Sliding Mode Control for Attitude Tracking of Thruster-Controlled Spacecraft

Yee-Jin Cheon

Abstract: Nonlinear pulse width modulation (PWM) controlled system is considered to achieve control performance of thruster-controlled spacecraft. The actual PWM controlled motions occur, very closely, around the average model trajectory. Furthermore nonlinear PWM controller design can be directly applied to thruster controlled spacecraft to determine thruster on-time. Sliding mode control for attitude tracking of three-axis thruster-controlled spacecraft is presented. Simulation results are shown which use modified Rodrigues parameters and sliding mode control law to achieve attitude tracking of a three-axis spacecraft with thrusters.

Keywords: sliding mode, pulse width modulation, modified Rodrigues parameters, nonlinear system, average model

I. Introduction

In a great variety of practical cases, the control signal in variable structure systems(VSS) is a pulse-modulated signal(in particular, pulse-amplitude, pulse-frequency or pulse-width modulated control signal). Variable structure systems in which the control signal is a train of pulses constitute a sub-class of such systems, which poses specific properties. A natural name for such system is referred to as sampled-data variable structure systems.

PWM controlled systems constitute a sub-class of nonlinear sampled-data control system. The sampled output error, which is difference between the desired and the actual system output signals, is translated into a pulse control signal whose pulse width is proportional to the error signal. Typically, PWM controlled systems, as VSS in sliding mode, are known as robust with respect to parameter variations of system and external perturbation signal[1].

Sira-Ramirez has studied a different design approach by using the geometric properties of average PWM controlled system response[4][5]. By allowing a simpler analysis of nonlinear PWM controlled systems through their average response, it is also known that actual PWM controlled system response shows sliding mode trajectories around integral manifolds of the average PWM controlled system model[1]. Furthermore, thruster-controlled spacecraft system constitutes a sub-class of nonlinear PWM controlled system. Therefore nonlinear PWM controller design can be directly applied to the attitude tracking of thruster-controlled spacecraft. In real application, it is very important to determine on-time of thruster because system response directly depends on it.

In this paper, sliding mode control problem for attitude tracking of thruster-controlled spacecraft is considered. Actual thruster on-time is given by PWM controller. Spacecraft control problem is considered based on modified Rodrigues parameters to achieve non-singular attitude description and minimal parameterization. The specification of nonlinear PWM controlled system is made on the basis of the average PWM model. The main contribution of this paper is utilization of nonlinear PWM controlled system that exactly describes the behavior of thruster-controlled spacecraft.

Manuscript received: Mar. 9, 2001., Accepted: Oct. 29, 2001.
Yee-Jin Cheon: Satellite Electronics Dept., Satellite Division, Korea Aerospace Research Institute (yjcheon@kari.re.kr)

II. Theoretical background

In this section, a review of variable structure control and dynamic equations of motions for a three-axis stabilized spacecraft is presented.

1. Attitude kinematics and dynamics

The attitude of spacecraft is assumed to be presented by quaternion. The quaternion representation is defined as

$$\beta_0 = \cos\frac{\theta}{2} \tag{1a}$$

$$\beta_i = n_i \sin\frac{\theta}{2}, \qquad i = 1,2,3 \tag{1b}$$

where n_i is an unit vector corresponding to the axis of rotation, and θ is the angle of rotation. For minimal parameterization of the attitude, we use modified Rodrigues parameters(MRP) which is derived by applying stereographic projection of the quaternions. The transformation from quaternion to MRP vector $\boldsymbol{\sigma}$ is given by [2].

