퍼지 및 지능시스템학회 논문지 2001, Vol. 11, No. 7, pp. 653-658

On the Design of the Observers of the Nonlinear Systems

Dong-Hwi Roh, Se-Yeon Park, Dong-Young Ryu, and Hong-Gi Lee

School of Electrical and Electronic Engineering, Chung-Ang University
221, Huksuk-Dong, Dongjak-Ku, Seoul 156-756, Korea

Abstract

In this paper, we find the necessary and sufficient conditions for the discrete time nonlinear system to be transformed into observable canonical form by state coordinates change. Unlike the continuous time case, our theorems give the desired state coordinates change without solving partial differential equations. Also, our approach is applicable to both autonomous systems and control systems by slight change of the definition of the vector field.

Key Words : Observer, Nonlinear systems, Autonomous systems, Control systems, Observer canonical forms, State coordinates change.

1. Introduction

Modern control of the nonlinear systems usually exploits the state feedback and state coordinate change. Since the states of the system are not always available, the observer is needed to control the systems. Unlike the linear case, the observer design for nonlinear systems is not simple. One way of the observer design is to use the same scheme as the linear one together with the state transformation.

Consider the following continuous and discrete autonomous nonlinear systems:

$$\dot{x} = f(x) \tag{1.1a}$$
$$y = h(x) \tag{1.1b}$$
$$x(t+1) = f(x(t)) \tag{1.2a}$$
$$y = h(x(t)) \tag{1.2b}$$

where $f(x)$ and $h(x)$ are smooth functions and $x \in R^n$ and $y \in R$ with $f(0)=0$ and $h(0)=0$. Suppose that there exists a state coordinate change $z=S(x)$ which transforms the above system into the following observable linear systems:

$$\dot{z} = Az + \gamma(y) \tag{1.3a}$$
$$y = Cz \tag{1.3b}$$
$$z(t+1) = Az(t) + \gamma(y(t)) \tag{1.4a}$$
$$y(t) = Cz(t) \tag{1.4b}$$

Then we can easily obtain the observer

$$\dot{\hat{z}} = (A+GC)z - Gy + \gamma(y) \tag{1.5}$$
$$\hat{z}(t+1) = (A+GC)z(t) - Gy(t) + \gamma(y(t)) \tag{1.6}$$

If we let $e(t) = \hat{z}(t) - z(t)$, we obtain

$$\dot{e} = (A+GC)e \tag{1.7}$$
$$e(t+1) = (A+GC)e(t) \tag{1.8}$$

and $\lim_{t\to\infty} e(t) = 0$ by choosing an appropriate matrix G. That is, if we can transform the nonlinear system (1.1) and (1.2) into almost linear system (1.3) and (1.4), then we can use the linear system theory to find an observer (1.5) and (1.6) for the system (1.3) and (1.4). The system (1.5) and (1.6) can also be observers of system (1.1) and (1.2) with additional map $\hat{x}(t) = S^{-1}(\hat{z}(t))$.

Similar arguments can be applied for control systems. Consider the following continuous and discrete nonlinear control systems:

$$\dot{x} = f(x) + g(x)u \tag{1.9a}$$
$$y = h(x) \tag{1.9b}$$
$$x(t+1) = f(x(t), u(t)) \tag{1.10a}$$
$$y = h(x(t)) \tag{1.10b}$$

where f, g, and h are smooth functions and $x \in R^n$ and $y \in R$ with $f(0)=0$ and $h(0)=0$. Suppose that there exists a state coordinate change $z = S(x)$ which transforms the above system into the following observable linear systems:

$$\dot{z} = Az + \gamma(y, u) \tag{1.11a}$$
$$y = Cz \tag{1.11b}$$
$$z(t+1) = Az(t) + \gamma(y(t), u(t)) \tag{1.12a}$$
$$y(t) = Cz(t) \tag{1.12b}$$

