)=\frac{\partial v(t,a)}{\partial x}=0\\&v(0,x)=v_0(x),\ v_t(0,x)=v_{t0}(x)\end{aligned}$$

for $\varphi\in V$ and $t>0$, where $q_i^*\in R^+$, $i=1,2,3$, and the input reference signal is assumed to satisfy

$g \in L_2(0,T\,;\,V^*)$ for all $T>0$.

3. Design of Control Laws: Stability

Consider a nominal control input, f^*, as follows:

$$\langle f^*,\varphi\rangle = \sigma_1(\theta_1^*\,;\,u,\varphi) + \sigma_2(\theta_2^*\,;\,u_t,\varphi) + \theta_3^*\langle g,\varphi\rangle \quad (14)$$

where $\theta_1^* = (q_1 - q_1^*)/q_3$, $\theta_2^* = (q_2 - q_2^*)/q_3$, and $\theta_3^* = q_3^*/q_3$. By substituting the nominal control input into (12), it is seen that (12) coincides with (13), i.e., the plant equation and the reference model equation become identical. But because q_i, $i=1,2,3$, are unknown, the values of θ_i^* are no t known. Hence, the following adaptive control law is introduced:

$$\langle f,\varphi\rangle = \sigma_1(\theta_1(t)\,;\,u,\varphi) + \sigma_2(\theta_2(t)\,;\,u_t,\varphi) + \theta_3(t)\langle g,\varphi\rangle \quad (15)$$

for each $t>0$, where $\theta_i(t) \in R$, $i=1,2,3$, denote adaptively updated estimates for θ_i^*, respectively.

The substitution of (15) into (12) yields the closed loop plant equation as

$$\begin{aligned}&\langle u_{tt},\varphi\rangle - \sigma_1(q_3\tilde{\theta}_1\,;\,u,\varphi) + \sigma_1(q_1^*\,;\,u,\varphi)\\ &\quad - \sigma_2(q_3\tilde{\theta}_2\,;\,u_t,\varphi) + \sigma_2(q_2^*\,;\,u_t,\varphi)\\ &\quad = q_3\tilde{\theta}_3\langle g,\varphi\rangle + q_3^*\langle g,\varphi\rangle \end{aligned} \quad (16)$$

where $\tilde{\theta}_i(t) = \theta_i(t) - \theta_i^*$, $i=1,2,3$, are the controller parameter estimation errors. In deriving (16), $q_i - q_3\theta_i = q_3(q_i - q_i^*)/q_3 + q_i^* - q_3\theta_i = q_i^* - q_3(\theta_i - \theta_i^*) = q_i^* - q_3\tilde{\theta}$, $i=1,2$, and $q_3\theta_3 = q_3\tilde{\theta}_3 + q_3\theta_3^*$ have been used. Note also that if $\tilde{\theta}_i(t) \to 0$, $i=1,2,3$, i.e., the controller parameter errors converge to zero, then the closed loop plant (16) is exactly the same as the reference model (13).

Let us define the state error as

$$e(t,x) = u(t,x) - v(t,x) \quad (17)$$

Then the following state error equation is obtained from (13) and (16).

$$\begin{aligned}&\langle e_{tt},\varphi\rangle - \sigma_1(q_3\tilde{\theta}_1\,;\,e,\varphi) - \sigma_1(q_3\tilde{\theta}_1\,;\,v,\varphi)\\ &\quad + \sigma_1(q_1^*\,;\,e,\varphi) - \sigma_2(q_3\tilde{\theta}_2\,;\,e_t,\varphi) \quad (18)\\ &- \sigma_2(q_3\tilde{\theta}_2\,;\,v_t,\varphi) + \sigma_2(q_2^*\,;\,e_t,\varphi) = q_3\tilde{\theta}_3\langle g,\varphi\rangle\\ &e(t,0) = \frac{\partial e(t,0)}{\partial x} = e(t,a) = \frac{\partial e(t,a)}{\partial x} = 0\\ &e(0,x) = e_0(x),\ e_t(0,x) = e_{t0}(x),\ \tilde{\theta}_i(0) = \tilde{\theta}_{i0},\ i=1,2,3\end{aligned}$$

By considering an appropriate Lyapunov function, the stability of the closed loop system together with the adaptation laws can be established. Now, a functional $V: [0,\infty) \to R^+$ is considered as

$$\begin{aligned}V(t) &= \frac{1}{2}\left\langle \begin{pmatrix} e \\ e_t\end{pmatrix}, \begin{pmatrix} e \\ e_t\end{pmatrix}\right\rangle_H + \frac{1}{2\gamma}(\tilde{\theta}_1^2 + \tilde{\theta}_2^2 + \tilde{\theta}_3^2)\\ &= \frac{1}{2}\{\sigma_1(q_1^*\,;\,e,\,e) + \langle e_t,\,e_t\rangle_H\\ &\quad + 2\mu\langle e,\,e_t\rangle_H + \sigma_2(\mu q_2^*\,;\,e,\,e)\} \quad (19)\\ &\quad + \frac{1}{2\gamma}(\tilde{\theta}_1^2 + \tilde{\theta}_2^2 + \tilde{\theta}_3^2)\end{aligned}$$

where $\gamma>0$ is an adaptive gain. Note that $e \in H_0^2(\Gamma)$ and $e_t \in L^2(\Gamma)$.

Differentiating (19) with respect to t along the trajectories of (18) yields

$$\begin{aligned}V(t) &= \sigma_1(q_1^*\,;\,e,\,e_t) + \langle e_t,\,e_{tt}\rangle_H + \mu\langle e_t,\,e_t\rangle_H\\ &\quad + \mu\langle e,\,e_{tt}\rangle_H + \sigma_2(\mu q_2^*\,;\,e,\,e_t)\\ &\quad + \frac{1}{\gamma}(\dot{\tilde{\theta}}_1\tilde{\theta}_1 + \dot{\tilde{\theta}}_2\tilde{\theta}_2 + \dot{\tilde{\theta}}_3\tilde{\theta}_3)\\ &= \sigma_1(q_1^*\,;\,e,\,e_t) + \sigma_1(q_3\tilde{\theta}_1\,;\,e,\,e_t)\\ &\quad + \sigma_1(q_3\tilde{\theta}_1\,;\,v,\,e_t) - \sigma_1(q_1^*\,;\,e,\,e_t)\\ &\quad + \sigma_2(q_3\tilde{\theta}_2\,;\,e_t,\,e_t) + \sigma_2(q_3\tilde{\theta}_2\,;\,v_t,\,e_t)\\ &\quad - \sigma_2(q_2^*\,;\,e_t,\,e_t) + q_3\tilde{\theta}_3\langle g,\,e_t\rangle\\ &\quad + \mu\langle e_t,\,e_t\rangle + \sigma_1(\mu q_3\tilde{\theta}_1\,;\,e,\,e)\\ &\quad + \sigma_1(\mu q_3\tilde{\theta}_1\,;\,v,\,e) - \sigma_1(\mu q_1^*\,;\,e,\,e) \quad (20)\\ &\quad + \sigma_2(\mu q_3\tilde{\theta}_2\,;\,e_t,\,e) + \sigma_2(\mu q_3\tilde{\theta}_2\,;\,v_t,\,e)\\ &\quad - \sigma_2(\mu q_2^*\,;\,e_t,\,e) + \mu q_3\tilde{\theta}_3\langle g,\,e\rangle\\ &\quad + \sigma_2(\mu q_2^*\,;\,e,\,e_t) + \frac{1}{\gamma}(\dot{\tilde{\theta}}_1\tilde{\theta}_1 + \dot{\tilde{\theta}}_2\tilde{\theta}_2 + \dot{\tilde{\theta}}_3\tilde{\theta}_3)\end{aligned}$$