$$\sigma_i = \frac{\beta_i}{\beta_0 + 1}, \qquad i = 1,2,3 \tag{2}$$

Using Equation (1), the modified Rodrigues parameters are written as

$$\boldsymbol{\sigma} = \tan\frac{\theta}{4}\hat{\boldsymbol{n}} \tag{3}$$

Studying Equation (3), it is evident that the MRP have a geometric singularity at $\theta = \pm 360$ degrees. But the non-uniqueness of the MRP allows one to avoid their singularities[2]. The MRP kinematic differential equation in vector form by using spacecraft's body angular velocity ($\boldsymbol{\omega}$), is

$$\dot{\boldsymbol{\sigma}} = \frac{1}{4}\left[(1-\boldsymbol{\sigma}^2)\boldsymbol{I}_{3\times3} + 2[\boldsymbol{\sigma}\times] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T\right]\boldsymbol{\omega} = B(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{4}$$

where the notation $\boldsymbol{\sigma}^2 = (\boldsymbol{\sigma}^T\boldsymbol{\sigma})$ is used, and $\boldsymbol{I}_{3\times3}$ is the 3×3 identity matrix. The cross matrix operator $[\boldsymbol{\sigma}\times]$ is defined by

$$[\boldsymbol{\sigma}\times] \equiv \begin{bmatrix} 0 & -\sigma_3 & \sigma_2 \\ \sigma_3 & 0 & -\sigma_1 \\ -\sigma_2 & \sigma_1 & 0 \end{bmatrix} \tag{5}$$

The inverse transformation of $B(\boldsymbol{\sigma})$ in Equation (4) in explicit vector form is given by:

$$B^{-1}(\sigma)=\frac{4}{(1+\sigma^2)^2}\left[(1-\sigma^2)I_{3\times3}-2[\sigma\times]+\sigma\sigma^T\right] \tag{6}$$

Let J be the rigid body inertia matrix, and u be the control torque vector, then Euler's rotational equations of motions for a rigid body are given by

$$\dot{\omega}=f(\omega)+J^{-1}u \tag{7}$$

where

$$f(\omega)=J^{-1}[J\omega\times]\omega \tag{8}$$

2. Variable structure control

The system presented as

$$\dot{x}_1=f_1(x_1,x_2) \tag{9a}$$

$$\dot{x}_2=f_2(x_1,x_2)+g(x_1,x_2)u \tag{9b}$$

is referred to as a *regular form* in variable structure control literature[3], where $x_1\in R^n$, $x_2\in R^m$, and $u\in R^m$. It is possible to specify the sliding manifold $s=0$ as [3]

$$s=x_2-s_0(x_1)=0 \tag{10}$$

where $s_0(x_1)$ is the solution of equation $s=0$ regarding to x_2. The equation of motion along the manifold $s=0$ in Equation (10), or $x_2=s_0(x_1)$ will be of the form.

$$\dot{x}_1=f_1(x_1,s_0(x_1)) \tag{11}$$

i.e. the sliding equation does not depend on the gradient of vector s and the $s_0(x_1)$ may be therefore chosen arbitrarily. As a result, we face a design problem for the system (9) with an m-dimensional control $s_0(x_1)$ rather than $(m+n)$-dimensional control. A switching surface can be found by designing a nonlinear feedback control law $x_2=s_0(x_1)$ for the system (9a). When designing control law for system (9a), $s_0(x_1)$ can be simply treated as control input to the system.
For especially spacecraft model, dynamic system model is given in state form below:

$$x_1=F(x_1)x_2 \tag{12a}$$

$$\dot{x}_2=f(x_1,x_2)+G(x_1,x_2)u \tag{12b}$$

Systems modeled by Equations (12a) and (12b) are also *regular form*. And the sliding surface is defined below:

$$s=x_2-s_0(x_1)=0 \tag{13a}$$

$$s_0(x_1)=F(x_1)^{-1}f_{1d}(x_1) \tag{13b}$$

where f_{1d} determines the desired evolution form of evolution of x_1.