Thus we can easily obtain the observer

$$\dot{\hat{z}} = (A+GC)z - Gy + \gamma(y) \tag{1.13}$$
$$\hat{z}(t+1) = (A+GC)z(t) - Gy(t) + \gamma(y(t)) \tag{1.14}$$

Since the first attempt in this direction was made by Krener and Isidori[1], many authors[2-9] have studied

Manuscript received December 21, 2001.
This work was supported by the Chung-Ang University Research Grants in 2000.

this problem. For continuous time systems the results are well summarized in [11]. For discrete time system Lee and Nam[6] obtained the conditions when the Jacobian of the vector field is nonsingular. Also, Lin and Byrnes[8] gave the algebraic conditions for discrete time autonomous systems. In [9], Moraal and Grizzle used a totally different approach of observer design from the others.

In this paper we obtain, in section 2, the discrete version of the results in [11] which is a geometric condition for continuous time systems, without any assumption. We show that our approach to autonomous system is also applicable to the systems with input by slight modification. Some illustrating examples are presented in section 3. Finally, conclusions are included in section 4.

The notations and definitions which are not clear in this paper can be found in [11-14].

2. Geometric Conditions for discrete time systems

In this section we state the geometric conditions for continuous time systems and we find the discrete version of those.

Theorem 1[10] : Consider continuous time nonlinear autonomous system (1.1). There exists a state coordinate change $z=S(x)$ transforming system (1.1) into a linear observer form

$$\dot{z}=\begin{bmatrix}0 & 0 & \cdots & 0 & 0\\ 1 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & 1 & 0\end{bmatrix}z+\begin{bmatrix}\gamma_1(y)\\ \gamma_2(y)\\ \vdots\\ \gamma_n(y)\end{bmatrix} \tag{2.1a}$$

$$y=[0\ 0\ \cdots\ 0\ 1]z \tag{2.1b}$$

if and only if

(i) $\operatorname{rank}\{dh,\ dL_fh,\cdots,\ dL_f^{n-1}h\}=n$,

(ii) $[ad_f^ir(x),ad_f^jr(x)]=0,\ 0\le i,j\le n-1$,

with $r(x)$ being the vector field solution of

$$\begin{bmatrix}\langle dh,r\rangle\\ \vdots\\ \langle dL_f^{n-2}h,r\rangle\\ \langle dL_f^{n-1}h,r\rangle\end{bmatrix}=\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix} \tag{2.2}$$

Theorem 2[10] : Consider continuous time nonlinear control system (1.9). There exists a state coordinate change $z=S(x)$ transforming system (1.9) into a linear observer form

$$\dot{z}=\begin{bmatrix}0 & 0 & \cdots & 0 & 0\\ 1 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & 1 & 0\end{bmatrix}z+\begin{bmatrix}\gamma_1(y,u)\\ \gamma_2(y,u)\\ \vdots\\ \gamma_n(y,u)\end{bmatrix} \tag{2.3a}$$

$$y=[0\ 0\ \cdots\ 0\ 1]z \tag{2.3b}$$

if and only if

(i) $\operatorname{rank}\{dh,\ dL_fh,\cdots,\ dL_f^{n-1}h\}=n$,

(ii) $[ad_f^ir(x),ad_f^jr(x)]=0,\ 0\le i,j\le n-1$,

(iii) $[g,ad_f^jr(x)]=0,\ 0\le j\le n-2$,

with $r(x)$ being the vector field solution of (2.2).