Therefore, we choose the adaptation laws for $\theta_i(t)$, i.e., $\dot{\theta}_i(t)$, which are equal to $\dot{\tilde{\theta}}_i(t)$, as

$$\begin{aligned}\dot{\theta}_1(t) &= -\gamma\{\sigma_1(q_3\,;\,e+v,e_t)\\ &\quad + \sigma_1(\mu q_3\,;\,e+v,e)\},\ \theta_1(0) = \theta_{10} \quad (21a)\\ \dot{\theta}_2(t) &= -\gamma\{\sigma_2(q_3\,;\,e_t+v_t,e_t)\\ &\quad + \sigma_2(\mu q_3\,;\,e_t+v_t,\,e)\},\ \theta_2(0) = \theta_{20} \quad (21b)\\ \dot{\theta}_3(t) &= -\gamma\{q_3\langle g,e_t\rangle + \mu q_3\langle g,e\rangle\},\ \theta_3(0) = \theta_{30} \quad (21c)\end{aligned}$$

Although the adaptation laws (21a)-(21c) contain the unknown parameter q_3, this is not a problem because rq_3 and $r\mu q_3$ are treated as adaptation gains.

Let μ satisfy

$$\mu < \min\{\alpha_1/|K^2 - \alpha_2|,\ 1,\ \alpha_2/K^2\} \quad (22)$$

where K, α_1, and α_2 are defined in (6) and condition (A2), respectively. Then, (20) with the adaptation laws (21a)-(21b), condition (A2), and (6) becomes

$$\dot{V}(t) = -\sigma_1(\mu q_1^*\,;\,e,\,e) - \sigma_2(q_2^*\,;\,e_t,\,e_t) + \mu\langle e_t,\,e_t\rangle$$

$$\leq -\alpha_1|e|_V^2-\alpha_2|e_t|_V^2+\mu|e_t|_H^2 \quad (23)$$
$$\leq -\alpha_1|e|_V^2-\alpha_2|e_t|_V^2+\mu K^2|e_t|_V^2$$
$$=-\alpha_1|e|_V^2-\{\alpha_2-\mu K^2\}|e_t|_V^2$$
$$\leq -c_1\left|\begin{pmatrix} e \\ e \end{pmatrix}\right|_V^2$$
$$\leq 0,$$

where $c_1>0$.

Therefore, the functional $V(t)$ given by (19) is nonincreasing, and is a Lyapunov function since $\dot{V}(t)$ is negative semidefinite. The existence of a Lyapunov function $V(t)$ implies that a set $E_\beta=\{(e,\ e_t,\ \tilde{q}_i)\ ;\ V(t)\leq\beta,\ \beta\in R^+\}$ is positive invariant (Walker, 1980). Hence we have $\left|\begin{pmatrix} e \\ e_t \end{pmatrix}\right|_V\leq\beta'$, $\left|\begin{pmatrix} e \\ e_t \end{pmatrix}\right|_H\leq\beta'$ and $|\tilde{q}_i|\leq\beta'$, for $\forall_t\geq 0$, where β' is a positive constant not depending on time t.

4. Well-Posedness of the Closed Loop System

In this section it will be shown that the nonlinear coupled equations (13), (18), and (21a, b, c) are well-posed, i. e. there exists a unique solution for the coupled system. To pose the problem in first-order form, define operators $A_1(q_1)$, $A_2(q_2)\in L(V,\ V^*)$ as

$$\langle A_i(q_i)\phi,\ \psi\rangle_{V*,V}=\sigma_i(q_i\ ;\ \phi,\psi), \quad (24)$$

for $q_i\in R$, $i=1,2$, and $\phi,\psi\in V$. $L(V,\ V^*)$ denotes the space of bounded linear operators $A_i(q_i)$, $i=1,2$, mapping from V into V^* (Showalter, 1977) and $\langle\cdot,\cdot\rangle_{V*,V}$ denotes the duality pairing between V^* and V. The existence of $A_1(q_1)$ and $A_2(q_2)$ is guaranteed by the boundedness of σ_1 and σ_2, respectively. Let the parameter space be denoted by $Q=R^3$ with an inner product $\langle\cdot,\ \cdot\rangle_Q$ and the corresponding induced norm be $|\cdot|_Q$. For the product spaces $\mathbf{H}=V\times H$ and $\mathbf{V}=V\times V$, which form a Gelfand triple $\mathbf{V}\mapsto\mathbf{H}\mapsto\mathbf{V}^*$ with the embeddings which are dense and continuous, we have that ,

$$|\varphi|_H\leq K_0|\varphi|_V \quad (25)$$

where $K_0>0$ and $\varphi\in\mathbf{V}$.

We first define $\mathbf{A}_1(q_3\tilde{\theta}):\mathbf{V}\to\mathbf{V}^*$ given by

$$\left\langle\mathbf{A}_1(q_3\tilde{\theta})\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix},\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\right\rangle_{\mathbf{V}^*,\mathbf{V}}=\langle A_1(q_3\tilde{\theta}_1)\varphi_1,\ \psi_1\rangle_{V*,V}$$
$$+\langle A_2(q_3\tilde{\theta}_2)\varphi_2,\ \psi_2\rangle_{V*,V}+\frac{1}{2}q_3\tilde{\theta}_3\langle g,\ \psi_2\rangle_{V*,V} \quad (26)$$

for $q_3\tilde{\theta}\in Q$ and $\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix},\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\in\mathbf{V}$. The operator in (26) satisfies

(B1) There exists $\alpha_3>0$ such that

$$\left|\left\langle\mathbf{A}_1(\tilde{q})\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix},\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\right\rangle_{\mathbf{V}^*,\mathbf{V}}\right|\leq\alpha_3|\tilde{q}|_Q\left|\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right|_{\mathbf{V}}\left|\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\right|_{\mathbf{V}},$$

(boundedness)

from (A1) and Schwarz's Inequality for inner products.

