論文

비대칭 연단보강재가 설치된 직교이방성 개방단면 압축재의 탄성국부좌굴

윤순종*, 정상균**

Elastic Local Buckling of Orthotropic Open Section Compression Members with Asymmetric Edge Stiffeners

S. J. Yoon*, S. K. Jeong**

ABSTRACT

This paper presents the results of an analytical investigation pertaining to the elastic local buckling behavior of asymmetric edge stiffened orthotropic open section structural member under uniform compression. The asymmetric edge stiffener is considered as a beam element neglecting its torsional rigidity. We suggested the analytical model of asymmetric edge stiffeners which is composed of a strip of flange plate, equal width of edge stiffener, and a plate attached at the flange end, and computed the moment of inertia of the stiffener about an axis through the centroid of the ensuing cross-section. Using the derived equation, the local buckling coefficients of asymmetrically edge stiffened orhtotropic I-section columns are predicted and the results are presented in a graphical form.

초 록

본 연구는 등분포 압축을 받는 비대칭 연단보강된 직교이방성 개방단면 구조용 부재의 탄성국부좌굴에 대한 이론적 연구의 결과이다. 비대칭 연단보강재는 비틀림강성을 무시한 보요소로 간주하였다. 플랜지판의 일부와 플랜지의 연단에 부착된 판으로 구성된 비대칭 연단보강재에 대한 이론적 해석 모델을 제시하였으며, 이 결과로부터 얻어지는 비대칭 연단보강재 단면의 도심을 지나는 축에 대한 단면2차모멘트를 계산하였다. 유도된 식을 사용하여 비대칭 연단보강된 직교이방성 I형 압축재의 국부좌굴계수를 구하였으며 그 결과를 그래프로 제시하였다.

1. INTRODUCTION

Composite materials, such as fiber reinforced plastics (FRP), are partially replacing conventional materials in civil engineering applications which include buildings, transportation systems, sewage and water treatment facilities, off-shore and on-shore structures[1].

This trend is expected to continue due to increasing demand for lightweight, high stiffness and/or strength, nonconducting, and noncorroding materials.

However, a lack of understanding of the fundamental behavior of composite members and a lack of standardized design criteria of these advanced construction materials under various types of service conditions have limited their application.

In recent years, many researches pertaining to analytical

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and experimental investigations for FRP structural members to establish design criteria have been conducted in numerous institutions[2].

Most of structural members used in civil engineering construction have open or closed thin-walled cross section composed of thin flat plate elements. In the design of thin-walled compression members, one of the stability problems to be considered is the local buckling. To date, the theoretical investigation on the local buckling of orthotropic thin-walled sections is relatively well established by many researchers; it is based on the fundamental equilibrium equations for compressed thin plates under various edge conditions, and takes into account the full interaction between the plate components when buckling occurs simultaneously. A problem of reinforcement of the flange free edges has not been considered extensively, because of analytical difficulties in deriving appropriate solutions. The practically applicable solutions are restricted to the case of which the free edges of the flange are reinforced with narrow beam stiffeners with negligible torsional rigidity[3].

Fig. 1 shows some typical types of flange stiffeners. Types (a) and (b) are plain stiffeners often used to strengthening a channel or an I-shape. Type (c) is a bulb, used to provide an extra torsional stiffness to an angle of extruded profile section.

The objective of this paper is to present an analytical solution for local buckling of orthotropic thin-walled I-section column in which the flanges are reinforced with the stiffener of type (a) in Fig. 1.

2. ASYMMETRIC EDGE STIFFENER EFFECT ON THE LOCAL BUCKLING

At present there is virtually no reliable data on the local buckling of edge stiffened section which may be useful for structural design engineers. Many analytical and experimental researches have been conducted by Chilver(1953)[4] and Bulson(1967)[3] for isotropic materials such as steel and/or aluminum alloy.