The nonlinear model for spacecraft motions can be characterized by Equation (4) and Equation (7), and is of *regular form* as Equation (12) with $x_1=\sigma$, $x_2=\omega$, $F=B$ and $G=J^{-1}$. The sliding manifold is defined as Equation (13) with $s_0(\sigma)=B^{-1}(\sigma)d(\sigma)$.

$$s=\omega-s_0(\sigma)=\omega-B^{-1}(\sigma)d(\sigma) \tag{14}$$

The $d(\sigma)$ determines the desired form of evolution σ, given by

$$d(\sigma)=\Lambda(\sigma-\sigma_d) \tag{15}$$

where σ_d is a desired final value of the attitude parameter, and Λ is a diagonal matrix with negative elements. Notice that if the state trajectories move onto a sliding manifold ($s=0$), then $\omega=B^{-1}(\sigma)d(\sigma)$ and the kinematic equation governing the attitude parameter evolution becomes

$$\dot{\sigma}=\Lambda(\sigma-\sigma_d) \tag{16}$$

Due to the propertiy of matrix Λ, the decoupled sliding motions and exponential convergence towards the final desired orientation are achieved. Simultaneously, spacecraft's angular velocity ω would tend to zero as desired.

III. Generalization about PWM controlled systems

Consider the nonlinear PWM controlled system, described by:

$$\dot{x}=h(x)=\begin{cases}h_1(x)+h_2(x)sign(e(t_k)) & \text{for } t_k<t\le t_k+\tau(x(t_k))\mathrm{T}\\ h_1(x) & \text{for } t_k+\tau(x(t_k))\mathrm{T}<t\le t_k+\mathrm{T}\end{cases} \tag{17}$$

where $h_1(x)$ and $h_2(x)$ are smooth vector fields defined in R^n and $e(t_k)$ is a known smooth function of x. The t_k is the k-th sampling time(initial instant of the k-th pulse). The duty ratio function $\tau(x)$ is determined in correspondence with the value of sampled state vector $x(t_k)$, and takes values in bounded interval [0 , 1]. The regions where $\tau(x)$ is fixed at either 0 or 1, constitute the saturation regions of the PWM controller. The sampling period T, also known as the duty cycle, is assumed to be constant, and sufficiently small as compared with the time constants associated with the dynamics of the controlled system.
The system in Equation (17) can be expressed, in terms of a switched position function u taking values in the discrete set {-1, 0, +1}, as:

$$\dot{x}=h_1(x)+uh_2(x) \tag{18}$$

where

$$u=\mathrm{PWM}\,\tau(e(t_k))=\begin{cases}sign(e(t_k)) & \text{for } t_k<t\le t_k+\tau(x(t_k))\mathrm{T}\\ 0 & \text{for } t_k+\tau(x(t_k))\mathrm{T}<t\le t_k+\mathrm{T}\end{cases} \tag{19}$$

Notice that the average PWM model of Equations (17) and (18), in the region where $0<\tau(x)<1$, simply described as following:

$$\dot{x}=h_1(x)+h_2(x)\tau(x)sign(e(x)) \tag{20}$$

Corollary 1 [1]

In those regions of the state space where $0<\tau(x)<1$, the state trajectories of the PWM systems represented in Equations

(17)-(19) exhibit a sliding mode behavior about integral manifolds of the average PWM model in Equation (20).

Consider an output error feedback PWM controlled system defined in R^n , and described by

$$\dot{x} = f(x) + g(x)\boldsymbol{u} \quad (21a)$$

$$y = z(x) \quad (21b)$$

$$\boldsymbol{e} = \boldsymbol{y}_d - \boldsymbol{y} \quad (21c)$$

$$\boldsymbol{u} = \text{MPWM}\,\tau(\boldsymbol{e}(t_k)) \quad (21d)$$

where f and g are smooth vector fields, z is output function. The control input $\boldsymbol{u}$ is a discontinuous control vector obtained as the output of a PWM excited by the output error $\boldsymbol{e}$. The sampling process is assumed to occur at fixed time interval T, in other words, $t_{k+1} = t_k + \text{T}$. M is the maximum allowable input magnitude. The PWM operator, $\text{PWM}\,\tau(\boldsymbol{e})$, characterizing an ON-OFF switch, is defined as in Equation (19) with $\tau(\boldsymbol{e}(t_k))$ as the error dependent duty ratio function defined by:

$$\tau(\boldsymbol{e}(t_k)) = \begin{cases} \alpha|\boldsymbol{e}(t_k)| & \text{for } |\boldsymbol{e}(t_k)| \le 1/\alpha \\ 1 & \text{for } |\boldsymbol{e}(t_k)| > 1/\alpha \end{cases} \quad (22)$$

with α is positive constant. Fig. 1 depicts block diagram of nonlinear PWM controlled system.

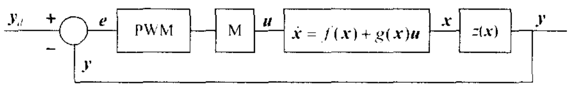


Fig. 1. Nonlinear PWM Controlled System.

Notice that

$$\tau(\boldsymbol{e}(t_k))sign(\boldsymbol{e}(t_k))$$
$$= sat(\boldsymbol{e}(t_k), \alpha) = \begin{cases} \alpha \boldsymbol{e}(t_k) & \text{for } |\boldsymbol{e}(t_k)| \le 1/\alpha \\ sign(\boldsymbol{e}(t_k)) & \text{for } |\boldsymbol{e}(t_k)| > 1/\alpha \end{cases} \quad (23)$$

It is known that the behavior of the infinite frequency sampled system is described by a nonlinear system which includes a continuous piece-wise smooth control input generated as the output of a memoryless nonlinear function of the saturation type[1]. As the sampling frequency 1/T tends to infinity, the description of the nonlinear system in Equation (21), an average model of PWM controlled system, will be the form of

$$\dot{x} = f(x) + g(x)\boldsymbol{v} \quad (24a)$$

$$y = z(x) \quad (24b)$$

$$\boldsymbol{e} = \boldsymbol{y}_d - \boldsymbol{y} \quad (24c)$$

$$\boldsymbol{v} = \text{M}sat(\boldsymbol{e}, \alpha) \quad (24d)$$

Fig. 2 depicts the average model of PWM controlled system corresponding to the system as shown in Fig. 1.

Fig. 2. Average model of PWM controlled system.

IV. Attitude tracking of thruster-controlled spacecraft

The thruster-controlled spacecraft system can be represented as nonlinear, output error feedback PWM controlled system described in Section III. Fig. 3 depicts nonlinear PWM controlled system for thruster-controlled spacecraft system.

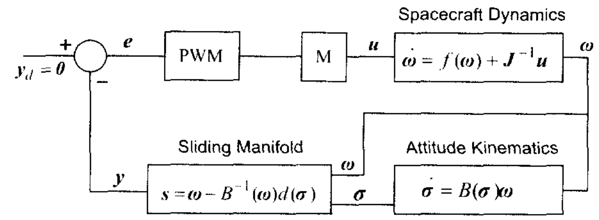


Fig. 3. Sliding mode control for thruster-controlled spacecraft system.

From the Equation (21), the spacecraft system can be described by

$$\dot{\omega} = f(\omega) + \boldsymbol{J}^{-1}\boldsymbol{u} \quad (25a)$$

$$y = s = \omega - B^{-1}(\sigma)d(\sigma) \quad (25b)$$

$$\boldsymbol{e} = \boldsymbol{y}_d - \boldsymbol{y} \quad (25c)$$

$$\boldsymbol{u} = \text{MPWM}\,\tau(\boldsymbol{e}(t_k)) \quad (25d)$$

$$\dot{\sigma} = B(\sigma)\omega \quad (25e)$$

It should be noted that the desired output, $\boldsymbol{y}_d$, becomes zero vector because the sliding manifold is selected as output of the system for feedback purpose. As described in Section II, if the output is driven to $y = 0$ and the state trajectories stay on the manifold ($s = 0$), then the controlled attitude parameters asymptotically converge to $\sigma = \sigma_d$ and spacecraft's angular velocity ω also asymptotically tends to zero.