(B2) The map $\tilde{q}\to\mathbf{A}_1(\tilde{q})\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}$ from Q into $\mathbf{V}^*$ is linear, (Q-linearity)

for each $\tilde{q}\in Q$ and $\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\in\mathbf{V}$, which follows from (A4).

Then, the error system (18) is rewritten as

$$\left\langle\begin{pmatrix}e_t\\ e_{tt}\end{pmatrix},\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right\rangle+\left\langle\begin{pmatrix}0 & -I\\ A_1(q_1^*) & A_2(q_2^*)\end{pmatrix}\begin{pmatrix}e\\ e_t\end{pmatrix},\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right\rangle$$
$$-\left\langle\mathbf{A}_1(q_3\tilde{\theta})\begin{pmatrix}v\\ v_t\end{pmatrix},\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right\rangle-\left\langle\mathbf{A}_1(q_3\tilde{\theta})\begin{pmatrix}e\\ e_t\end{pmatrix},\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right\rangle=0 \quad (27)$$

To obtain a strong form of the first order system that is appropriate for control purposes, a system operator $\mathbf{A}_0(q^*):Dom(\mathbf{A}_0(q^*))\subset\mathbf{H}\to\mathbf{H}$ is defined as follows:

$$Dom(\mathbf{A}_0(q^*))$$
$$=\left\{\Theta=\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}\in\mathbf{V}:A_1(q_1^*)\phi_1+A_2(q_2^*)\phi_2\in H\right\} \quad (28a)$$

$$\mathbf{A}_0(q^*)=\begin{pmatrix}0 & -I\\ A_1(q_1^*) & A_2(q_2^*)\end{pmatrix} \quad (28b)$$

The operator in (28b) satisfies the following conditions:

(B3) There exists $\alpha_4>0$ such that

$$\left|\left\langle\mathbf{A}_0(q^*)\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix},\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\right\rangle_{\mathbf{V}^*,\mathbf{V}}\right|\leq\alpha_4|q^*|_Q\left|\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right|_{\mathbf{V}}\left|\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\right|_{\mathbf{V}},$$

(boundedness)

for $\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix},\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\in\mathbf{V}$ from condition (A1) and Schwarz's Inequality for inner products, (B4) There exists a $\rho_0>0$ for which

$$\mathrm{Re}\left\langle \mathbf{A}_0(q^*)\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix},\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right\rangle_{\mathbf{V}^*,\mathbf{V}}\geq\rho_0\left|\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right|_{\mathbf{V}}^2,\ \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\in\mathbf{V},$$

(V-coercivity)

from condition (A2).

The operator $-\mathbf{A}_0(q^*)$ restricted to the subspace $Dom(\mathbf{A}_0(q^*))$ is the infinitesimal generator of an analytic semigroup, $\{T_0(t): t\geq 0\}$, of bounded linear operators on $\mathbf{H}$ (Banks and Ito, 1988). From (25) and (B4) we have that

$$|T_0(t)\varphi|\leq e^{-\rho_0 K_0^{-2}t}|\varphi|,\ \varphi\in\mathbf{H} \qquad (29)$$

The state error system (18) as a first order system is then given by

$$\langle\mathbf{e}_t,\ \varphi_0\rangle+\langle\mathbf{A}_0(q^*)\mathbf{e},\ \varphi_0\rangle-\langle\mathbf{A}_1(q_3\tilde{\theta})\mathbf{v},\ \varphi_0\rangle -\langle\mathbf{A}_1(q_3\tilde{\theta})\mathbf{e},\ \varphi_0\rangle=0 \qquad (30)$$

where $\mathbf{e}=\begin{pmatrix}e\\ e_t\end{pmatrix}\in\mathbf{H}$, $\mathbf{v}=\begin{pmatrix}v\\ v_t\end{pmatrix}\in\mathbf{H}$,

and $\varphi_0=\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\in\mathbf{V}$

The adaptive laws (21a)-(21c) are rewritten as

$$\dot{\tilde{\theta}}_1\tilde{\theta}_1+\gamma\{\langle A_1(q_3\tilde{\theta}_1)v,e_t\rangle+\langle A_1(q_3\tilde{\theta}_1)e,e_t\rangle +\mu\langle A_1(q_3\tilde{\theta}_1)v,e\rangle+\mu\langle A_1(q_3\tilde{\theta}_1)e,e\rangle\}=0,$$
$$\dot{\tilde{\theta}}_2\tilde{\theta}_2+\gamma\{\langle A_2(q_3\tilde{\theta}_2)v_t,e_t\rangle+\langle A_2(q_3\tilde{\theta}_2)e_t,e_t\rangle +\mu\langle A_2(q_3\tilde{\theta}_2)v_t,e\rangle+\mu\langle A_2(q_3\tilde{\theta}_2)e_t,e\rangle\}=0,$$
$$\dot{\tilde{\theta}}_3\tilde{\theta}_3+\gamma\{q_3\tilde{\theta}_3\langle g,e_t\rangle+\mu q_3\tilde{\theta}_3\langle g,e\rangle\}=0 \qquad (31a, b, c)$$

Let $\mathbf{A}_2(q_3\tilde{\theta}):\mathbf{V}\rightarrow\mathbf{V}^*$ be defined by

$$\left\langle\mathbf{A}_2(q_3\tilde{\theta})\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix},\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\right\rangle_{\mathbf{V}^*,\mathbf{V}} =\langle A_1(q_3\tilde{\theta}_1)\varphi_1,\psi_1\rangle_{V*,V}+\langle A_2(q_3\tilde{\theta}_2)\varphi_2,\psi_1\rangle_{V*,V} +\frac{1}{2}q_3\tilde{\theta}_3\langle g,\ \psi_1\rangle_{V*,V} \qquad (32)$$

then the adaptive laws (31a, b, c) are written as

$$\langle\dot{\tilde{\theta}},\ \tilde{\theta}\rangle_Q+\gamma\{\langle\mathbf{A}_1(q_3\tilde{\theta})\mathbf{v},\ \mathbf{e}\rangle+\langle\mathbf{A}_1(q_3\tilde{\theta})\mathbf{e},\ \mathbf{e}\rangle +\mu[\langle\mathbf{A}_2(q_3\tilde{\theta})\mathbf{v},\ \mathbf{e}\rangle+\langle\mathbf{A}_2(q_3\tilde{\theta})\mathbf{e},\ \mathbf{e}\rangle]\}=0 \qquad (33)$$

where $\tilde{\theta}=(\tilde{\theta}_1,\ \tilde{\theta}_2,\ \tilde{\theta}_3)$. It is noted that the operator $\mathbf{A}_2(\tilde{q})$ also satisfies

(B5) There exists $\alpha_5>0$ such that

$$\left|\left\langle\mathbf{A}_2(\tilde{q})\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix},\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\right\rangle_{\mathbf{V}^*,\mathbf{V}}\right|\leq\alpha_5|\tilde{q}|_Q\left|\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right|_{\mathbf{V}}\left|\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}\right|_{\mathbf{V}},$$

(boundedness)

from condition (A1) and Schwarz's Inequality for inner products.

