Design of an Omni-directional Mobile Robot with 3 Caster Wheels

Wheekuk Kim, Do Hyung Kim, Byung-Ju Yi, Bum-Jae You, and Sung-Il Yang

Abstract: In this paper, design of a 3-degree-of-freedom mobile robot with three caster wheels is performed. Initially, kinematic modeling and singularity analysis of the mobile robot is performed. It is found that the singularity can be avoided when the robot has more than two wheels on which two active joints are located. Optimal kinematic parameters of mobile robots with three active joint variables and with four active joint variables are obtained and compared with respect to kinematic isotropic index of the Jacobian matrix of the mobile robot which is functions of the wheel radius and the length of steering link.

Keywords: design, isotropic index, kinematic analysis, mobile robot

I. Introduction

Typical types of wheels used to mobile robots could be expressed into the following four types of wheels : conventional wheel, centered orientable wheel, off-centered orientable wheel("caster wheel"), Swedish wheel(Campion, et al., 1996). Either the conventional wheel or the centered orientable wheel could be modeled kinematically as a two degrees-of-freedom serial chain when the motion along the rotation axle of the wheel is constrained due to the friction. Either the caster wheel or Swedish wheel could be modeled kinematically as a three degrees-of-freedom serial chain.

For the mobile robot to have omni-directional characteristics on the plane, only wheels with three degrees of freedom must be employed to mobile robots. Generally, it is known that either Swedish wheels or other type of "omni-directonal wheels" are very sensitive to road conditions such that its operation speed is reduced significantly, compared to conventional wheels.

In case of the caster wheel, a steering link and joint is attached to the conventional wheel to exert three degrees of freedom. Therefore, similarly to the conventional wheel, it is not sensitive to road conditions either. However, differently from the other wheels, it has configuration dependent singularities.

Thus, in this paper, firstly, kinematic analysis of the mobile robot is performed via. the transfer method of general coordinates, which is a systematic method in modeling parallel mechanisms with respect to various input variables(Campion, et al., 1996). Secondly, singularity analysis of the mobile robot is performed by investigating the determinant of the Jacobian matrix between the various sets of active joint variables and the output variables of the mobile robot. Lastly, kinematic characteristics of mobile robots with three active joint variables and with four active joint variables are investigated and compared via. kinematic isotropic index of the Jacobian matrix of the mobile robot with respect to design parameters such as the wheel radius and the length of steering link, and with respect to operating conditions such as maximum actuator velocities and the maximum output velocities.

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II. Kinematic modeling of the omni-directional robot with three caster

Assume that the motion of the mobile robot is constrained to the plane and there exist no sliding friction and skidding friction but there is friction such that rotation of the wheel about vertical axis is allowed easily.

Denote $(\hat{X}\ \hat{Y}\ \hat{Z})$ and $(\hat{x}_b\ \hat{y}_b\ \hat{z}_b)$ as the reference frame fixed to ground and the body frame fixed to the body of the mobile robot the origin of which is located at O_b, respectively. Note that $\hat{Z} = \hat{z}_b$. And denote the frame $(x_c\ y_c\ z_c)$ as the wheel contact frame the origin of the which is located to the contact point between the wheel and ground as shown in Fig. 1. Define the output velocity of the mobile robot as

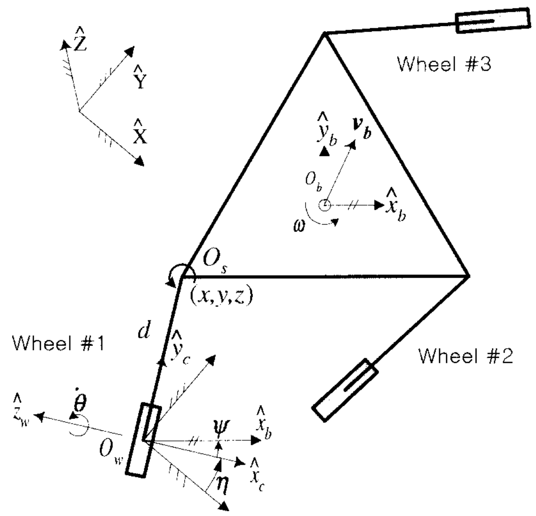


Fig. 1. A mobile robot with three caster wheels.

$$\dot{u} = (v_{bx}, v_{by}, \omega), \tag{1}$$

where $v_b = (v_{bx}, v_{by})^T$ and ω represents the translational velocity of the point O_b and the angular velocity of the body frame about vertical axis.