Theorem 1 [1]

The closed loop PWM controlled system (21) is asymptotically stable toward the manifold $\boldsymbol{e} = 0$, if and only if the average PWM system (24) is asymptotically stable toward such a manifold.

Theorem 2 [1]

If the control law $\boldsymbol{u} = \text{M}sign(\boldsymbol{e})$ creates a sliding regime locally around the manifold $\boldsymbol{e} = \boldsymbol{y}_d - \boldsymbol{y} = \boldsymbol{y}_d - z(x)$, then there exists a sufficiently high gain α of the average PWM operator such that the state trajectories of the average PWM system stabilize toward $\boldsymbol{e} = 0$.

V. Simulation

An example of a three-axis spacecraft maneuver is presented and the simultaneous reorientation of all axes is considered here, and the following inertia matrix is taken from the reference [6].

$$J = diag[114 \quad 86 \quad 87] \quad \text{kg} - \text{m}^2$$

The initial conditions for the angular velocity are set to zero, and also the initial conditions for the modified Rodrigues parameters are set to zero, and given by the following:

$$\sigma(t_0) = [0 \quad 0 \quad 0]^T$$

$$\omega(t_0) = [0 \quad 0 \quad 0]^T$$

The desired attitude parameters are given by

$$\sigma_d = [-0.1 \quad 0.5 \quad 1.0]^T$$

The matrix Λ is set to $-0.015\boldsymbol{I}_{3\times 3}$ $\sec^{-1}$ and the high gain parameter α is chosen as 50, repectively. The sampling frequency in PWM controlled system is set to 1 sample per 0.25 second. Also, the maximum torque of thruster is limited to 1 N-m.

Plots of switching function trajectories are shown in Figs. 4 through 6. Also the comparisons between the angular velocity trajectories of PWM controlled and average model are shown in Figs. 9 through 11. From the Figs. 4 through 6 and 9 through 11, one can clearly know that the actual PWM controlled motions occur, very closely, around the average model trajectory. Plots of the PWM model's angular velocity and the average model's angular velocity trajectories are shown in Figs. 7 and 8, repectively. Fig. 12 depicts time trajectories of modified Rodrigues parameters, and shows that spacecraft rotates from the initial position to the desired position. Also, plots of PWM controller's on-time trajectories are show in Fig. 13.

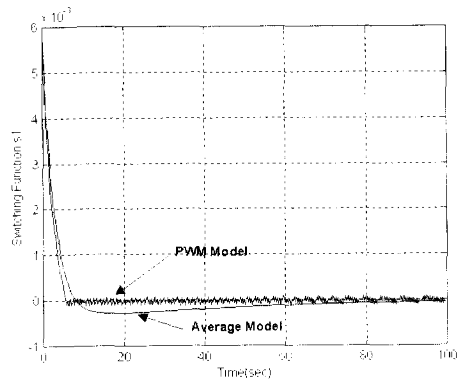


Fig. 4. Plot of switching function trajectories(s_1).

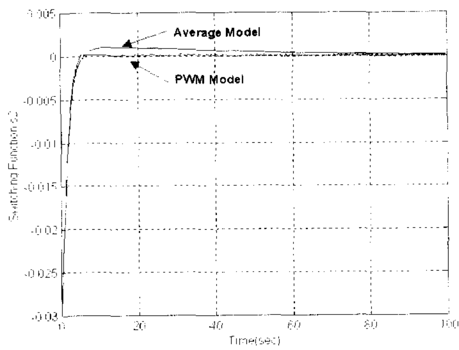


Fig. 5. Plot of switching function trajectories(s_2).