We now turn our attention to discrete time autonomous system (1.2) in order to find the discrete version of Theorem 1. For the discrete time system, we define the following notation:

$$\hat{F}^1(x,w)=F(x,w)$$
$$\hat{F}^\ell(x,w)=F(\hat{F}^{\ell-1}(x,w),0),\ \ \ell\ge 2$$

We assume that

$$\operatorname{rank}\{dh,d(h\circ f),\cdots,d(h\circ f^{n-1})\}=n \tag{2.4}$$

Under this assumption we define the vector field $F(x,w):R^{n+1}\to R^n$ by

$$h\circ\hat{F}^i(x,w)=h\circ f^i(x),\ \ 1\le i\le n-1 \tag{2.5a}$$

$$h\circ\hat{F}^n(x,w)=h\circ f^n(x)+w \tag{2.5b}$$

Vector field $F(x,w)$ plays the same role as the vector field $r(x)$ in Theorem 1 and can be found by solving algebraic equation (2.5) or by the following procedure:

Under the assumption (2.4), consider state coordinate change

$$\xi=\begin{bmatrix}h\circ f^{n-1}(x)\\ \vdots\\ h\circ f(x)\\ h(x)\end{bmatrix}=\bar{S}(x) \tag{2.6}$$

and obtain the dynamic equation for the new state ξ:

$$\xi(t+1)=\begin{bmatrix}\alpha(\xi)\\ \xi_1\\ \vdots\\ \xi_{n-2}\\ \xi_{n-1}\end{bmatrix}=\bar{f}(\xi(t)) \tag{2.7a}$$

$$y(t)=\xi_n(t) \tag{2.7b}$$

Now the vector field

$$F(x)=\bar{f}(\bar{S}(x))+\begin{bmatrix}w\\ 0\\ \vdots\\ 0\end{bmatrix} \tag{2.8}$$

will satisfies (2.5).

Now we define the following composite functions:

$$\hat{F}^1(x(t),w(t))\triangleq F(x(t),w(t)) \tag{2.9a}$$

$$\hat{F}^i(x(t),w(t))\triangleq F(\hat{F}^{i-1}(x(t),w(t)),0),\ \ i\ge 2 \tag{2.9b}$$

$$\mathcal{F}(w^1,\ w^2,\cdots,w^{n+1})\triangleq F(\cdots F(F(0,w^{n+1}),w^n),\cdots,w^1) \tag{2.10a}$$

$$\Psi(w^1,w^2,\ \cdots,w^n)\triangleq\mathcal{F}(w^1,w^2,\cdots,w^n,0)=F(\cdots F(F(0,w^n),w^{n-1}),\cdots,w^1) \tag{2.10b}$$

Theorem 3 : Consider discrete time nonlinear autonomous system (1.2). There exists a state coordinate change $z=S(x)$ transforming system (1.2) into a linear observer form

$$z(t+1)=\begin{bmatrix}0&0&\cdots&0&0\\1&0&\cdots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&1&0\end{bmatrix}z(t)+\begin{bmatrix}\gamma_1(y(t))\\\gamma_2(y)\\\vdots\\\gamma_n(y)\end{bmatrix}\qquad(2.11a)$$

$$y(t)=[0\ 0\ \cdots\ 0\ 1]z(t)\qquad(2.11b)$$

if and only if

(i) $\operatorname{rank}\{dh,\ dL_f h,\cdots,\ dL_f^{n-1}h\}=n$,

(ii) $\mathcal{F}_*(\frac{\partial}{\partial w^i})$, $1\le i\le n$ are well-defined vector fields.

Furthermore, $\xi=S(x)=\Psi^{-1}(x)$ is a desired state coordinates change.

Proof:

(Necessity); Suppose that there exists $z=S(x)$ which satisfies (2.11). Condition (i) is obvious since the observability is invariant under state coordinates change. By (2.4), it is easy to see that

$$z_n=h(x)\qquad(2.12a)$$

$$\begin{aligned}z_i&=h\circ f^{n-i}(x)-\sum_{\ell=i+1}^{n-1}\gamma_{\ell+1}(h\circ f^{\ell-i}(x))-\gamma_{i+1}(h(x))\\&=h\circ f^{n-i}(x)-\sum_{\ell=i}^{n-1}\gamma_{\ell+1}(h\circ f^{\ell-i}(x)),\quad 1\le i\le n-1\end{aligned}\qquad(2.12b)$$

$$h\circ f^n(x)-\sum_{\ell=1}^{n-1}\gamma_{\ell+1}(h\circ f^{\ell}(x))-\gamma_1(h(x))=0\qquad(2.12c)$$