Consider one caster wheel out of three wheels attached to the mobile robot. Let $\dot{\theta}$ and $\dot{\eta}$ represent the rotational velocity about the wheel axis $\hat{z}_w$ and the rotational velocity about $\hat{z}$, respectively. r, d, and ϕ denotes the radius of the wheel.

the length of the steering link and the relative angular velocity between the steering link and $\hat{x}_b$, respectively. v_w represents the translational velocity of the point O_w, $\overrightarrow{O_w O_s}$ and $\overrightarrow{O_s O_b}$ represents the position vector from O_w to O_s on the steering axis and from O_s to O_b in body frame respectively.

Now, the translational velocity $v_b = (v_{bx}\ v_{by})^T$ and the rotational velocity ω of the mobile robot can be expressed as

$$v_b = v_\omega + \dot{\eta}\hat{Z} \times \overrightarrow{O_w O_s} + \omega \hat{z}_b \times \overrightarrow{O_s O_b}, \tag{2}$$

$$\omega = \dot{\eta} + \dot{\varphi}, \tag{3}$$

where

$$v_w = \dot{\theta}\hat{z}_w \times r\hat{z}_b, \tag{4}$$

$$\hat{z}_w = -\cos\varphi\,\hat{x}_b + \sin\varphi\,\hat{y}_b, \tag{5}$$

$$\overrightarrow{O_w O_s} = d\sin\varphi\,\hat{x}_b + d\cos\varphi\,\hat{y}_b, \tag{6}$$

$$\overrightarrow{O_s O_b} = -(x\hat{x}_b + y\hat{y}_b + z\hat{z}_b), \tag{7}$$

$$\hat{x}_c = \cos\varphi\,\hat{x}_b - \sin\varphi\,\hat{y}_b, \tag{8}$$

$$\hat{y}_c = \sin\varphi\,\hat{x}_b + \cos\varphi\,\hat{y}_b. \tag{9}$$

The above equation can be rewritten, with respect to the output variables $u = (v_{bx}, v_{by}, \omega)^T$ of the mobile robot and the active joint velocity vector $\dot{\phi} = (\dot{\eta}, \dot{\theta}, \dot{\varphi})^T$, as

$$\dot{u} = [G_\phi^u]\dot{\phi}, \tag{10}$$

where

$$[G_\phi^u] = \begin{bmatrix} -d\cos\varphi + y & r\sin\varphi & y \\ d\sin\varphi - x & r\cos\varphi & -x \\ 1 & 0 & 1 \end{bmatrix}. \tag{11}$$

When the matrix $[G_\phi^u]$ is nonsingular, the inverse relationship of (10) can be found by taking the inverse of the matrix $[G_\phi^u]$ as

$$\dot{\phi} = [G_u^\phi]\dot{u}, \tag{12}$$

where

$$[G_u^\phi] = \frac{1}{dr}\begin{bmatrix} -r\cos\varphi & r\sin\varphi & rx\sin\varphi + ry\cos\varphi \\ d\sin\varphi & d\cos\varphi & dx\cos\varphi - dy\sin\varphi \\ r\cos\varphi & -r\sin\varphi & dr - rx\sin\varphi - ry\cos\varphi \end{bmatrix}, \tag{13}$$

Now, let ${}_i\dot{\phi} = (\dot{\eta}_i, \dot{\theta}_i, \dot{\varphi}_i)^T$ $(i = 1,2,3)$ denote joint velocity vector of the i^{th} wheel. And denote the positions of three wheels in the body frame as $(-l/2, -a)$, $(l/2, -a)$ and $(0, b)$, respectively. Then by inserting three position coordinate values into (10)-(13), the velocity relationship between the output of the mobile robot and the joint variables of each of three wheels can be written, respectively, as

$$\dot{u} = [{}_iG_\phi^u]\,{}_i\dot{\phi} \quad (i = 1,2,3), \tag{14}$$

where

$$[{}_1G_\phi^u] = \begin{bmatrix} -d\cos\varphi_1 - a & r\sin\varphi_1 & -a \\ d\sin\varphi_1 + \frac{l}{2} & r\cos\varphi_1 & \frac{l}{2} \\ 1 & 0 & 1 \end{bmatrix}, \tag{15}$$

$$[{}_2G_\phi^u] = \begin{bmatrix} -d\cos\varphi_2 - a & r\sin\varphi_2 & -a \\ d\sin\varphi_2 - \frac{l}{2} & r\cos\varphi_2 & -\frac{l}{2} \\ 1 & 0 & 1 \end{bmatrix}, \tag{16}$$

$$[{}_3G_\phi^u] = \begin{bmatrix} -d\cos\varphi_3 + b & r\sin\varphi_3 & b \\ d\sin\varphi_3 & r\cos\varphi_3 & 0 \\ 1 & 0 & 1 \end{bmatrix}. \tag{17}$$