(B6) The map $\tilde{q}\rightarrow\mathbf{A}_2(\tilde{q})\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}$ from Q into $\mathbf{V}^*$ is linear. (Q-linearity)

The reference model system (12) is rewritten as

$$\left\langle\begin{pmatrix}v_t\\ v_{tt}\end{pmatrix},\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right\rangle+\left\langle\begin{pmatrix}0 & -I\\ A_1(q_1^*) & A_2(q_2^*)\end{pmatrix}\begin{pmatrix}v\\ v_t\end{pmatrix},\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right\rangle=q_3^*\left\langle\begin{pmatrix}0\\ g\end{pmatrix},\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\right\rangle \qquad (34)$$

Then the model system (34) is given by

$$\langle\mathbf{v}_t,\ \varphi_0\rangle+\langle\mathbf{A}_0(q^*)\mathbf{v},\ \varphi_0\rangle=q_3^*\langle g_0,\ \varphi_0\rangle \qquad (35)$$

where $g_0=\begin{pmatrix}0\\ g\end{pmatrix}$. The solution to the initial value problem (35) is given by

$$\mathbf{v}(t)=T_0(t)\mathbf{v}_0+\int_0^t T_0(t-s)g_0(s)\,ds,\ t\geq 0 \qquad (36)$$

First, the following regularity result for the reference model (35) is established.

Theorem 1: For the reference model given by (35), the following results hold:

(i) If $g_0\in L_\infty(0,\ \infty;\mathbf{V})$ and $\mathbf{v}_0\in\mathbf{V}$, then $\mathbf{v}\in L_\infty(0,\ \infty;\mathbf{V})$.

(ii) If $g_0\in L_2(0,\ \infty;\mathbf{V}^*)$, then $\mathbf{v}\in L_\infty(0,\ \infty;\mathbf{H})\cap L_2(0,\ \infty;\mathbf{V})$.

Proof: Statement (i) follows immediately from (29) and (36). To verify (ii), for almost every $t>0$ we have that

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}|\mathbf{v}(t)|_{\mathrm{H}}^2&=\langle-\mathbf{A}_0\mathbf{v}(t)+g_0(t),\ \mathbf{v}(t)\rangle\\ &\leq-\rho_0|\mathbf{v}(t)|_{\mathrm{V}}^2+|g_0(t)|_{\mathrm{V}*}|\mathbf{v}(t)|_{\mathrm{V}}\\ &=-\frac{\rho_0}{2}|\mathbf{v}(t)|_{\mathrm{V}}^2-\frac{1}{2\rho_0}\left(\rho_0|\mathbf{v}(t)|_{\mathrm{V}}-|g_0(t)|_{\mathrm{V}*}\right)^2+\frac{1}{2\rho_0}|g_0(t)|_{\mathrm{V}*}^2\\ &\leq-\frac{\rho_0}{2}|\mathbf{v}(t)|_{\mathrm{V}}^2+\frac{1}{2\rho_0}|g_0(t)|_{\mathrm{V}*}^2\end{aligned} \qquad (37)$$

Integrating both sides of (37) from 0 to t yields:

$$|\mathbf{v}(t)|_{\mathrm{H}}^2+\rho_0\int_0^t|\mathbf{v}(s)|_{\mathrm{V}}^2ds\leq|\mathbf{v}(0)|_{\mathrm{H}}^2+\frac{1}{\rho_0}\int_0^t|g_0(s)|_{\mathrm{V}*},\ t>0.$$

Thus, we have

$$|\mathbf{v}(t)|_{\mathrm{H}}^2+\rho_0\int_0^t|\mathbf{v}(s)|_{\mathrm{V}}^2ds\leq|\mathbf{v}(0)|_{\mathrm{H}}^2+\frac{1}{\rho_0}|g_0(t)|_{L_2(0,\infty,\mathrm{V}*)}^2,\ t>0,$$

from which the second assertion is immediately obtained.

We now combine equations (30), (33), and

(35) in a first-order system as

$$\langle \mathbf{e}_t,\varphi_0\rangle+\langle \mathbf{A}_0(q^*)\mathbf{e},\varphi_0\rangle-\langle \mathbf{A}_1(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\varphi\rangle =0,\ \mathbf{e}(0)=\mathbf{e}_0 \quad (38a)$$

$$\langle \mathbf{v}_t,\varphi_0\rangle+\langle \mathbf{A}_0(q^*)\mathbf{v},\varphi_0\rangle=q_3^*\langle g_0,\varphi_0\rangle,\ \mathbf{v}(0)=\mathbf{v}_0 \quad (38b)$$

$$\langle \tilde{\theta}_t,\tilde{\theta}\rangle_Q+\gamma\{\langle \mathbf{A}_1(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\mathbf{e}\rangle +\mu\langle \mathbf{A}_2(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\mathbf{e}\rangle\}=0,\ \tilde{\theta}(0)=\tilde{\theta}_0 \quad (38c)$$

We consider the system (38a)-(38c) written as

$$\langle \mathbf{e}_t,\varphi_0\rangle+\langle \mathbf{v}_t,\varphi_0\rangle+\langle \tilde{\theta}_t,\tilde{\theta}\rangle_Q+\langle \mathbf{A}_0(q^*)\mathbf{e},\varphi_0\rangle +\langle \mathbf{A}_0(q^*)\mathbf{v},\varphi_0\rangle+\gamma\langle \lambda\tilde{\theta},\tilde{\theta}\rangle_Q =\langle \mathbf{A}_1(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\varphi_0\rangle+q_3^*\langle g_0,\varphi_0\rangle+\gamma\langle \lambda\tilde{\theta},\tilde{\theta}\rangle_Q -\gamma\{\langle \mathbf{A}_1(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\mathbf{e}\rangle+\mu\langle \mathbf{A}_2(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\mathbf{e}\rangle\} \quad (39)$$