From this relationship and (2.5), we obtain

$$\begin{aligned}&z_i\circ F(x,w)\\&=h\circ f^{n-i}(F(x,w))-\sum_{\ell=i}^{n-1}\gamma_{\ell+1}(h\circ f^{\ell-i}(F(x,w))\\&=h\circ f^{n-i+1}(x)-\sum_{\ell=i}^{n-1}\gamma_{\ell+1}(h\circ f^{\ell-i+1}(x))\\&=z_{i-1}+\gamma_i(h(x)),\quad 2\le i\le n\\&z_1\circ F(x,w)\\&=h\circ f^{n-1}(F(x,w))-\sum_{\ell=1}^{n-1}\gamma_{\ell+1}(h\circ f^{\ell-1}(F(x,w))\\&=h\circ f^n(x)+w-\sum_{\ell=1}^{n-1}\gamma_{\ell+1}(h\circ f^{\ell}(x))\\&=\gamma_1(h(x))+w\end{aligned}$$

Now, if we let

$$\hat{F}(z,w)=S\circ F(S^{-1}(z),w)\text{or}$$
$$F(x,w)=S^{-1}\circ\hat{F}(S(x),w),$$

it follows

$$\begin{aligned}\hat{F}(z,w)&=\begin{bmatrix}0&0&\cdots&0&0\\1&0&\cdots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&1&0\end{bmatrix}z+\begin{bmatrix}\gamma_1(z_n)\\\gamma_2(z_n)\\\vdots\\\gamma_n(z_n)\end{bmatrix}+\begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}w\\&=A_0z+\gamma(y)+bw\end{aligned}$$

Therefore it is easy to see that

$$\hat{F}(\cdots\hat{F}(\hat{F}(0,w^{n+1}),w^n),\cdots,w^2)=\begin{bmatrix}w^2\\w^3\\\vdots\\w^{n+1}\end{bmatrix}$$

$$\begin{aligned}&\hat{\mathcal{F}}(w^1,w^2,\cdots,w^{n+1})\\&=\hat{F}(\cdots\hat{F}(\hat{F}(0,w^{n+1}),w^n),\cdots,w^1)\\&=\begin{bmatrix}w^1+\gamma_1(w^{n+1})\\w^2+\gamma_2(w^{n+1})\\\vdots\\w^n+\gamma_n(w^{n+1})\end{bmatrix}\end{aligned}$$

which implies $\hat{\mathcal{F}}_*(\frac{\partial}{\partial w^i})$, $1\le i\le n$ are well-defined vector fields. Since

$$\mathcal{F}(w^1,w^2,\cdots,w^{n+1})=S^{-1}\circ\hat{\mathcal{F}}(w^1,w^2,\cdots,w^{n+1}),$$

condition (ii) is satisfied.

(Sufficiency); Suppose that condition (i) and (ii) are satisfied. Let $\xi=S(x)=\Psi^{-1}(x)$ and

$$\begin{aligned}\xi(t+1)&=\Psi^{-1}\circ F(\Psi(\xi),w)\\&=\hat{F}(\xi,w)\end{aligned}$$

If we let

$$Y^i=\mathcal{F}_*(\frac{\partial}{\partial w^i}),\quad 1\le i\le n,$$

then $\{Y^1,Y^2,\cdots,Y^n\}$ are linearly independent vector fields and it is easy to see that

$$Y^i=\mathcal{F}_*(\frac{\partial}{\partial w^i})=\Psi(\xi)_*(\frac{\partial}{\partial\xi^i}),\quad 1\le i\le n,$$