According to the results of Bulson's analytical investigations, it is found that if the stiffeners are so narrow they are not advantageous or even reduce slightly the initial buckling stress. In contrast, if the width of stiffener is large in comparison with the flange width, it will be stiff enough to hold the stiffener and flange junction straight during buckling, so that the stiffener can itself be treated as a plate element. Therefore, it is necessary to determine appropriate dimensions of edge stiffeners.

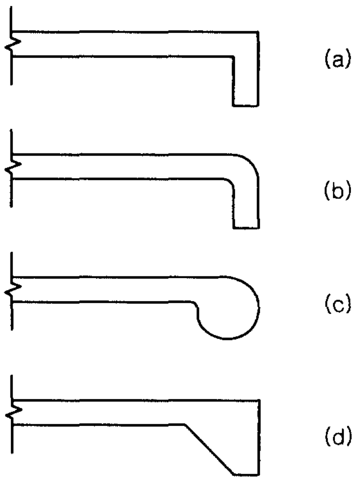


Fig. 1 Types of asymmetric edge stiffeners.

In the analysis, the axis of bending of the edge stiffener has been assumed to coincide with the mid-plane of the flange plate. But in practice, this assumption is only applicable for the case of symmetric stiffener. When the flange is stiffened by the asymmetric stiffener, a conservative recommendation is given by Bulson (1967)[3] that narrow portion of flange, equal width of stiffener, is included in the stiffener and estimating the moment of inertia with respect to the centroidal axis of the resulting section as shown in Fig. 2.

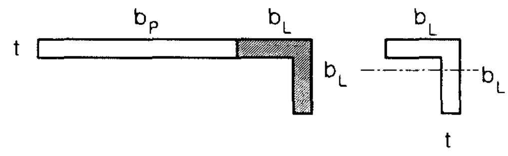


Fig 2. Stiffener cross section.

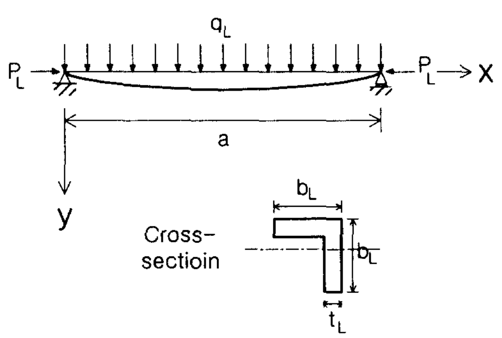


Fig. 3 Asymmetric edge stiffener under axial and lateral forces.

The stiffener considered in this paper is a narrow rectangular section beam element with negligible torsional rigidity. The equilibrium equation of asymmetric stiffener shown in Fig. 3 under axial compression P_L and lateral force q_L is given by Yoon[5];

$$E_{11L} I_L \frac{\partial^4 w_L}{\partial x^4} + P_L \frac{\partial^2 w_L}{\partial x^2} - q_L = 0 \tag{1}$$

where E_{11L} is the elastic modulus of stiffener in axial direction and q_L is the lateral force acting on the stiffener which is transmitted from the flange. I_L is the moment of inertia of the stiffener about an axis through the centroid as shown in Fig. 2.

Assuming that the deflection curve of stiffener along the longitudinal direction as a sinusoidal function and introducing dimensionless parameters $\xi = x/a$, $\eta = y/b$, Eq. (1) can be written in the form;

$$b_L D_{11L} \frac{1}{a^4} \frac{\partial^4 w_L}{\partial \xi^4} + A_L \sigma_x \frac{1}{a^2} \frac{\partial^2 w_L}{\partial \xi^2} - q_L = 0 \tag{2}$$

in which A_L is the area of stiffener ($b_L \cdot t_L$), σ_x is the axial stress (P_L/A_L) and w_L is the deflection curve of stiffener which can be taken in the form considering boundary conditions;

$$w_L = A_i \sin m\pi\xi \tag{3}$$

In Eq. (4), A_i is an arbitrary constant expressing the amplitude of deflection of stiffener.