Note that in the above eqs., the determinant of the matrix $[{}_iG_u^\phi](i = 1,2,3)$ is $-dr$. Thus, unless and $r = 0$ or $d = 0$, the inverse velocity relationship of (14) an be found as

$${}_i\dot{\phi} = [{}^iG_u^\phi]\dot{u}, \tag{18}$$

where

$$[{}^iG_u^\phi] = [{}_iG_\phi^u]^{-1}, \tag{19}$$

$$[{}^1G_u^\phi] = \frac{1}{dr}\times\begin{bmatrix} -r\cos\varphi_1 & r\sin\varphi_1 & -\frac{lr}{2}\sin\varphi_1 - ar\cos\varphi_1 \\ d\sin\varphi_1 & d\cos\varphi_1 & -\frac{dl}{2}\cos\varphi_1 + ad\sin\varphi_1 \\ r\cos\varphi_1 & -r\sin\varphi_1 & dr + \frac{lr}{2}\sin\varphi_1 + ar\cos\varphi_1 \end{bmatrix}, \tag{20}$$

$$[{}^2G_u^\phi] = \frac{1}{dr}\times\begin{bmatrix} -r\cos\varphi_2 & r\sin\varphi_2 & \frac{lr}{2}\sin\varphi_2 - ar\cos\varphi_2 \\ d\sin\varphi_2 & d\cos\varphi_2 & \frac{dl}{2}\cos\varphi_2 + ad\sin\varphi_2 \\ r\cos\varphi_2 & -r\sin\varphi_2 & dr - \frac{lr}{2}\sin\varphi_2 + ar\cos\varphi_2 \end{bmatrix}, \tag{21}$$

$$[{}^3G_u^\phi] = \frac{1}{dr}\times\begin{bmatrix} -r\cos\varphi_3 & r\sin\varphi_3 & -br\cos\varphi_3 \\ d\sin\varphi_3 & d\cos\varphi_3 & -bd\sin\varphi_3 \\ r\cos\varphi_3 & -r\sin\varphi_3 & dr - br\cos\varphi_3 \end{bmatrix}. \tag{22}$$

As an example, suppose that rolling variables (θ_1, θ_2) of the first and the second wheels and the steering variable φ_3 of the third wheel are selected as active input joint vector such as $\phi_a = (\theta_1, \theta_2, \varphi_3)^T$. The inverse input/output velocity relationship of the mobile robot can be found by taking the second equation from (20) and from (21), and the third equation from (22), as

$$\dot{\phi}_a = [G_u^a]\dot{u}, \tag{23}$$

where

$$[G_u^a] = \frac{1}{dr}\begin{bmatrix} d\sin\varphi_1 & d\cos\varphi_1 & -\frac{dl}{2}\cos\varphi_1 + ad\sin\varphi_1 \\ d\sin\varphi_2 & d\cos\varphi_2 & \frac{dl}{2}\cos\varphi_2 + ad\sin\varphi_2 \\ r\cos\varphi_3 & -r\sin\varphi_3 & dr - dr\cos\varphi_3 \end{bmatrix}. \tag{24}$$

Likewise, when the matrix of (24) is nonsingular, the input/output velocity relationship of the mobile robot can be found by taking the inverse of the matrix as

$$\dot{u} = [G_a^u]\dot{\phi}_a, \tag{25}$$

where

$$[G_a^u] = [G_u^a]^{-1}. \tag{26}$$

III. Singularity analysis

Recall in kinematic analysis of the mobile robot performed in section II, that it is assumed, all matrices $[{}_iG_\phi^u]$ (for $i=1,2,3$) and $[G_u^u]$ are not singular. Particularly, for the case of input joint vector of $\phi_a=(\theta_1,\theta_2,\varphi_3)^T$, the determinants of $[{}_iG_\phi^u]$ $(i=1,2,3)$ turn out to be -dr. Thus unless the radius of the wheel and/or the length of the steering link is 0, their inverse matrices always exist. Particularly, when d=0, the wheel becomes an orientable conventional wheel.