Since $F(\Psi(\xi),w)=\mathcal{F}(w,\xi)$, $\xi\in R^n$, we obtain

$$\begin{aligned}\hat{F}(\xi,w)_*(\frac{\partial}{\partial w})&=(\Psi^{-1}\circ F(\Psi(\xi),w))_*(\frac{\partial}{\partial w})\\&=(\Psi^{-1}\circ\mathcal{F}(w,\xi))_*(\frac{\partial}{\partial w})\\&=(\Psi^{-1})_*Y^1=\frac{\partial}{\partial\xi_1}\end{aligned}$$

Similarly, we can show that

$$\begin{aligned}\hat{F}(\xi,w)_*(\frac{\partial}{\partial\xi_i})&=(\Psi^{-1}\circ\mathcal{F}(w,\xi))_*(\frac{\partial}{\partial\xi_i})\\&=(\Psi^{-1})_*Y^{i+1}\\&=\frac{\partial}{\partial\xi_{i+1}},\quad 1\le i\le n-1\end{aligned}$$

Therefore we have

$$\hat{F}(\xi,w)=\begin{bmatrix}w+\gamma_1(\xi_n)\\\xi_1+\gamma_1(\xi_n)\\\vdots\\\xi_{n-1}+\gamma_1(\xi_n)\end{bmatrix}\qquad(2.13)$$

Also note that by (2.5),

$$\hat{h}\circ\hat{\mathcal{F}}^i(x,w)=\hat{h}\circ\hat{f}^i(x),\quad 1\le i\le n-1\qquad(214a)$$

$$\hat{h}\circ\hat{\mathcal{F}}^n(x,w)=\hat{h}\circ\hat{f}^n(x)+w\qquad(2.14b)$$

$$\frac{\partial}{\partial w}\hat{h}\circ\hat{\mathcal{F}}^i(x,w)=0,\quad 1\le i\le n-1\qquad(2.15a)$$

$$\frac{\partial}{\partial w}\hat{h}\circ\hat{\mathcal{F}}^n(\xi,w)=1\qquad(2.15b)$$

which implies

$$y=h(\Psi(\xi))=\hat{h}(\xi)=\xi_n\qquad(2.16)$$

Therefore, by (2.13) and (2.14), it is easy to see that

$$\hat{f}(\xi)=\begin{bmatrix}\gamma_1(\xi_n)\\ \xi_1+\gamma_1(\xi_n)\\ \vdots\\ \xi_{n-1}+\gamma_1(\xi_n)\end{bmatrix} \quad (2.17)$$

Same arguments can be applied for discrete time control system (1.10) in order to find the discrete version of Theorem 2. We assume that

$$\text{rank}\{dh, d(h\circ\hat{f}(x,0)),\cdots,d(h\circ\hat{f}^{n-1}(x,0))\}=n$$

Under this assumption we define the vector field $F(x,u,w): R^{n+2}\to R^n$ by

$$h\circ\hat{F}^i(x,u,w)=h\circ\hat{f}^i(x,u),\ 1\le i\le n-1 \quad (2.18a)$$
$$h\circ\hat{F}^n(x,u,w)=h\circ\hat{f}^n(x,u)+w \quad (2.18b)$$

Here we use the following definition:

$$\hat{F}^1(x,u,w)=F(x,u,w)$$
$$\hat{F}^{\ell}(x,u,w)=F(\hat{F}^{\ell-1}(x,u,w),0,0),\ \ell\ge 2$$

Now we define the following composite functions:

$$\mathcal{F}(w^1,w^2,\cdots,w^{n+1},u) \triangleq F(\cdots F(F(0,0,w^{n+1}),0,w^n),\cdots),u,w^1) \quad (2.19a)$$
$$\Psi(w^1,w^2,\cdots,w^n) \triangleq \mathcal{F}(w^1,w^2,\cdots,w^n,0,0) = F(\cdots F(F(0,0,w^n),0,w^{n-1}),\cdots),0,w^1) \quad (2.19b)$$