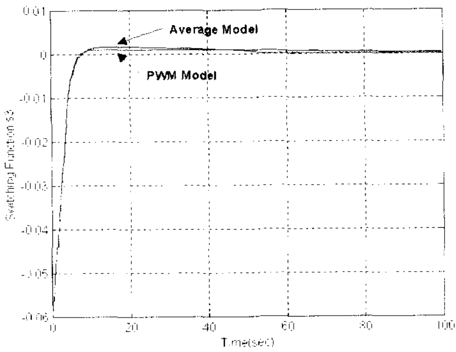


Fig. 6. Plot of switching function trajectories(s_3).

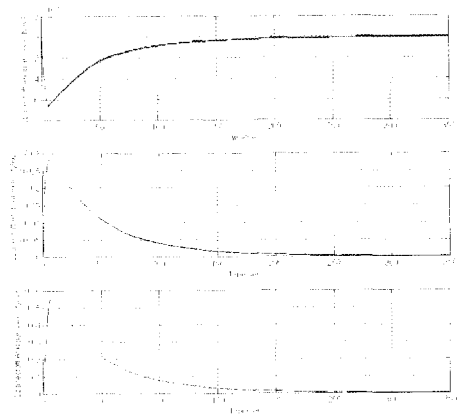


Fig. 7. Plot of angular velocity trajectories (PWM Model).

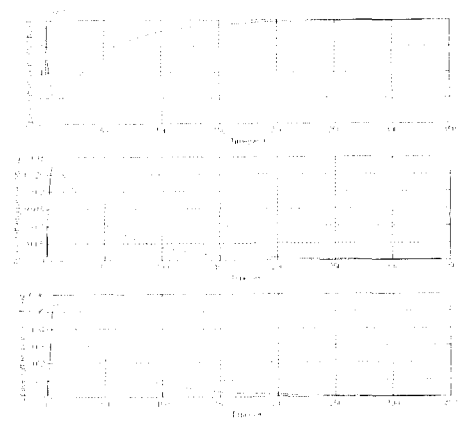


Fig. 8. Plot of angular velocity trajectories (average model)

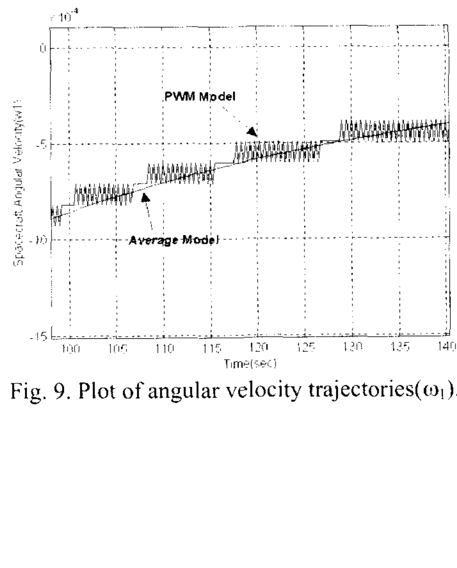


Fig. 9. Plot of angular velocity trajectories(ω_1).

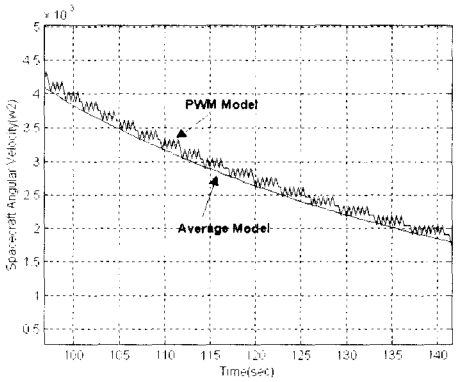


Fig. 10. Plot of angular velocity trajectories(ω_2).