The determinant of the matrix in (24) can be obtained as

$$\det([G_u^a])=\frac{1}{dr^2}[\sin(\varphi_2-\varphi_1)\{d-(a+b)\cos\varphi_3\}+\frac{l}{2}\cos\varphi_1\cos(\varphi_3-\phi_2)+\frac{l}{2}\cos\varphi_2\cos(\varphi_3-\varphi_1)\cdot \quad (27)$$

The conditions that the determinant of (27) becomes represent the singularity configuration of the mobile robot with respect to the input joint vector $\phi_a=(\theta_1,\theta_2,\varphi_3)^T$. Fig. 2(a) represents one of these configurations. Geometrically, singularity configurations are identified by checking if a common intersection point among the following three lines exists : 1) the line passing the center line of the steering link of the first wheel, 2) the line passing the center line of the steering link of the second wheel, 3) the line passing through the wheel axis of the third wheel. Fig. (2b) represents the other singularity configuration. In this singularity configuration, those three lines are parallel one another, implying a common intersection point exist at ∞. It can be easily noted that at the singularity configuration in Fig. (2a), the rotational motion about a common intersection point is not possible. Likewise, for the case of Fig. (2b), the translational motion in the orthogonal direction to the three parallel lines is not possible.

Based on the above result, the simple procedure of identifying singularity configurations without obtaining Jacobian matrix corresponding to selected input joint vector could be summarized as follows: 1) at singularity configurations of the mobile robot with respect to the input joint vector there exists either a common intersection point among three lines or three lines in parallel. Those three lines are either line passing through the center line of the steering link when θ_i is set as a input joint variable, or line passing through the wheel axis of the j^{th} wheel when θ_i is set as a input joint variable. As an example, Fig. 3 represents singularity configurations of the mobile robot with input joint vector $\phi_a=(\theta_1,\varphi_2,\theta_3)^T$.

Based on this method of finding singularity configurations, it can be easily noted that every non-redundantly actuated mobile robot with three actuated joints has singularity configurations. Even for the cases of redundantly actuated mobile robot with four active input joints such as $\phi_a=(\theta_i,\varphi_i,\theta_i,\theta_k)^T$, $(\theta_i,\varphi_i,\theta_j,\varphi_k)^T$, $(\theta_i,\varphi_i,\varphi_j,\theta_k)^T$, or $(\theta_i,\varphi_i,\varphi_j,\varphi_k)^T$. the mobile robot has unavoidable singularity configurations through redundant actuation. Fig. 4 represents the singularity configurations for cases of $\phi_a=(\theta_1,\varphi_1,\theta_2,\theta_3)^T$, $(\theta_1,\varphi_1,\theta_2,\varphi_3)^T$, $(\theta_1,\varphi_1,\theta_2,\theta_3)^T$, $(\theta_1,\varphi_1,\varphi_2,\varphi_3)^T$. As can be seen from Fig. 4, there exists a common intersection point of four lines corresponding to four input joint variables.

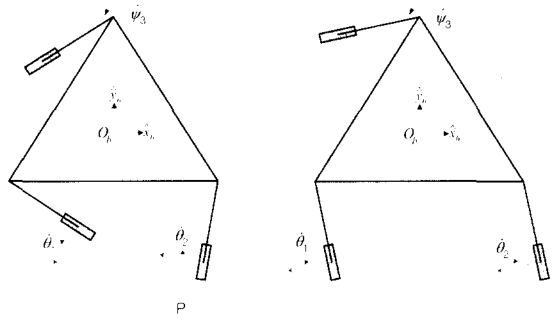


Fig. 2. Singularity configuration for $\phi_a=(\theta_1,\theta_2,\varphi_3)^T$.

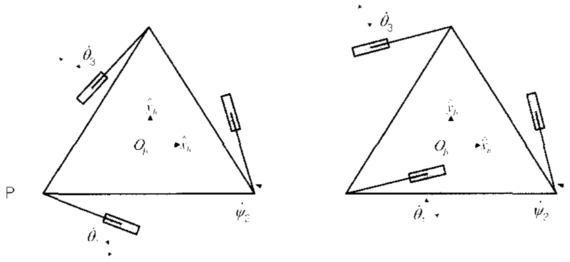
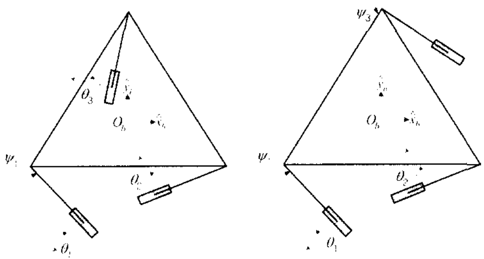
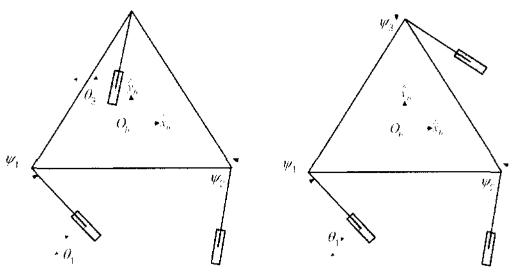


Fig. 3. Singularity configuration for $\phi_a=(\theta_1,\varphi_2,\theta_3)^T$.