Let $\mathbf{H}_0=\mathbf{H}\times\mathbf{H}\times Q$ be endowed with the inner product

$$\langle(\varphi_1,\ \psi_1,\ q_1),(\varphi_2,\ \psi_2,\ q_2)\rangle_{\mathbf{H}_0} =\langle \varphi_1,\ \varphi_2\rangle_{\mathbf{H}}+\langle \psi_1,\ \psi_2\rangle_{\mathbf{H}}+\langle q_1,\ q_2\rangle_Q \quad (40)$$

where $(\varphi_i,\ \psi_i,\ q_i)\in\mathbf{H}_0$, $i=1,2$ and let $|\cdot|_{\mathbf{H}_0}$ denote the corresponding induced norm. Also, let $\mathbf{V}_0=\mathbf{V}\times\mathbf{V}\times Q$ be endowed with the inner product

$$\langle((\varphi_1,\ \psi_1,\ q_1),(\varphi_2,\ \psi_2,\ q_2)\rangle_{\mathbf{V}_0} =\langle \varphi_1,\ \varphi_2\rangle_{\mathbf{V}}+\langle \psi_1,\ \psi_2\rangle_{\mathbf{V}}+\langle q_1,\ q_2\rangle_Q \quad (41)$$

where $(\varphi_i,\ \psi_i,\ q_i)\in\mathbf{V}_0$, $i=1,2$ and let $|\cdot|_{\mathbf{V}_0}$ denote the corresponding induced norm. Then $\mathbf{V}_0$ and $\mathbf{H}_0$ are Hilbert spaces, and $\mathbf{V}_0$ is densely and continuously embedded in $\mathbf{H}_0$. It follows that

$$\mathbf{V}_0 \mapsto \mathbf{H}_0 \mapsto \mathbf{V}_0^* \quad (42)$$

with the embeddings which are dense and continuous.

For $\gamma>0$, define the linear operator $A_\lambda:\mathbf{V}_0\to\mathbf{V}_0^*$ by

$$\langle A_\lambda x,\ \Phi\rangle_{\mathbf{V}_0^*,\mathbf{V}_0}=\langle \mathbf{A}_0(q^*)\mathbf{e},\ \varphi_0\rangle +\langle \mathbf{A}_0(q^*)\mathbf{v},\ \varphi_0\rangle+\gamma\langle \lambda\tilde{\theta},\ \tilde{\theta}\rangle_Q \quad (43)$$

where $x=(\mathbf{e},\ \mathbf{v},\ \tilde{\theta})\in\mathbf{H}_0$, $\Phi=(\varphi_0,\ \varphi_0,\ \tilde{\theta})\in\mathbf{V}_0$. In the above definition, $\langle\cdot,\cdot\rangle_{\mathbf{V}_0^*,\mathbf{V}_0}$ denotes the duality pairing between $\mathbf{V}_0^*$ and $\mathbf{V}_0$.

For $\lambda>0$, define $G_\lambda: R^+\times\mathbf{H}_0\to\mathbf{V}_0^*$ by

$$\langle G_\lambda(t,\ x),\Phi\rangle_{\mathbf{V}_0^*,\mathbf{V}_0}=\langle \mathbf{A}_1(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\varphi_0\rangle +q_3^*\langle g_0,\varphi_0\rangle+\gamma\langle \lambda\tilde{\theta},\ \tilde{\theta}\rangle_Q -\gamma\{\langle \mathbf{A}_1(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\mathbf{e}\rangle +\mu\langle \mathbf{A}_2(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\mathbf{e}\rangle\} \quad (44)$$

For $\varphi\in\mathbf{V}$, define the operator $B_i(\varphi):Q\to\mathbf{V}^*$ by

$$\langle B_i(\varphi)q,\psi\rangle=\langle \mathbf{A}_i(q)\varphi,\psi\rangle \text{ for } i=1,2 \quad (45)$$

where $\psi\in\mathbf{V}$, $q\in Q$. (B1) and (B5) imply that for $\varphi\in\mathbf{V}$, $B_i(\varphi):L(Q,\mathbf{V}^*)$ with $|B_1(\varphi)|\le\alpha_3|\varphi|_{\mathbf{V}}$ and $|B_2(\varphi)|\le\alpha_5|\varphi|_{\mathbf{V}}$. For $\varphi\in\mathbf{V}$, let $B_i(\varphi)'\in L(\mathbf{V},Q)$ denote the adjoint of $B_i(\varphi)$. For φ, $\psi\in\mathbf{V}$, we then have

$$\langle B_i(\varphi)'\psi,q\rangle_Q=\langle B_i(\varphi)q,\psi\rangle=\langle \mathbf{A}_i(q)\varphi,\psi\rangle \text{ for } i=1,2 \quad (46)$$

That is,

$$\langle \mathbf{A}_i(q_3\tilde{\theta})\{\mathbf{e}+\mathbf{v}\},\varphi_0\rangle=\langle B_i(\mathbf{e}+\mathbf{v})q_3\tilde{\theta},\varphi_0\rangle \text{ for } i=1,2.$$

Therefore, (44) is written as

$$\langle G_\lambda(t,x),\Phi\rangle_{\mathbf{V}_0^*,\mathbf{V}_0}=\langle B_1(\mathbf{e}+\mathbf{v})q_3\tilde{\theta},\varphi_0\rangle+q_3^*\langle g_0,\varphi_0\rangle +\gamma\{\langle\lambda\tilde{\theta}-B_1(\mathbf{e}+\mathbf{v})'\mathbf{e},q_3\tilde{\theta}\rangle_Q-\mu\langle B_2(\mathbf{e}+\mathbf{v})\mathbf{e}',q_3\tilde{\theta}\rangle_Q\} \quad (47)$$

Thus, we consider the system (38a)-(38c) written as

$$\langle x_t(t),\Phi\rangle_{\mathbf{V}_0^*,\mathbf{V}_0}+\langle A_\lambda x(t),\Phi\rangle_{\mathbf{V}_0^*,\mathbf{V}_0} =\langle G_\lambda(t,x(t),\Phi\rangle_{\mathbf{V}_0^*,\mathbf{V}_0},x(0)=x_0 \quad (48)$$

where $\lambda>0$, and for each $t\ge0$, $x(t)=(\mathbf{e},\mathbf{v},\tilde{\theta})$, and $\Phi\in\mathbf{V}_0$

To establish the existence of a unique solution to the system (48), we establish the existence of a unique strong solution to the initial value problem in $\mathbf{H}_0$ given by

$$x_t(t)+A_\lambda x(t)=G_\lambda(t,x(t)),\ x(0)=x_0 \quad (49)$$

Note that $Dom(A_\lambda)$ is independent of $\lambda>0$ and that for $\lambda>0$, $-A_\lambda$ is the infinitesimal generator of a uniformly exponentially stable analytic semigroup, $\{T_\lambda(t):t\ge0\}$. Thus, in the usual manner, a unique mild or generalized solution to the system (49) is obtained as

$$x(t)=T_\lambda(t)x_0+\int_0^t T_\lambda(t-s)G_\lambda(s,x(s))\,ds \quad (50)$$

5. Tracking and Parameter Errors Convergence

Theorem 2: Let be defined by (22). Then, from (19), (23) and (50) the following results hold:

$$\lim_{t\to\infty}|e(t)|_H=0 \text{ and } \lim_{t\to\infty}|e_t(t)|_V=0 \quad (51)$$

Proof: Due to the length of the paper, the proof is omitted. However, the assertion can be easily proved by applying Theorem 1 of (Hong, 1997).