Lateral force q_L can then be expressed in the form;

$$q_L = A_i \left(\frac{m\pi}{\lambda}\right)^2 \left\{ b_L D_{11L} \left(\frac{m\pi}{\lambda}\right) - A_L \sigma_x \right\} \sin m\pi\xi \tag{4}$$

where, λ is the aspect ratio (a/b_L) of plate attached at the free edge of flange.

3. LOCAL BUCKLING OF ORTHOTROPIC OPEN SECTION COLUMN WITH ASYMMETRIC EDGE STIFFENERS

Local buckling equation of edge stiffened columns presented is the same one available in the references published by authors[6, 7]. Hence, for brevity, only the relevant equations are quoted herein.

Consider a short edge stiffened structural I-shape member of length a, which is composed of five plate components as shown in Fig. 4. This member has two common junctions meeting three plate components and 4 joints meeting the flange plate and the asymmetric edge stiffener.

In addition to the basic assumptions for the local buckling analysis of structural shapes given by Bulson(1955)[8], some assumptions are adopted at the joints meeting the stiffener and flange plate:

(1) the outward deflection of flange plate is equal to the deflection of the stiffener

(2) the intensity of shear force transmitted from the flange plate to the stiffener contributes equilibrium

(3) there is equilibrium of moments about joint and the torsional rigidity of stiffener can be negligibly small.

Considering above assumptions, we can find the local buckling equation of edge stiffened I-shape orthotropic compression members as follows[6]:

$$\left(\frac{SSY}{FSY}\right)_1 + \frac{1}{\delta_2 \omega_2^3}\left[\frac{SFR - SS \cdot LFR}{FFR - FS \cdot LFR}\right]_2 + \frac{1}{\delta_3 \omega_3^3}\left[\frac{SFR - SS \cdot LFR}{FFR - FS \cdot LFR}\right]_3 = 0 \tag{5}$$

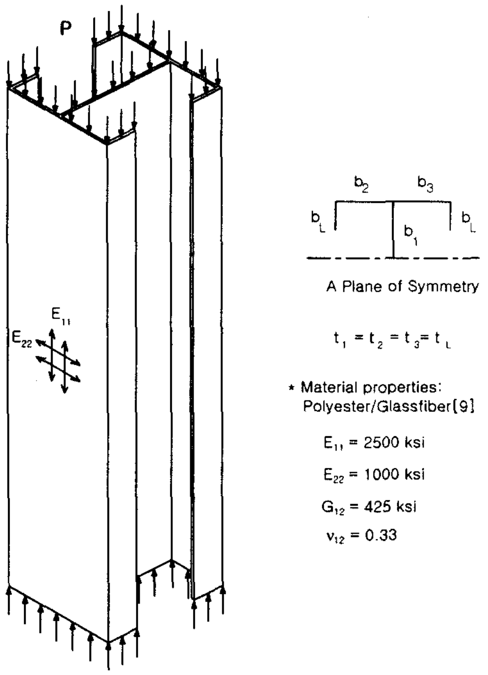


Fig. 4 Edge stiffened orthotropic I-shape section column.

where, SSY, FSY, SFR, FFR, SS, and FS are the transcendental form of buckling equation of plate with the following boundary conditions in unloaded edges, respectively[6, 7]:

SSY : Simply supported at $\eta = 0$,
A plane of symmetry at $\eta = 1/2$

FSY : Fixed support at $\eta = 0$,
A plane of symmetry at $\eta = 1/2$

SFR : Simply supported at $\eta = 0$,
Free at $\eta = 1$

FFR : Fixed support at $\eta = 0$,
Free at $\eta = 1$

SS : Simply supported at $\eta = 0$,
Simply supported at $\eta = 1$

FS : Fixed support at $\eta = 0$,
Simply supported at $\eta = 1$

In addition, LFR is the equation relating to the stiffener.

