

Expanding Generalized Hadamard Matrices over G^m by Substituting Several Generalized Hadamard Matrices over G

Jong-Seon No and Hong-Yeop Song

Abstract: Over an additive abelian group G of order g and for a given positive integer λ , a generalized Hadamard matrix $GH(g, \lambda)$ is defined as a $g\lambda \times g\lambda$ matrix $[h(i, j)]$, where $1 \leq i \leq g\lambda$ and $1 \leq j \leq g\lambda$, such that every element of G appears exactly λ times in the list $h(i_1, 1) - h(i_2, 1), h(i_1, 2) - h(i_2, 2), \dots, h(i_1, g\lambda) - h(i_2, g\lambda)$, for any $i_1 \neq i_2$. In this paper, we propose a new method of expanding a $GH(g^m, \lambda_1) = B = [B_{ij}]$ over G^m by replacing each of its m -tuple B_{ij} with $B_{ij} \oplus GH(g, \lambda_2)$ where $m = g\lambda_2$. We may use $g^m\lambda_1$ (not necessarily all distinct) $GH(g, \lambda_2)$'s for the substitution and the resulting matrix is defined over the group of order g .

Index Terms: Generalized Hadamard matrices, difference matrices, p^m -ary m -sequences.

I. INTRODUCTION

A Hadamard matrix, H_λ , of order λ is a $\lambda \times \lambda$ matrix with elements $+1$'s and -1 's such that $H_\lambda \cdot H_\lambda^T = \lambda I_\lambda$, where I_λ is the identity matrix of order λ [1]–[3]. This implies that any two distinct rows of H_λ are orthogonal. For this reason and many others, Hadamard matrices have been studied in many different but related such areas as wireless communication systems engineering, coding theory, and statistical design theory [3]–[8].

The symbol alphabet can be generalized to a group of order ≥ 2 . In this case, the notion of orthogonality should be suitably modified as in the following definition [4], [9]–[13]. We will restrict our discussion to abelian groups written additively in this paper.

Definition 1: Let G be an additive abelian group of order g . Let $u = (u_1, u_2, \dots, u_{g\lambda})$ and $v = (v_1, v_2, \dots, v_{g\lambda})$ where $u_i, v_j \in G$. Then u and v are said to be *difference-balanced* if every element of G appears exactly λ times in the list of componentwise differences $u_1 - v_1, u_2 - v_2, \dots, u_{g\lambda} - v_{g\lambda}$.

Definition 2: Let G be an additive abelian group of order g . For a positive integer λ , a generalized Hadamard matrix $GH(g, \lambda)$ is a $g\lambda \times g\lambda$ matrix over G in which any two distinct rows are difference-balanced.

Remark 1: $GH(2, \lambda/2)$ is a (binary) Hadamard matrix of order λ .

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J.-S. No is with the School of Electrical Engineering and Computer Science, Seoul National University, Seoul, 151-744, Korea, e-mail: jsno@snu.ac.kr.

H.-Y. Song is with Dept. of Electrical and Electronics Engineering, Yonsei University, Seoul, 120-749, Korea, e-mail: hysong@yonsei.ac.kr.

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Fig. 1. A Hadamard matrix of order 8 from Sylvester construction.

For $\lambda = 1$, an easy example of $GH(g, 1)$ is the group table of the cyclic group of order $g \geq 1$. For any other pair of g and λ , see [4], [9]–[12] for specific constructions.

One of the well-known constructions for (binary) Hadamard matrices was originally from Sylvester [1], [2], [4]. It works as follows: If there exist Hadamard matrices H_m and $H_k = [h_{ij}]$ of orders m and k , respectively, then the matrix obtained by replacing each $h_{ij} = \pm 1$ with $\pm H_m$ is a Hadamard matrix of order mk . A Hadamard matrix of order 8 from Sylvester construction is shown in Fig. 1. Observe that H_8 in Fig. 1 has 16 blocks of order 2 and these blocks are either H_2 or $-H_2$, exclusively. Similarly, there is a construction for generalized Hadamard matrices $GH(g, g\lambda_1\lambda_2)$ over G of order g assuming that there exist $B = [B_{ij}] = GH(g, \lambda_1)$ and $C = GH(g, \lambda_2)$, both over G [10]. It replaces each element B_{ij} of B with $B_{ij} + C$, and the blocks are of the form $a + C$ for $a \in G$. We will also call this Sylvester's method for generalized Hadamard matrices.