Theorem 2 implies that the control objective, i.e., asymptotic tracking, is achieved. Indeed, all signals in the closed loop are bounded and model following is achieved. In addition to the state error convergence to zero, it is also desirable to have an adaptive system to provide parameter error convergence as well, i.e., the controller parameters $\theta_i(t)$, $i=1,2,3$, should converge to the true parameters $\theta_i^*(t)$, respectively, as $t\to\infty$. Parameter error convergence is important in the sense that the adaptive system provides robustness in the presence of disturbance. In order to establish this, the following persistency of excitation condition is introduced.

Definition 3: The reference model (38b) or the triple $\{\mathbf{A}_0, g_0, \mathbf{v}_0\}$ consisting of the reference model dynamics operator $\mathbf{A}_0$, the input reference signal g_0, and the initial state of the reference model $\mathbf{v}_0$, will be said to be persistently exciting, or, sufficiently rich, if there exist positive constants τ_0, δ_0, and ε_0, such that for each $p\in Q$ with $|p|_Q=1$ and $t\geq 0$ sufficiently large, there exists $\bar{t}\in[t,t+\tau_0]$ for which

$$\left\|\int_{\bar{t}}^{\bar{t}+\delta_0}\mathbf{A}_1(p)\mathbf{u}(\tau)\,d\tau\right\|_*\geq\varepsilon_0 \tag{52}$$

where $\mathbf{u}=\mathbf{e}+\mathbf{v}$ is the closed loop state of the plant as given by (38a)-(38c).

Theorem 4: If either $g_0\in L_2(0,\infty;\mathbf{V}^*)$ or $g_0\in L_\infty(0,\infty;\mathbf{V})$ and $\mathbf{v}_0\in\mathbf{V}$, and if the reference model (38b) is persistently exciting, then $\lim_{t\to\infty}|\tilde{\theta}(t)|_Q=0$.

Proof: In this proof we assume that $|\cdot|=|\cdot|_H$, $\|\cdot\|=|\cdot|_V$ and $\|\cdot\|_*=|\cdot|_{V*}$. If $g_0\in L_2(0,\infty;\mathbf{V}^*)$, then Theorem 1 implies that $\mathbf{u}\in L_2(0,\infty;\mathbf{V})$. Lyapunov equation (19) above implies that $\lim_{t\to\infty}|\tilde{\theta}(t)|_Q$ exists. If we assume that $\lim_{t\to\infty}|\tilde{\theta}(t)|_Q\neq 0$, then there exists $\{t_k\}_{k=1}^{\infty}$, an increasing sequence of positive numbers for which $\lim_{k\to\infty}t_k=\infty$ and

$$|q_3\tilde{\theta}(t_k)|_Q\geq\delta,\ k=1,2,\cdots \tag{53}$$

for some $\delta>0$. If the reference model (38b) is persistently exciting, it then follows from (B1) and (B5) that for each $k=1,2,\cdots$ and some $\bar{t}_k\in[t_k,t_k+\tau_0]$, we have

$$0<\delta\varepsilon_0\leq|q_3\tilde{\theta}(t_k)|_Q\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\mathbf{A}_0\left(\frac{q_3\tilde{\theta}(t_k)}{|q_3\theta(t_k)|_Q}\right)\mathbf{u}(t)\,dt\right\|_*$$

$$=\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\mathbf{A}_1(q_3\tilde{\theta}(t_k))\mathbf{u}(t)\,dt\right\|_* \tag{54}$$

$$\leq\alpha_3|q_3\tilde{\theta}|_{L_\infty(0,\infty,Q)}\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|\mathbf{u}(t)\|^2dt\right\}^{1/2}$$

Letting $k=\infty$ in (54), and using the fact that $\mathbf{u}\in L_2(0,\infty;\mathbf{V})$ implies that

$$\lim_{k\to\infty}\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|\mathbf{u}(t)\|^2dt=0.$$

A contradiction is obtained.

Now suppose that $g_0\in L_\infty(0,\infty;\mathbf{V})$ and $\mathbf{v}_0\in\mathbf{V}$. We first recall that Theorem 1 implies that $\mathbf{v}\in L_\infty(0,\infty;\mathbf{V})$. Now, for $t_2>t_1$, (38a), (B3), and (25) imply that

$$\left\|\int_{t_1}^{t_2}\mathbf{A}_1(q_3\tilde{\theta}(t))\mathbf{u}(t)\,dt\right\|_*$$
$$=\left\|\int_{t_1}^{t_2}\mathbf{A}_1(q_3\tilde{\theta}(t))\{\mathbf{e}(t)+\mathbf{v}(t)\}dt\right\|_*$$
$$\leq\|\mathbf{e}(t_2)\|_*+\|\mathbf{e}(t_1)\|_*+\int_{t_1}^{t_2}\|\mathbf{A}_0\mathbf{e}(t)\|_*dt$$
$$\leq K_0|\mathbf{e}(t_2)|+K_0|\mathbf{e}(t_1)|+\alpha_4(t_2-t_1)^{1/2}\left\{\int_{t_1}^{t_2}\|\mathbf{e}(t)\|^2dt\right\}^{1/2} \tag{55}$$