Theorem 4 : Consider discrete time nonlinear autonomous system (1.10). There exists a state coordinate change $z=S(x)$ transforming system (1.10) into a linear observer form

$$z(t+1)=\begin{bmatrix}0&0&\cdots&0&0\\1&0&\cdots&0&0\\ \vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&1&0\end{bmatrix}z(t)+\begin{bmatrix}\gamma_1(y,u)\\ \gamma_2(y,u)\\ \vdots\\ \gamma_n(y,u)\end{bmatrix} \quad (2.20a)$$
$$y(t)=[0\ 0\ \cdots\ 0\ 1]\,z(t) \quad (2.20b)$$

if and only if

(i) $\text{rank}\{dh, d(h\circ\hat{f}(x,0)),\cdots,d(h\circ\hat{f}^{n-1}(x,0))\}=n$

(ii) $\mathcal{F}_*(\frac{\partial}{\partial w^i})$, $1\le i\le n$ are well-defined vector fields.

Furthermore, $\xi=S(x)=\Psi^{-1}(x)$ is a desired state coordinates change.

The proof of Theorem 4 is almost the same as that of Theorem 3. Thus it will be omitted.

Condition (ii) of Theorem 3 and 4 can be easily checked by the following equivalent condition:

(ii)′ $[\frac{\partial}{\partial w^i},\ker(\mathcal{F}_*)]\subset\ker(\mathcal{F}_*)$, $1\le i\le n$

3. Examples

In this section, a couple of examples are given to explain the effectiveness of our conditions and scheme.

Example 1 : Consider the following discrete time nonlinear autonomous system:

$$\begin{bmatrix}x_1(t+1)\\x_2(t+1)\\x_3(t+1)\end{bmatrix}=\begin{bmatrix}0\\x_1(t)+[x_2(t)-x_3(t)^2]^2\\x_2(t)-x_3(t)^2\end{bmatrix}$$
$$y(t)=x_1(t)+x_3(t)=h(x(t))$$

Then, since

$$h=x_1+x_3,$$
$$h\circ f=x_2-x_3^2,$$
$$h\circ f^2=x_1,$$

observability condition (i) is satisfied. Also by (2.5) vector field $F(x,w)$ satisfies the following equations:

$$F_1+F_3=x_2-x_3^2$$
$$F_2-F_3^2=x_1,$$
$$F_1=w,$$

Thus we have

$$F=\begin{bmatrix}w\\x_1+(x_2-x_3^2-w)^2\\x_2-x_3^2-w\end{bmatrix}$$

From this, we obtain the following composite functions:

$$F(0,w^4)=\begin{bmatrix}w^4\\(w^4)^2\\-w^4\end{bmatrix}$$
$$F(F(0,w^4),w^3)=\begin{bmatrix}w^3\\w^4+(w^3)^2\\-w^3\end{bmatrix}$$
$$F(F(F(0,w^4),w^3),w^2)=\begin{bmatrix}w^2\\w^3+(w^4-w^2)^2\\w^4-w^2\end{bmatrix}$$
$$\Psi(w^1,w^2,w^3)=F(F(F(0,w^3),w^2),w^1)=\begin{bmatrix}w^1\\w^2+(w^3-w^1)^2\\w^3-w^1\end{bmatrix}$$
$$\mathcal{F}(w^1,w^2,w^3,w^4)=F(F(F(F(0,w^4),w^3),w^2),w^1)=\begin{bmatrix}w^1\\w^2+(w^3-w^1)^2\\w^3-w^1\end{bmatrix}$$

Since

$$\frac{\partial}{\partial w}\mathcal{F}=\begin{bmatrix}1&0&0&0\\-2(w^3-w^1)&1&2(w^3-w^1)&0\\-1&0&1&0\end{bmatrix}$$

we can get

$$\ker(\mathcal{F}_*)=span\{\frac{\partial}{\partial w^4}\}$$

which implies that

$$[\frac{\partial}{\partial w^i},\ker(\mathcal{F}_*)]\subset\ker(\mathcal{F}_*),\ 1\le i\le 3$$