(a) $\phi_a=(\theta_1,\varphi_1,\theta_2,\theta_3)^T$ (b) $\phi_a=(\theta_1,\varphi_1,\theta_2,\varphi_3)^T$.



(c) $\phi_a=(\theta_1,\varphi_1,\varphi_2,\theta_3)^T$ (d) $\phi_a=(\theta_1,\varphi_1,\varphi_2,\varphi_3)^T$

Fig. 4. Singularity configuration for four active joints.

However, for the cases of the mobile robot with four actuated joints two of which are attached at one wheel, the other two joints of which are attached at the other wheel, the singularity configurations could be avoided through appropriate redundant actuation. There are only three input joint sets such as $(\theta_1,\varphi_1,\theta_2,\varphi_2)$, $(\theta_1,\varphi_1,\theta_3,\varphi_3)$, and $(\theta_2,\varphi_2,\theta_3,\varphi_3)$. This three input joint sets coincide with the minimal admissible motorization sets that are suggested by Campion et al. [1996]. They mentioned that these three sets are able to move the mobile robot in any direction on the plane

VI. Kinematic characteristics

In this section, optimal locations of the active joints and optimal design parameters are investigated. There are six attachable locations of the active joints to the mobile robot. For convenience of comparison purpose, it is assumed that the mobile robot has symmetric structure to secure better isotropic property. Thus, each of three steering joints is located at the vertices of the equilateral triangle. The active input joint set of the mobile robot with symmetric structure could be categorized as the following six sets: ($\phi_{ai} = (\phi_1,\phi_2,\phi_3)^T$, $(\theta_1,\phi_2,\phi_3)^T$, $(\theta_1,\phi_1,\phi_2)^T$, $(\theta_1,\theta_{21},\varphi_3)^T$, $(\theta_1,\theta_2,\theta_3)^T$.) However, noting the input joint set of $(\phi_1,\phi_2,\phi_3)^T$ does not contain a rolling joint of the wheel, the workspace of the mobile robot is very small and limited. Thus, this set is not considered in the following comparative analysis. Therefore, in this section kinematic characteristics of the mobile robot with these remaining five sets which contains more than one rolling joint of the wheel are investigated and compared.

1. Kinematic performance index

Kinematic design parameters and locations of active input joint set to optimize kinematic isotropic characteristics of the omni-directional mobile robot are investigated. For that purpose, the kinematic isotropic index, implying the degree of uniform input-to-output velocity transmission ratio of the mobile robot at the given configuration and defined as below(Yi and Kim, 1994), are used in simulation:

$$\sigma_I = \frac{\sigma_{\min}([G_a^u])}{\sigma_{\max}([G_a^u])}, \tag{28}$$

where the matrix $[G_a^u]$ represents Jacobian matrix between the input joint vector and the output vector of the mobile robot. And $\sigma_{\min}([G_a^u])$ and $\sigma_{\max}([G_a^u])$ represents the minimum and the maximum singular value of the matrix $[G_a^u]$, respectively.

Noting that kinematic isotropic index given in (28) represents the local isotropic properties at the given configuration, the global kinematic isotropic index, defined as below(Yi and Kim, 1994), are used to compare the kinematic characteristics of the mobile robot with various active joint sets.

$$\sigma_{GI} = \frac{\int \sigma_I(G_{ai}^u])dW}{\int dW}, \tag{29}$$

where $\sigma_I([G_{ai}^u])$ and $W = \int dW$ represents the isotropic index for the Jacobian matrix between the active joint vector and the output of the mobile robot, and the workspace with respect to the steering joint variables of the mobile robot$(\varphi_1,\varphi_2,\varphi_3)$, respectively.