The detail expression of these equations is presented in Appendix. In Eq. (5), subscript 1, 2, and 3 after the parenthesis denote each plate component, δ_2, δ_3 are the width ratio of plate 2 and plate 1, plate 3 and plate 1, respectively, and ω_2, ω_3 are the thickness ratio of plate 2 and plate 1, plate 3 and plate 1, respectively. Since all of the plate component in a cross-section buckles simultaneously and the buckling stress of each plate component can be assumed to the same, we can find the buckling coefficient to be used to evaluate the buckling stress σ.

Following Timoshenko's expression[10] the local buckling stress can be estimated by using Eq. (6) in which k_1 is the buckling coefficient of web plate (plate 1).

$$\sigma = k_1 \cdot \frac{\pi^2\sqrt{E_{11}E_{22}}}{12(1-\nu_{12}\nu_{21})\left(\frac{b_i}{t_i}\right)^2}, \quad i = 1, 2, 3 \qquad (6)$$

In the local buckling mode, the curve of buckling coefficient of column (or plate component) is similar to the garland curve. Therefore, only the minimum value of the buckling coefficient which is independent from the length of column is practically important.

The minimum buckling coefficients k_1 for concentrically loaded stiffened orthotropic column are shown in Fig. 5. The minimum buckling coefficients for the isotropic I-shape column obtained by substituting isotropic material properties in the above equations are shown in Fig. 6.

In Fig. 5 and Fig. 6, the dotted line indicates the minimum buckling coefficient of I-section column without edge stiffeners.

The isotropic material used in this paper is A36 carbon steel (General structural purposed in civil and construction fields) suggested in ASTM A3. In this theoretical formulation, if the width of stiffener is less than 1/10 of the column web width, the local buckling strength is smaller than that of unstiffened I-section column. If the width of stiffener is larger than 4/10 of the column web width, the stiffener behaves like a plate element and the local buckling strength is decreased. Therefore, it can be concluded that the practical range of stiffener and column web width ratio is 0.1 - 0.3.

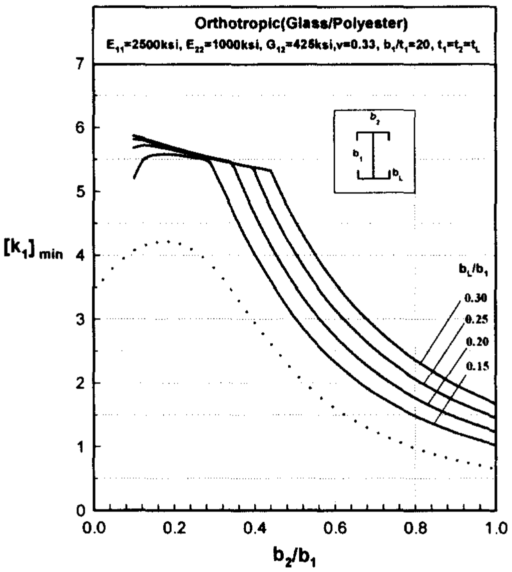


Fig. 5 The minimum buckling coefficient of orthotropic I-section column with asymmetric edge stiffeners

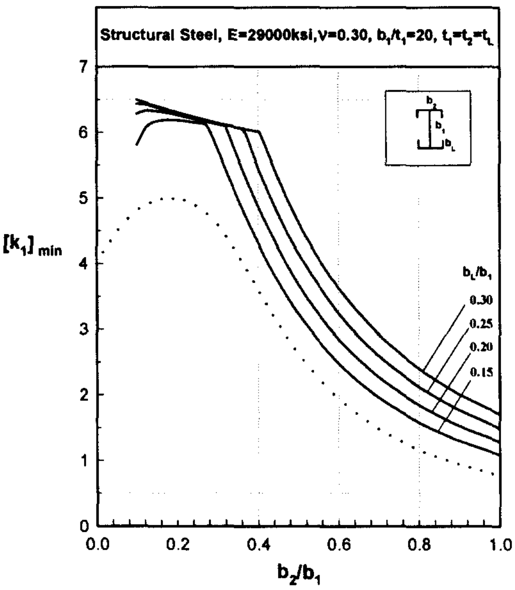


Fig. 6 The minimum buckling coefficient of isotropic I-section column with asymmetric edge stiffeners.