Therefore, condition (ii) of Theorem 3 is satisfied. Also, the desired state coordinates change is

$$z = S(x) = \Psi^{-1}(x) = \begin{bmatrix} x_1 \\ x_2 - x_3^2 \\ x_1 + x_3 \end{bmatrix}$$

and the new states of the system satisfies the following observer canonical form:

$$z(t+1) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} z(t)$$
$$y(t) = [0 \ 0 \ 1] z(t)$$

Example 2 : Consider the following discrete time nonlinear control system:

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = \begin{bmatrix} x_1^2 + u^2 \\ x_1 \end{bmatrix} = f(x, u)$$
$$y(t) = x_2(t) = h(x(t))$$

Then, since

$$h = x_2,$$
$$h \circ f = x_1,$$

observability condition (i) is satisfied. Also by (2.18) vector field $F(x, w)$ satisfies the following equations:

$$F_2 = x_1$$
$$F_1 = x_1^2 + u^2 + w,$$

Thus we have

$$F = \begin{bmatrix} x_1^2 + u^2 + w \\ x_1 \end{bmatrix}$$

From this, we obtain the following composite functions:

$$\Psi(w^1, w^2) = F(F(0,0,w^2),0,w^1) = \begin{bmatrix} (w^2)^2 + w^1 \\ w^2 \end{bmatrix}$$

$$\mathcal{F}(w^1, w^2, w^3) = F(F(F(F(0,0,w^3),0,w^2),u,w^1) = \begin{bmatrix} [(w^3)^2 + w^2]^2 + u^2 + w^1 \\ (w^3)^2 + w^2 \end{bmatrix}$$

Since

$$\frac{\partial}{\partial w}\mathcal{F} = \begin{bmatrix} 1 & 2[(w^3)^2 + w^2] & 4w^3[(w^3)^2 + w^2] \\ 0 & 1 & 2w^3 \end{bmatrix}$$

we can get

$$\ker(\mathcal{F}_*) = span\{ -2w^3 \frac{\partial}{\partial w^2} + \frac{\partial}{\partial w^3} \}$$

which implies that

$$[\frac{\partial}{\partial w^i}, \ker(\mathcal{F}_*)] \subset \ker(\mathcal{F}_*), \ 1 \le i \le 2 = n$$

Therefore, condition (ii) of Theorem 3 is satisfied. Also, the desired state coordinates change is

$$z = S(x) = \Psi^{-1}(x) = \begin{bmatrix} x_1 - x_2^2 \\ x_2 \end{bmatrix}$$

and the new states of the system satisfies the following observer canonical form:

$$z(t+1) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} z(t) + \begin{bmatrix} u(t)^2 \\ y(t)^2 \end{bmatrix}$$
$$y(t) = [0 \ 1] z(t)$$

4. Conclusions

In this paper, we have found the necessary and sufficient conditions for the discrete time nonlinear system to be transformed into observable canonical form by state coordinates change. The scheme we use can be considered as the discrete version of the well-known continuous time results. However, unlike the continuous time case, our theorems give the desired state coordinates change without solving partial differential equations. This phenomenon can be also found in linearization problem.[10] Finally, our approach has been shown to be applicable to both the systems without input and those with input, by slight change of the definition of the vector field.

References

[1] A. J. Krener and A. Isidori, "Linearization by output injection and nonlinear observers", *System & Control Letters*, vol. 3 pp.47-52, 1983.

[2] C. Bestle and M. Zeitz, "Canonical form observer design for nonlinear time variable systems," *Int. J. Control*, vol. 38, pp. 419-431, 1983.

[3] A. J. Kerner and W. Respondek, "Nonlinear observers with linearizable error dynamics," *SIAM J. Control*, vol. 23, pp. 197-216, 1985.