Depending on the capacity of the actuators attached to the mobile robot, the magnitude of the maximum joint velocity may be different. And depending on the desired operation conditions of the mobile robot, the different magnitude of maximum velocity along various output motion direction are required. To take these operation conditions into account, the velocity relationship in (25) could be normalized as follows:

$$\dot{u}^* = [G_{a*}^{u*}]\dot{\phi}_a^*, \tag{30}$$

where

$$[G_{a*}^{u*}] = \begin{bmatrix} \frac{1}{v_{bx\max}} & 0 & 0 \\ 0 & \frac{1}{v_{by\max}} & 0 \\ 0 & 0 & \omega_{\max} \end{bmatrix} [G_a^u] \begin{bmatrix} \phi_{1\max} & 0 & 0 \\ 0 & \phi_{2\max} & 0 \\ 0 & 0 & \phi_{3\max} \end{bmatrix}. \tag{31}$$

and

$$\dot{\phi}_a^* = \left(\frac{\dot{\phi}_1}{\dot{\phi}_{1\max}}, \frac{\dot{\phi}_2}{\dot{\phi}_{2\max}}, \frac{\dot{\phi}_3}{\dot{\phi}_{3\max}} \right)^T. \tag{32}$$

$$\dot{u}^* = \left(\frac{v_{bx}}{v_{bx\max}}, \frac{v_{by}}{v_{by\max}}, \frac{\omega}{\varpi_{\max}} \right)^T. \tag{33}$$

and $\dot{\phi}_{i\max}$, $v_{bx\max}$, $v_{by\max}$, $\omega_{\max}$ represents maximum rotational velocity of i^{th} input joint , the maximum velocity along the $\hat{x}_b$ and $\hat{y}_b$ direction, and the maximum rotational velocity of the mobile robot, respectively.

2. Analysis on kinematic characteristics

In the following simulations, comparative analysis of the mobile robots with respect to the locations of the active input joints are performed. Global kinematic characteristics of the mobile robot are approximated by computing the value of kinematic isotropic index at configurations formed with the interval of 5° for each steering angles $(\varphi_1,\varphi_2,\varphi_3)$ where the interval of5° is selected based on preliminary simulation results that finer intervals do not improve the accuracy of the kinematic characteristics too much but does increase computational burden significantly.

As operational conditions of the mobile robot, the ratio of maximum and minimum actuated joint velocity and the ratio of maximum translational velocity-to-maximum angular velocity of the mobile robot are selected and they are denoted as $\dot{\varphi}_{i\max} / \dot{\theta}_{i\max}$ and $v_{bx\max} / \omega_{\max}$, respectively. For convenience, it is assumed that actuated rolling velocity of each of the wheels, angular velocity of each of the steering joints, and maximum translational velocity of the mobile robot along the direction of $\hat{x}_b$ and $\hat{y}_b$ are assumed as follows: $v_{bx\max} = v_{by\max}$, $\dot{\phi}_{1\max} = \dot{\phi}_{2\max} = \dot{\phi}_{3\max}$

$$\dot{\theta}_{1\max} = \dot{\theta}_{2\max} = \dot{\theta}_{3\max}. \tag{34}$$

Also, without loss of generality, both values of $\dot{\theta}_{i\max}$ and $\omega_{\max}$ are set to be 1. Five different values of these two ratios $\dot{\varphi}_{i\max} / \dot{\theta}_{i\max}$ and $v_{bx\max} / \omega_{\max}$ are considered in the simulation.

There are four design parameters, i.e., radius of the wheel(r), length of the steering link(d), lateral length of the equilateral triangle(l) formed by connecting three connection point of the wheel to the body of the mobile robot and radius of the internal circle to the triangle(a). Note that $b = 2a$, $l = 2\sqrt{3}a$. In the following simulation, varied range of r and d is from $r = 0.1a$ to $r = 10a$, and from $d = 0.1a$ to $d = 10a$, respectively. The interval between neighboring values of those two parameters are divided into 20 sub-sections, i.e., interval of r from $0.1a$ to $1a$, and interval of d from $0.1a$ to $1a$. For convenience, the magnitude of a is set to have unit value.

Values of global isotropic index of the mobile robot are obtained by using isotropic index of the normalized matrix in (31) with respect to each input joint set as follows:

$$\sigma_{GI} = \frac{\int \sigma_I([G_{ai^*}^{u^*}])dW}{\int dW}, \tag{35}$$

Table 1 shows the results from simulation with respect to five different input joint sets. And from the table it can be seen that two input joint sets, $(\theta_1,\varphi_1,\varphi_2)^T$ and $(\theta_1,\varphi_1,\theta_2)^T$, have better global isotropic performance than other sets. As an example, For given operation conditions such as $\dot{\varphi}_{i\max}/\dot{\theta}_{i\max} = 1$ and $v_{bx\max}/\omega_{\max} = 0.5$ the mobile robot with the values of design parameters($r = 0.1a$, and $d = 0.1a$) and actuated by input joint set $(\theta_1,\varphi_1,\theta_2)^T$ have the best global isotropic index value of $\sigma_{GI} = 0.369$. Fig. 5 shows the contour

Table 1. Global isotropic index for various operation objectives.