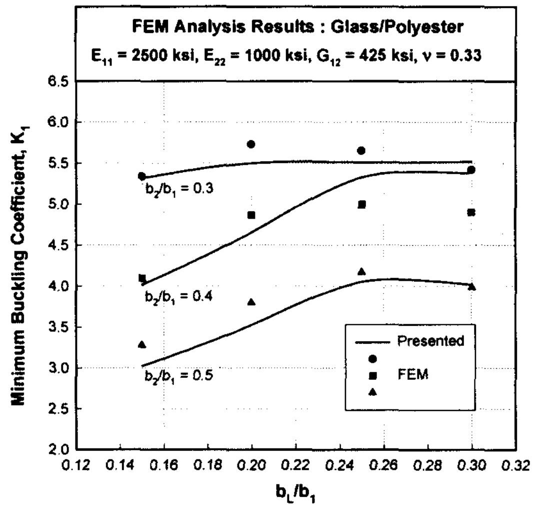


Fig. 7 Comparison of results (orthotropic)

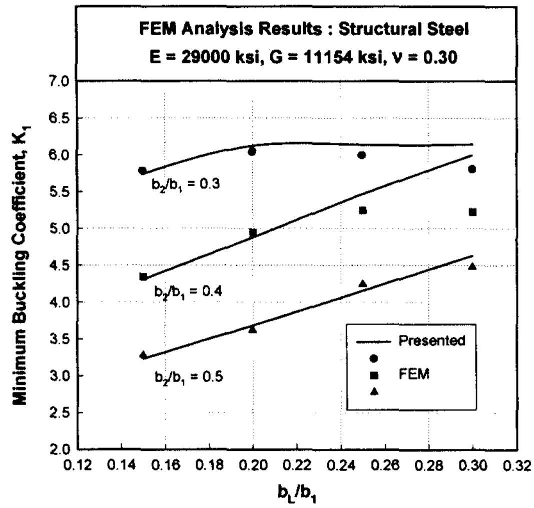
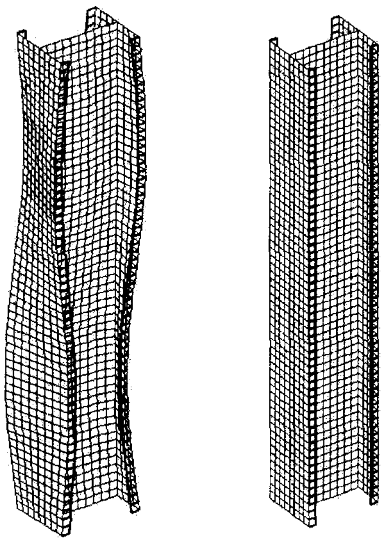


Fig. 8 Comparison of results (isotropic).

4. FINITE ELEMENT ANALYSIS

To evaluate the local buckling load of orthotropic I-section compression members with asymmetric edge stiffeners, the finite element (FEM) analysis was also performed using



$\underline{b_2/b_1 = 0.4,\ b_L/b_1 = 0.2}$

- Total length of member = 1.52 m
- # of plate element = 1560
- # of beam element = 240

Fig. 9 FE model with buckled shape

GTSTRUDL[11]. The finite element used in this analysis is a stretching bending hybrid quadrilateral element (SBHQ6) which is generally used for the plate buckling analysis. This element has five degrees of freedom (DOF) and one dummy DOF (deriving DOF) at each node. Total of nine column specimens are modeled to perform the local buckling analysis and each element dimension is 1 in. × 1 in. with uniform thickness t of 0.5 in. for all elements of the columns.

Comparison of results is shown graphically in Fig. 7 for orthotropic material and Fig. 8 for isotropic material, respectively.