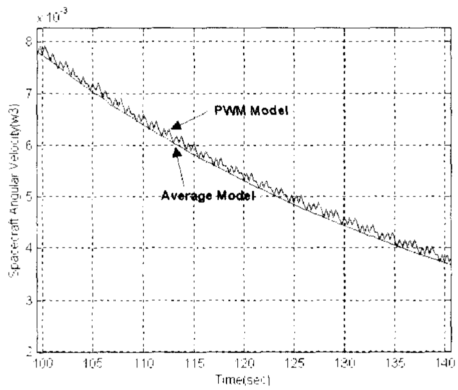


Fig. 11. Plot of angular velocity trajectories(ω_3).

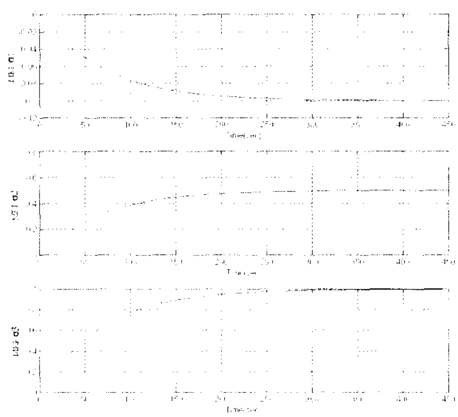


Fig. 12. Plot of modified rodrigues parameters trajectories (PWM model).

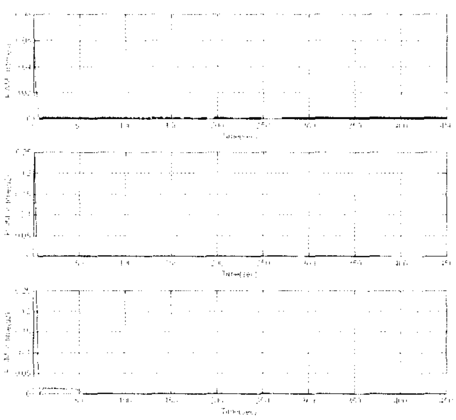


Fig. 13. Plot of PWM on-time trajectories.

VI. Conclusions

The problem of sliding mode control for attitude tracking of three-axis thruster-controlled spacecraft is considered based on nonlinear pulse width modulation controlled system and modified Rodrigues parameters. PWM controller outputs give on-time of thrusters. The sliding motion of PWM controlled system occurs, very closely, around sliding manifolds of the average model of PWM controlled system. Simulation results show the performance of sliding mode controller.

References

[1] H. Sira-Ramirez, "Nonlinear pulse width modulation controller design," *Chapter 3 in the Variable Structure Control for Robotics and Aerospace Applications*, Elsevier, 1993.

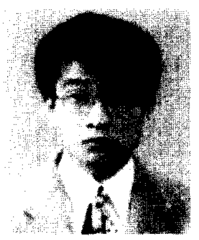
[2] H. Schaub, *Novel Coordinates for Nonlinear Multibody Motion with Applications to Spacecraft Dynamics and Control*, PhD thesis, Texas A&M Univ., May 1998.

[3] V. I. Utkin, *Sliding Modes in Control and Optimization*, Springer-Verlag, Heidelberg, 1992.

[4] H. Sira-Ramirez, "A geometric approach to pulse-width-Modulated control in nonlinear dynamical systems," *IEEE Trans. on Automat. Control*, pp. 184-187, 1989.

[5] H. Sira-Ramirez, "Differential geometric methods in Variable-Structure control," *Int. J. Control*, vol. 48, no. 4, pp. 1359-1390, 1988.

[6] T. A. W. Dwyer and H. Sira-Ramirez, "Variable-Structure control of spacecraft attitude maneuvers," *J. Guidance*, vol. 11, no. 3, pp. 262-270, May-June, 1988.



Yee-Jin Cheon

He was born in Korea on January 15, 1970. He received B.S and M.S degrees from School of Electrical Engineering and Computer Science, Kyungpook National University, Korea in 1993 and 1995. He is a senior researcher of Korea Aerospace Research Institute, Korea.His research interests are real-time software for control and variable structure control of spacecraft attitude tracking.