$\frac{v_{max}}{\omega_{max}}$ / $\frac{\dot\theta_{max}}{\dot\varphi_{max}}$	0.1	0.5	1	5	10
0.1	[0.7|10] (5)<0.33>	[0.1|0.9] (4)<0.34>	[0.1|0.9] (4)<0.25>	[0.2|1.6] (4)<0.04>	[0.2|1.6] (4)<0.03>
0.5	[3.9|10] (5)<0.33>	[0.1|0.2] (4)<0.37>	[0.1|0.2] (4)<0.25>	[0.8|1.3] (4)<0.04>	[1|1.6] (4)<0.03>
1	[7.9|10] (5)<0.32>	[0.1|0.1] (4)<0.37>	[0.1|0.1] (4)<0.25>	[1.6|1.3] (4)<0.04>	[2|1.6] (4)<0.03>
5	[10|3.2] (5)<0.14>	[0.5|0.1] (4)<0.37>	[0.5|0.1] (4)<0.25>	[7.9|1.3] (4)<0.04>	[10|1.6] (4)<0.03>
10	[10|1.3] (5)<0.12>	[1|0.1] (4)<0.37>	[1|0.1] (4)<0.25>	[10|0.9] (4)<0.04>	[10|0.9] (4)<0.03>

* Optimal set (Type)[r|d] < σ_{GI} >
Type 1: $\dot\theta_1,\dot\theta_2,\dot\theta_3$; Type 2: $\dot\theta_1,\dot\theta_2,\dot\varphi_3$; Type 3: $\dot\theta_1,\dot\varphi_2,\dot\varphi_3$;
Type 4: $\dot\theta_1,\dot\varphi_1,\dot\theta_2$; Type 5: $\dot\theta_1,\dot\varphi_1,\dot\varphi_2$

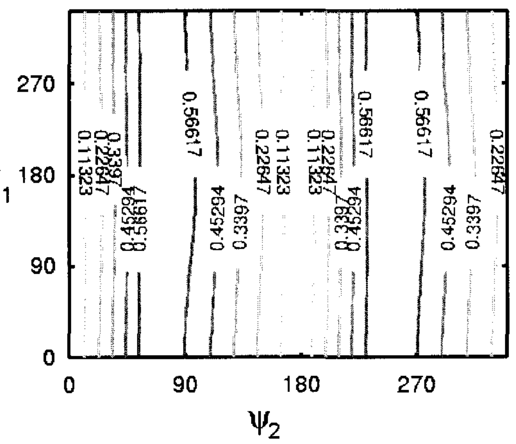


Fig. 5. Isotropic properties when, $r = 0.1a, d = 0.1a$.

plot of the mobile robot with such design parameters for such operation conditions. In the figure, the contour values represent the average value of isotropic index along φ_3 such as $\frac{\int \sigma_I([G_u^a])d\varphi_3}{\int d\varphi_3}$.

V. Kinematic characteristics of redundantly actuated mobile robot

As discussed, the mobile robot could avoid singularity when there are more than two wheels each of which is attached with two active joints. In this section, kinematic characteristics of that type of mobile robots are investigated with respect to operation conditions and design parameters. Due to symmetry, only one set of input joint vector need to be considered.

Velocity relationship between input joint vector ($\phi_a = \theta_1,\varphi_1,\theta_2,\varphi_2$) and the output of the mobile robot is found from (20)-(22) by taking equations corresponding to input joint variables as below:

$$\dot{\phi}_a = [G_u^a]\dot{u}, \tag{36}$$

where

$$[G_u^a] = \frac{1}{dr}\begin{bmatrix} -r\cos\varphi_1 & r\sin\varphi_1 & -\frac{lr}{2}\sin\varphi_1 - ar\cos\varphi_1 \\ d\sin\varphi_1 & d\cos\varphi_1 & -\frac{dl}{2}\cos\varphi_1 + ad\sin\varphi_1 \\ -r\cos\varphi_2 & r\sin\varphi_2 & \frac{lr}{2}\sin\varphi_2 - ar\cos\varphi_2 \\ d\sin\varphi_2 & d\cos\varphi_2 & \frac{dl}{2}\cos\varphi_2 + ad\sin\varphi_2 \end{bmatrix}. \tag{37}$$

Kinematic characteristics of the mobile robots actuated by four input joints are obtained by following the similar procedure in section IV.