In the finite element analysis, boundary conditions of the column ends were modeled as follows:

(1) the rotation due to bending is allowed at both ends

(2) the rotation due to torsion is restrained at both ends

(3) the out of plane deflection is restrained at both ends

(4) the axial deflection is allowed at loaded end but restrained at unloaded end.

Also one entire section of the column with buckled mode shape after performing finite element analysis is shown in Fig. 9. Element dimension is 25.4 mm × 25.4 mm and uniform thickness t = 5.08 mm for all columns.

5. CONCLUSION

This paper presented a closed-form solution for the local buckling analysis of fiber-reinforced polymeric I-section column with asymmetric edge stiffeners subjected to uniform compression. In derivation of equation the conservative model of asymmetric edge stiffener was utilized.

In this model, the edge stiffener is composed of a strip of flange plate which has the same width with the edge stiffener and plate attached at the free edge of flange. The moment of inertia was estimated at the centroid of ensuing cross-section. Using the local buckling equation, the practical range of asymmetrical stiffener width (10 % - 30 % of web plate width) shown in a graphical form is suggested. To verify the applicability of the solution suggested in this study, the graphical form of minimum buckling coefficient of isotropic (such as structural steel) columns is presented. These results are agreed well with those in published document.

The finite element method is also used to find the local buckling load and buckled mode shape. The results differ by 1% to 9% from the presented theoretical solutions.

ACKNOWLEDGMENT

This research was supported by the 2000 Hong-Ik University Academic Research Support Fund.

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APPENDIX

$$SSY = -\alpha\beta(\alpha^2+\beta^2)\cosh\frac{\alpha}{2}\cos\frac{\beta}{2}$$

$$FSY = -\alpha\beta\left(\alpha\sinh\frac{\alpha}{2}\cos\frac{\beta}{2}+\beta\cosh\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$$

$$SFR = (\alpha^2+\beta^2)(\beta R^2\sinh\alpha\cos\beta-\alpha S^2\cosh\alpha\sin\beta)$$

$$FFR = -2\alpha\beta\chi\psi-\alpha\beta(\chi^2+\psi^2)\cosh\alpha\cos\beta - (\beta^2\chi^2-\alpha^2\psi^2)\sinh\alpha\sin\beta$$

$$SS = (\alpha^2+\beta^2)\sinh\alpha\sin\beta$$

$$FS = -(\alpha^2+\beta^2)\sinh\alpha\sin\beta-2\alpha\beta(1-\cosh\alpha\cos\beta)$$

$$LFR = \left(\frac{m\pi}{\lambda}\right)^2\left\{b_1 D_{11L}\left(\frac{m\pi}{\lambda}\right)-A_1\sigma_x\right\}$$

$$\alpha = \frac{m\pi Q_2}{\lambda}\sqrt{1+\sqrt{1-\left(\frac{Q_1}{Q_2}\right)^4\left(1-\frac{k\cdot\lambda}{m^2Q_1^2}\right)}}$$

$$\beta = \frac{m\pi Q_2}{\lambda}\sqrt{-1+\sqrt{1-\left(\frac{Q_1}{Q_2}\right)^4\left(1-\frac{k\cdot\lambda}{m^2Q_1^2}\right)}}$$

$$Q_1 = \left(\frac{D_{11}}{D_{22}}\right)^{\frac{1}{4}}$$

$$Q_2 = \left(\frac{D_{12}+2D_{66}}{D_{22}}\right)^{\frac{1}{2}}$$

$$R = \alpha^2-\nu_{12}\left(\frac{m\pi}{\lambda}\right)^2$$

$$S = \beta^2-\nu_{12}\left(\frac{m\pi}{\lambda}\right)^2$$

UNIT CONVERSION

1 ksi = 1 kip/in^2 = 70.31 kg/cm^2
1 kip = 1,000 lb = 453.6 kg
1 in = 2.54 cm