Also, from (38c), (B1) and (B5) it follows that

$$|\tilde{\theta}(t_2)-\tilde{\theta}(t_1)|_Q=\sup_{|p|_Q\leq 1}|\langle\tilde{\theta}(t_2)-\tilde{\theta}(t_1),p\rangle_Q|$$
$$=\sup_{|p|_Q\leq 1}\left|\left\langle\int_{t_1}^{t_2}\tilde{\theta}_t(t)\,dt,p\right\rangle_Q\right|$$
$$\leq\int_{t_1}^{t_2}\sup_{|p|_Q\leq 1}\gamma|\langle\mathbf{A}_1(p)\{\mathbf{e}(t)+\mathbf{v}(t)\},\mathbf{e}(t)\rangle|dt$$
$$+\int_{t_1}^{t_2}\sup_{|p|_Q\leq 1}\gamma\mu|\langle\mathbf{A}_2(p)\{\mathbf{e}(t)+\mathbf{v}(t)\},\mathbf{e}(t)\rangle|dt$$
$$\leq\gamma\alpha_3\int_{t_1}^{t_2}\{\|\mathbf{e}(t)\|+\|\mathbf{v}(t)\|\}\|\mathbf{e}(t)\|dt$$
$$+\gamma\mu\alpha_5\int_{t_1}^{t_2}\{\|\mathbf{e}(t)\|+\|\mathbf{v}(t)\|\}\|\mathbf{e}(t)\|dt$$
$$\leq\gamma\alpha_3\int_{t_1}^{t_2}\|\mathbf{e}(t)\|^2dt+\gamma\alpha_3\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}$$
$$\int_{t_1}^{t_2}\|\mathbf{e}(t)\|dt+\gamma\mu\alpha_5\int_{t_1}^{t_2}\|\mathbf{e}(t)\|^2dt$$
$$+\gamma\mu\alpha_5\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}\int_{t_1}^{t_2}\|\mathbf{e}(t)\|dt$$
$$\leq\gamma\alpha_3\int_{t_1}^{t_2}\|\mathbf{e}(t)\|^2dt+\gamma\alpha_3\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}$$
$$(t_2-t_1)^{1/2}\left\{\int_{t_1}^{t_2}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}$$
$$+\gamma\mu\alpha_5\int_{t_1}^{t_2}\|\mathbf{e}(t)\|^2dt+\gamma\mu\alpha_5\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}$$
$$(t_2-t_1)^{1/2}\left\{\int_{t_1}^{t_2}\|\mathbf{e}(t)\|^2dt\right\}^{1/2} \tag{56}$$

Once again assume that $\lim_{t\to\infty}|\tilde{q}(t)|_Q\neq 0$, and let $\{t_k\}_{k=1}^{\infty}$ be an increasing sequence of positiv e numbers for which $\lim_{k\to\infty} t_k=\infty$ and for which (53) holds for some $\delta>0$. Assume further that the reference model (38b) is persistently exciting, and for each , $k=1,2,\cdots$, let $\bar{t}_k\in[t_k, t_k+\tau_0]$ be such that

$$\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\mathbf{A}_1\left(\frac{q_3\tilde{\theta}(t_k)}{|q_3\tilde{\theta}(t_k)|_Q}\right)\mathbf{u}(t)\,dt\right\|_*\geq\varepsilon_0. \quad (57)$$

Then, using (53), (55), (56), (57), and (B1), (B2), (B5) and (B6), we obtain the estimate

$$0<\delta\varepsilon_0\leq|q_3\tilde{\theta}(t_k)|_Q\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\mathbf{A}_1\left(\frac{q_3\tilde{\theta}(t_k)}{|q_3\tilde{\theta}(t_k)|_Q}\right)\mathbf{u}(t)\,dt\right\|_*$$
$$=\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\mathbf{A}_1(q_3\tilde{\theta}(t_k))\,\mathbf{u}(t)\,dt\right\|_*$$
$$\leq\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\mathbf{A}_1(q_3\tilde{\theta}(t))\,\mathbf{u}(t)\,dt\right\|_*$$
$$+\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\mathbf{A}_1(q_3\tilde{\theta}(t_k)-q_3\tilde{\theta}(t))\{\mathbf{e}(t)+\mathbf{v}(t)\}dt\right\|_*$$
$$\leq K_0|\mathbf{e}(\bar{t}_k+\delta_0)|+K_0|\mathbf{e}(\bar{t}_k)|+\alpha_4\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}$$
$$+\alpha_3\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}q_3|\tilde{\theta}(t)-\tilde{\theta}(t_k)|_Q\{\|\mathbf{e}(t)\|+\|\mathbf{v}(t)\|\}dt$$
$$\leq K_0|\mathbf{e}(\bar{t}_k+\delta_0)|+K_0|\mathbf{e}(\bar{t}_k)|+\alpha_4\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}$$
$$+\alpha_3q_3|\tilde{q}(\bar{t}_k+\tau_0+\delta_0)-\tilde{q}(\bar{t}_k)|_Q\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}$$
$$\{\|\mathbf{e}(t)\|+\|\mathbf{v}(t)\|\}dt$$
$$\leq K_0|\mathbf{e}(\bar{t}_k+\delta_0)|+K_0|\mathbf{e}(\bar{t}_k)|$$
$$+\alpha_4\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}+\alpha_3q_3\times$$
$$\left[\begin{array}{l}\gamma\alpha_3\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt+\gamma\alpha_3\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}\\ \sqrt{\tau_0+\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}\\ +\gamma\mu\alpha_5\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt+\gamma\mu\alpha_5\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}\\ \sqrt{\tau_0+\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}\end{array}\right]$$
$$\times\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\{\|\mathbf{e}(t)\|+\|\mathbf{v}(t)\|\}dt$$
$$\leq K_0|\mathbf{e}(\bar{t}_k+\delta_0)|+K_0|\mathbf{e}(\bar{t}_k)|$$
$$+\alpha_4\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}+\gamma\alpha_3q_3\times$$
$$\alpha_3\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt+\alpha_3\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}$$
$$\sqrt{\tau_0+\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}$$
$$+\mu\alpha_5\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt+\mu\alpha_5\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}$$
$$\sqrt{\tau_0+\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}$$
$$\times\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}+\delta_0\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)} \quad (58)$$

Now consider the functional given by (19) ; (23) can then be rewritten as

$$\begin{aligned}\dot{V}(t)&=-\sigma_1(\mu q_1^*\,;e,e)-\sigma_2(q_2^*\,;e_t,e_t)+\mu(e_t,e_t)\\ &\leq-\alpha_1|e|_V^2-\alpha_2|e_t|_V^2+\mu|e_t|_H^2 \qquad (59)\\ &\leq-\alpha_1|e|_V^2-\alpha_2|e_t|_V^2+K^2\mu|e_t|_V^2\\ &=-\alpha_1|e|_V^2-\{\alpha_2-K^2\mu\}|e_t|_V^2\\ &\leq-c_1\left|\begin{pmatrix}e\\ e_t\end{pmatrix}\right|_V^2\end{aligned}$$