Note, however, that since the matrix in (37) are function of ϕ_1 and ϕ_2 only, kinematic isotropic index is independent of ϕ_3. Global kinematic isotropic indices are investigated with respect to the radius of the wheel and the length of the steering link. Table 2 shows the results for different operation conditions and with optimal values of design parameters. Particularly, Fig. 6 shows the isotropic contour plot of the mobile robot with optimal global isotropic characteristics. The mobile robot has the global isotropic index of $\sigma_{GI} = 0.577$ for operation conditions $\dot\phi_{\max}/\dot\theta_{\max} = a$, $\omega_{\max}/v_{\max} = 0.5a$. As expected, the redundantly actuated mobile robot has improved kinematic characteristics over the mobile robot with three active joints.

Similar simulation is performed for a mobile robot having 4 redundant actuators. This case implies that the three wheels are driven by two actuators, respectively. The simulation result is that the value of the global isotropic index becomes maximum when the ratio of r/d is equal to 1 specifically for $\dot\varphi_{\max}/\dot\theta_{\max} = 1$ and $\omega_{\max}/v_{\max} = 0.1\sim10$.

Dynamic optimal design can be also performed which redistributes the masses and inertia of the mobile system. However, this is understood a very complicate procedure since there are many dynamic parameters. Instead, optimal actuator sizing will be more practical way of dynamic design. Optimal actuator sizes are defined as the maximum actuator force required to satisfy the given specification of operational performances such as maximum load, maximum acceleration, and maximum velocity of mobile platform, assuming that the kinematic and the dynamic parameters are already defined. The methodology introduced in Lee, et.al (1997) can be employed for actuator sizing.

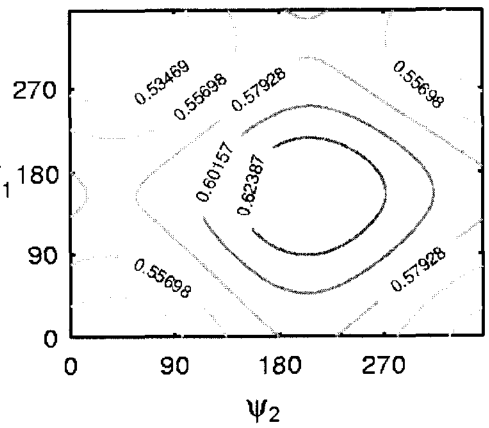


Fig. 6. Isotropic properties when, $r = 0.1a, d = 0.1a$.

Table 2. Global isotropic index for various operation objectives of redundantly actuated.

[illegible]	0.1	0.5	1	5	10
0.1	[0.8|10] <0.453>	[0.1|0.9] <0.526>	[0.1|0.9] <0.368>	[0.1|0.9] <0.08>	[0.1|0.9] <0.04>
0.5	[3.9|10] <0.452>	[0.1|0.2] <0.575>	[0.1|0.2] <0.401>	[0.1|0.2] <0.086>	[0.1|0.2] <0.043>
1	[7.9|10] <0.453>	[0.1|0.1] <0.577>	[0.1|0.1] <0.402>	[0.1|0.1] <0.086>	[0.1|0.1] <0.043>
5	[10|6.3] <0.201>	[0.5|0.1] <0.577>	[0.5|0.1] <0.402>	[0.5|0.1] <0.086>	[0.5|0.1] <0.043>
10	[10|1] <0.178>	[1|0.1] <0.577>	[1|0.1] <0.402>	[1|0.1] <0.086>	[1|0.1] <0.043>

* Optimal set [r|d] < σ_{GI} >

VI. Conclusions

In this paper, design of the omni-directional mobile robot with three caster wheels is performed. Firstly, kinematic models of the 3-degrees-of- freedom mobile robot with 3 caster wheels are derived and its singularity configuration with respect to the locations of active input joints via. its corresponding Jacobian matrix are identified. Secondly, kinematic isotropic characteristics of the mobile robot are investigated and compared with respect to the various operation conditions and design parameters. Also, the kinematic characteristics of the redundantly actuated mobile root are investigated and it is found that as expected, the redundantly actuated mobile robot has improved kinematic characteristics over the mobile robot with three active joints.

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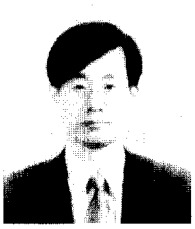

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