In this paper, we propose a new method of expanding a $GH(g^m, \lambda_1) = B = [B_{ij}]$ over G^m by replacing each of its m -tuple B_{ij} with $B_{ij} \oplus GH(g, \lambda_2)$ where $m = g\lambda_2$ and where the operation \oplus will soon be defined in Section II. We may use $g^m\lambda_1$ (not necessarily all distinct) $GH(g, \lambda_2)$'s for the substitution and the resulting matrix is defined over the group of order g .

We will prove the main construction in Section II and give some examples. An example over F_2 is explicitly given using a 4-ary m -sequence of length 15. A brief remark is given in Conclusion.

II. MAIN CONSTRUCTION

Let G be a group of order g . Given a generalized Hadamard matrix $GH(g, \lambda)$ over G , one can transform into another by (i) interchanging any two rows (columns, resp.), and/or (ii) adding

$a \in G$ to every element of a row (column, resp.). Two generalized Hadamard matrices which differ only by some combination of these four operations are said to be *equivalent*. For a given generalized Hadamard matrix we can find an equivalent one in which the top row and the left-most column consist entirely of 0's, the identity of G . Such a generalized Hadamard matrix is called *normalized*. Clearly the remaining rows (if any) must contain every element of G exactly λ times.

Note that if G is a group of order g , then all the m -tuples of elements of G form a group of order g^m with respect to componentwise addition. We will denote this group by G^m throughout this section. We will use \oplus as the operation of G^m . With slight abuse of notation, we will also use \oplus as in the following definition:

Definition 3: (Operation \oplus) Let $X = (x_1, x_2, \dots, x_m)$ be an m -tuple over G and C be a $k \times m$ matrix over G . Denote the rows of C by $\underline{C}_1, \underline{C}_2, \dots, \underline{C}_k$. Then

$$X \oplus C = X \oplus \begin{bmatrix} \underline{C}_1 \\ \underline{C}_2 \\ \vdots \\ \underline{C}_k \end{bmatrix} = \begin{bmatrix} X \oplus \underline{C}_1 \\ X \oplus \underline{C}_2 \\ \vdots \\ X \oplus \underline{C}_k \end{bmatrix} = \begin{bmatrix} x_1 + c_{11}, x_2 + c_{12}, \dots, x_m + c_{1m} \\ x_1 + c_{21}, x_2 + c_{22}, \dots, x_m + c_{2m} \\ \vdots \\ x_1 + c_{k1}, x_2 + c_{k2}, \dots, x_m + c_{km} \end{bmatrix}.$$

The above operation plays the key role in our main construction. As a result of the operation $X \oplus C$, each row \underline{C}_i of C is replaced with $X \oplus \underline{C}_i$. If we put it into another way, in every column of C , for example, in j -th column, the element c_{ij} is replaced with $x_j + c_{ij}$ for $1 \leq i \leq k$, where x_j is the j -th component of X . This proves the following Lemma:

Lemma 1: Let G be a group of order g and let $m = g\lambda$. If X is an m -tuple over G and C is a $GH(g, \lambda)$ over G , then $X \oplus C$ is a $GH(g, \lambda)$ over G .

Theorem 1: (Main) We assume that there exists a generalized Hadamard matrix $B \triangleq [B_{ij}] \triangleq GH(g^m, \lambda_1)$ over G^m where G is a group of order g . We also assume that there exist M generalized Hadamard matrices $C^{(1)}, C^{(2)}, \dots, C^{(M)}$, not necessarily all distinct, all of which are $GH(g, \lambda_2)$ over G . If $M = g^m \lambda_1$ and $m = g\lambda_2$, then the matrix H over G of size $mM \times mM$ obtained by replacing B_{ij} with $B_{ij} \oplus C^{(i)}$ is a generalized Hadamard matrix $GH(g, g^m \lambda_1 \lambda_2)$ over G .

Proof: The resulting matrix H looks like the following:

$$H = \begin{bmatrix} B_{11} \oplus C^{(1)} & B_{12} \oplus C^{(1)} & \dots & B_{1M} \oplus C^{(1)} \\ B_{21} \oplus C^{(2)} & B_{22} \oplus C^{(2)} & \dots & B_{2M} \oplus C^{(2)} \\ B_{31} \oplus C^{(3)} & B_{32} \oplus C^{(3)} & \dots & B_{3M} \oplus C^{(3)} \\ \vdots & \vdots & \dots & \vdots \\ B_{M1} \oplus C^{(M)} & B_{M2} \oplus C^{(M)} & \dots & B_{MM} \oplus C^{(M)} \end{bmatrix}. \tag{1}$$

The matrix H has a natural partition of M^2 submatrices each of size $m \times m$. Thus, we will call successive m rows of H a row-block. That is, H has M row-blocks and each row-block contains M submatrices of size $m \times m$. All we have to show is that any two distinct rows of H of length $L (\triangleq mM)$ over G are difference-balanced in the sense of Definition 1. We will distinguish two cases: (I) two rows from the same row-block and (II) two rows from different row-blocks.