[4] X. Xia and W. Gao, "Nonlinear observer design by observer canonical forms," *Int. J. Control*, vol. 47, pp. 1081-1100, 1988.

[5] X. Xia and W. Gao, "Nonlinear observer design by observer error linearization," *SIAM. J. Control*, vol. 27, pp. 199-216, 1989.

[6] W. Lee and K. Nam, "Observer design for autonomous discrete-time systems," *System & Control Letters*, vol. 17, pp. 49-58, 1991.

[7] S. T. Chung and J. W. Grizzle, "Sampled-data observer error linearization," *Automatica*, vol. 36, pp. 997-1007, 1990.

[8] W. Lin and C. I. Byrnes, "Remarks on linearization of discrete-time autonomous systems and nonlinear observer design," *System & Control Letters*, vol. 25, pp. 31-40, 1995.

[9] P. E. Moraal and J. W. Grizzle, "Observer design for nonlinear systems with discrete-time measurements," *IEEE Trans. Autom. Control*, vol. 40, pp. 395-404, 1995

[10] H. G. Lee, A, Arapostathis and S. I. Marcus, "Linearization of discrete-time systems," *Int. J. Control*, vol. 45, pp. 1803-1822, 1987.

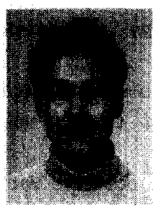
[11] R. Marino and P. Tomei, *Nonlinear Control Design*,

Prentice Hall, 1995.
[12] A. Isidori, *Nonlinear Control Systems*, 3rd ed., Springer-Verlag, 1995.
[13] H. Nijmeijer and A. J. van der Schaft, *Nonlinear Dynamical Control Systems*, Springer-Verlag New York Inc., 1990.
[14] H. G. Lee, *Linearization of Nonlinear Control Systems*, Chung-Ang University Press, Seoul, 2001.

Dong-Hwi Roh

Dong-Hwi Roh was born on October 18, 1975. He received the B.S. degree and M.S. degree in Department of Control and Instrumentation Engineering from Chung-Ang University, Seoul, Korea, in 1998 and 2000, respectively. His research interests include nonlinear control theory and intelligent control.

Phone : +82-2-820-5317
Fax : +82-2-817-0292
E-mail : mizi@nextlink.co.kr



Se-Yeon Park

Se-Yeon Park was born on July 23, 1974. He received the B.S. degree in Department of Control and Instrumentation Engineering from Chung-Ang University, Seoul, Korea, in 2001. His research interests include Nonlinear control theory and genetic algorithm.

Phone : +82-2-820-5317
Fax : +82-2-817-0292
E-mail : ppsy23@dreamwiz.com

Dong-Young Ryu

Dong-Young Ryu was born on August 19, 1974. He received the B.S. degree in Department of Information and Communication Engineering from Suwon University, Suwon, Korea, in 2001. His research interests include Nonlinear Control theory and wavelet theory.

Phone : +82-2-820-5317
Fax : +82-2-817-0292
E-mail : ryudy@hanmail.net

Hong-Gi Lee

Hong-Gi Lee was born November 28. 1958. He received the B.S. and M.S. degrees in Department of Electronics Engineering from Seoul National University, Seoul Korea, in 1981 and 1983, respectively. And he received Ph. D. degree in Department of Electrical and Computer Engineering from the University of Texas at Austin in 1986. From August 1986 to February 1989, he was with department of Electrical and Computer Engineering at Louisiana State University. Since March 1989, he has been with School of Electrical and Electronic Engineering at the Chung-Ang University, where he is currently a Professor. His research interests include nonlinear control system theory, robotics, stochastic systems, and intelligent control systems. He is a member of IEEE, KITE, KIEE, and ICASE.

Phone : +82-2-820-5317
Fax : +82-2-817-0292
E-mail : hglee@cau.ac.kr