From (59) we have that

$$V(t)+c_0\int_0^t|\mathbf{e}(s)|_V^2ds\leq\zeta_0,\ t\geq0, \quad (60)$$

where $c_0\in R^+$ and $\zeta_0=V(0)$. (60) implies that for any $L>0$ $\lim_{t\to\infty}\int_t^{t+L}\|\mathbf{e}(s)\|^2ds=0$. Therefore, letting $k\to\infty$ in (58), Theorem 2 and (60) imply that

$$0<\delta\varepsilon_0$$
$$\leq K_0\lim_{k\to\infty}|\mathbf{e}(\bar{t}_k+\delta_0)|+K_0\lim_{k\to\infty}|\mathbf{e}(\bar{t}_k)|$$
$$+\alpha_4\sqrt{\delta_0}\left\{\lim_{k\to\infty}\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}+\gamma\alpha_3q_3\times$$
$$\left[\lim_{k\to\infty}\alpha_3\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt+\alpha_3\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}\right.$$
$$\sqrt{\tau_0+\delta_0}\left\{\lim_{k\to\infty}\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}$$
$$+\lim_{k\to\infty}\mu\alpha_5\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt+\mu\alpha_5\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}$$
$$\left.\sqrt{\tau_0+\delta_0}\left\{\lim_{k\to\infty}\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}\right]$$
$$\times\left[\sqrt{\delta_0}\left\{\lim_{k\to\infty}\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|\mathbf{e}(t)\|^2dt\right\}^{1/2}+\delta_0\|\mathbf{v}(t)\|_{L_\infty(0,\infty;V)}\right]$$
$$=0,$$

which is a contradiction, and the theorem is proved.

6. Numerical Simulations

In this section, the proposed MRAC scheme developed in Sections 2-5 is simulated for the shape control of a flexible beam. The flexible beam is assumed to have Kelvin-Voigt damping (c_DI), both ends fixed, a constant flexural rigidity (EI), and uniform mass per unit length (ρ). ρ, EI, and c_DI are unknown, but for simulation purposes the values are taken as $5Kg/m$, $1.5Nm^2$,

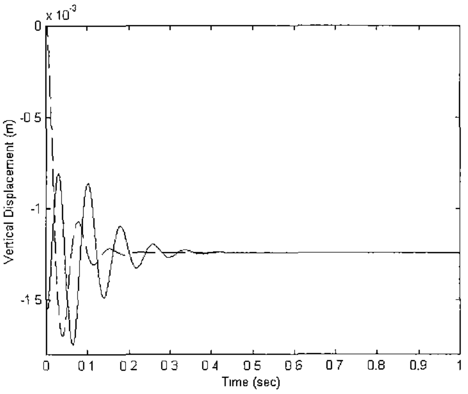

Fig. 2 Vertical displacements at the middle point of the beam: solid line for plant (12) and dashed line for reference model (13)

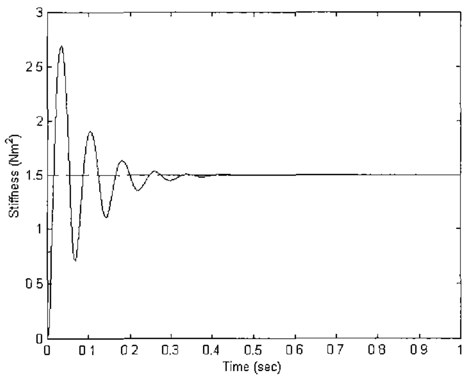

Fig. 3 Convergence to the true stiffness coefficient $EI=1.5\ Nm^2$

and $0.033Nm^2$, respectively. The reference model parameters are chosen as $q_1^*=2.727$, $q_2^*=0.019$, $q_3^*=0.909$. A reference input $g(x,t)=10+2\sin(10\pi t)+\cos(5\pi t)N$ is applied, assuming that the response $\{v,v_t\}$ are the desired characteristics. The adaptive gains in (21a-c) are selected to be $\gamma=200$ and $\mu=0.02$.

Figure 2 shows the deflection of the beam at the middle point. The solid line indicates the deflection of the actual beam and the dashed line refers that of the reference model. It is seen that the difference between these two lines becomes zero, i.e., the convergence of the state error $e(x,t)$ to zero is shown. Figures 3~Figures 5 show the convergence of the tuning plant parameters to the actual values $EI=1.5$, $c_DI=0.033$, $\rho=5$, respectively.

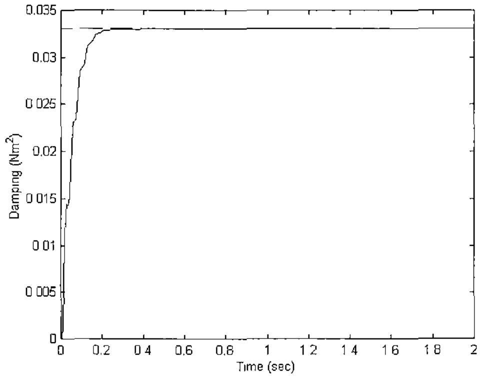

Fig. 4 Convergence to the true damping coefficient $c_DI=0.033\ Nm^2$

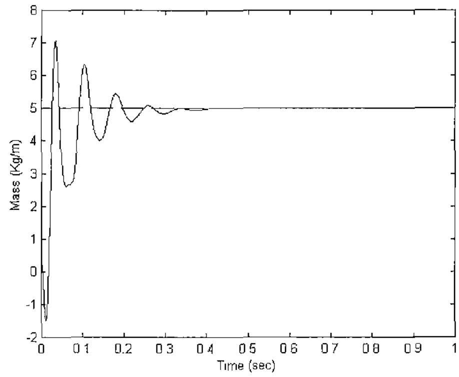

Fig. 5 Convergence to the true mass density $\rho=5\ Kg/m$

7. Conclusions

In this paper, the direct model reference adaptive control for a flexible structure, i.e. an Euler-Bernoulli beam with Kelvin-Voigt damping, was investigated. The stability of the closed loop system together with proposed adaptation laws was established by considering an appropriate Lyapunov function. The well-posedness of the closed loop system was established by treating the closed loop system as a semilinear system with a linear component of the dynamics being the infinitesimal generator of an analytic semigroup. It was shown through the Lyapunov redesign approach that the state error actually converges asymptotically to zero. With the additional assumption on the reference model that the input signal is persistently exciting, parameter-error convergence was also established. Even though distributed sensing and actuation were assumed in this work, the authors believe that the approximation of the control law derived by using the original PDE, rather than approximating the PDE and deriving a control law, can perform better because spillover problems appearing in modal control can be eliminated. The issue of finite-dimensionalization of a distributed control law is an important research topic. Other issues related to the stability and performance of finite-dimensionalized controllers are also left for future work.

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