(CASE I) By Lemma 1, each of the $m \times m$ submatrices $B_{ij} \oplus C^{(i)}$ for all i and j is a $GH(g, \lambda_2)$ over G . Therefore, any two distinct rows of H from the same row-block are difference-balanced. In this case, note that we used the assumption that each $C^{(i)}$ is a generalized Hadamard matrix.

(CASE II) Now assume $i \neq j$ and consider x -th row of i -th row-block and y -th row of j -th row-block for some x, y with $1 \leq x \leq m$ and $1 \leq y \leq m$. If we denote x -th row of $C^{(i)}$ and y -th row of $C^{(j)}$ by $\underline{C}_x^{(i)}$ and $\underline{C}_y^{(j)}$, respectively, then the two rows look like the following at the bottom of this page. Note that $B_{ik}, B_{jk}, \underline{C}_x^{(i)}$, and $\underline{C}_y^{(j)}$ are m -tuples over G . If we regard two rows in (2) as vectors of length M over G^m , then they are difference-balanced over G^m since i -th row and j -th row of B are difference-balanced. These differences over G^m are, letting $\underline{C}_x^{(i)} \ominus \underline{C}_y^{(j)} \triangleq D$,

$$B_{ik} \ominus B_{jk} \oplus D, \quad \text{for } k = 1, 2, \dots, M. \tag{3}$$

Note that D is an m -tuple over G and is only a constant bias of the differences $B_{ik} \ominus B_{jk}$, regardless of k . On the other hand, $B_{ik} \ominus B_{jk}$ is computed componentwise, so we may concentrate on the components of the differences in (3). Similarly, we may regard two rows in (2) as vectors of length mM over G , and consider their componentwise differences over G . They are exactly the same as the differences listed in (3) except now that we are looking at the components. They are difference-balanced over G since in the list of M m -tuples over G in which every m -tuple of G^m appears exactly λ_1 times, every element of G appears exactly mM/g times. Note, in this case, that we used the assumption that B is a generalized Hadamard matrix over G^m . \square

index	row			
x-th row of i -th row-block	$B_{i1} \oplus \underline{C}_x^{(i)}$	$B_{i2} \oplus \underline{C}_x^{(i)}$	\dots	$B_{iM} \oplus \underline{C}_x^{(i)}$
y-th row of j -th row-block	$B_{j1} \oplus \underline{C}_y^{(j)}$	$B_{j2} \oplus \underline{C}_y^{(j)}$	\dots	$B_{jM} \oplus \underline{C}_y^{(j)}$

(2)

Remark 2: A generalized Hadamard matrix over a group is a square array. A rectangular array with the same condition on the list of differences of any two distinct rows is called a difference matrix [10]. Our main construction applies easily to the construction of difference matrices, since the proof of Theorem 1 does not use the fact that either B or C is a square array. Therefore, our method directly applies to difference matrices B over G^m and C 's over G to construct larger size difference matrix over G .

Remark 3: We may use in the construction all the same $C^{(i)}$'s, some different but equivalent $C^{(i)}$'s, all distinct $C^{(i)}$'s, or all inequivalent $C^{(i)}$'s. We note that the construction method is different from those by Sylvester even if $C^{(i)}$'s are all the same.

Example 1: We take G to be the additive group of the finite field F_p of p elements for some prime p . For example, we take $G = \{0, 1, 2\}$ to be the additive group of integers mod 3 so that $p = g = 3$. If we have $B = GH(3^m, \lambda_1)$ over G^m and M (not necessarily distinct) $GH(3, \lambda_2)$'s over G where $m = 3\lambda_2$ and $M = 3^m \lambda_1$, the construction gives a $GH(3, mM/3)$ over G .

Example 2: If we have $B = GH(p^m, \lambda_1)$ over F_{p^m} and M (not necessarily distinct) $GH(p, \lambda_2)$'s over F_p where $m = p\lambda_2$ and $M = p^m \lambda_1$, the construction gives a $GH(p, mM/p)$ over F_p . Here, we consider the elements of F_{p^m} as m -tuples over F_p .

For example, consider the case $p = 2$. If $\{b_t | t = 0, 1, 2, \dots, N-1\}$ is a 2^m -ary maximal-length linear feedback shift register sequence [14]–[18] of period $N = 2^{mn} - 1$ for some n , then the $(N+1) \times (N+1)$ matrix given by

$$B \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & b_0 & b_1 & b_2 & \dots & b_{N-1} \\ 0 & b_1 & b_2 & b_3 & \dots & b_0 \\ 0 & b_2 & b_3 & b_4 & \dots & b_1 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & b_{N-1} & b_0 & b_1 & \dots & b_{N-2} \end{bmatrix} \quad (4)$$

Table 1. Substitution Rule for $GH(2, 16)$ over F_2 where $B = GH(4, 4)$ over F_4 .

for the top half of B				for the bottom half of B			
replace 0	with	$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$		replace 0	with	$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$	
replace 1	with	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$		replace 1	with	$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$	
replace β	with	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$		replace β	with	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	
replace β^2	with	$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$		replace β^2	with	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	

is a generalized Hadamard matrix $GH(g^m, \lambda_1) = GH(2^m, 2^{m(n-1)})$ over the additive group of F_{2^m} . Here, 0 and b_t in (4) can be represented [17] as m -tuples over F_2 . Let $M = N + 1 = 2^{mn}$. If we have M (not necessarily all distinct) binary Hadamard matrices $GH(2, m/2)$ over F_2 , then we can construct a $GH(2, mM/2) = GH(2, m2^{mn-1})$ over F_2 , which is a binary Hadamard matrix of size $m2^{mn} \times m2^{mn}$.

Specifically, let $m = 2$ and $n = 2$. Denote $F_4 = \{0, 1, \beta, \beta^2\}$. Then, b_t in (5) is a 4-ary m -sequence of period 15 at the bottom of this page. This gives a $GH(4, 4)$ over F_4 given as (6) at the bottom of this page.

Denote 0, 1, β, β^2 in F_4 by 2-tuples 00, 01, 10, 11 over F_2 , respectively. Suppose we use $C^{(i)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ for $1 \leq i \leq 8$,

and its transpose as $C^{(i)}$ for $9 \leq i \leq 16$, then a 32×32 binary Hadamard matrix is obtained if we replace 0, 1, β, β^2 in B according to Table 1.

III. CONCLUDING REMARKS

A feature of the main construction is that we replace each m -tuple B_{ij} of B with $B_{ij} \oplus C^{(i)}$ for all i . A direction of future research is to investigate equivalence of two generalized

$$t \mid 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \quad (5)$$

$$b_t \mid 0 \ 1 \ 1 \ \beta^2 \ 1 \ 0 \ \beta \ \beta \ 1 \ \beta \ 0 \ \beta^2 \ \beta^2 \ \beta \ \beta^2$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 \\ 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 \\ 0 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 \\ 0 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 \\ 0 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 \\ 0 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 \\ 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 \\ 0 & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta \\ 0 & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta \\ 0 & \beta & 0 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta \\ 0 & 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta \\ 0 & \beta^2 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 \\ 0 & \beta^2 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 \\ 0 & \beta & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 \\ 0 & \beta^2 & 0 & 1 & 1 & \beta^2 & 1 & 0 & \beta & \beta & 1 & \beta & 0 & \beta^2 & \beta^2 & \beta \end{bmatrix} \quad (6)$$

Hadamard matrices obtained by the proposed construction using some different sets of $C^{(i)}$'s.

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Jong-Seon No received the B.S. and M.S.E.E. degrees in electronic engineering from Seoul National University, Seoul, Korea, in 1981 and 1984, respectively, and the Ph.D. degree in electrical engineering from the University of Southern California, Los Angeles, in 1988. He was a Senior MTS at Hughes Network Systems from February 1988 to July 1990. He was also an Associate Professor in the Department of Electronic Engineering, Konkuk University, Seoul, Korea, from September of 1990 to July 1999. He joined the faculty of the School of Electrical Engineering and Computer Science, Seoul National University, Seoul, Korea, in August of 1999, where he is currently working as an associate professor. His area of research interests includes error-correcting codes, pseudo-noise (PN) sequences, difference sets, cryptography, and wireless communication systems.



Hong-Yeop Song received his BS degree in Electronic Engineering from Yonsei University in 1984. MSEE and Ph.D degrees from University of Southern California, Los Angeles, CA in 1986 and 1991, respectively, specializing in the area of communication theory and coding. After spending 2 years as a research staff in the Communication Sciences Institute at USC, he joined Qualcomm Inc., San Diego, CA in 1994 as a senior engineer and worked in a team researching and developing North American CDMA Standards for PCS and cellular air-interface. Finally, he joined the Dept. of Electrical and Electronics Engineering at Yonsei University, Seoul, Korea in 1995, and is currently working as an associate professor. His area of research interest includes spread spectrum communication and the application of discrete mathematics into various communication and coding problems. He is a member of IEEE, MAA(Mathematical Association of America), IEEK, KICS, and KIISC.