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Wideband Time-Frequency Symbols and their Applications

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Abstract

We generalize the wideband P0-Weyl symbol (P0WS) and the wideband spreading function (WSF) using the generalized warping function. The new generalized P0WS and WSF are useful for analyzing systems and communication channels producing generalized time shifts. We also investigate the relationship between the affine Weyl symbol (AWS) and the P0WS. By using specific warping functions, we derive new P0WS and WSF as analysis tools for systems and communication channels with non-linear group delay characteristics. The new P0WS preserves specific types of changes imposed on random processes. The new WSF provides a new interpretation of outputs of system and communication channel as weighted superpositions of non-linear time shifts on the input. It is compared to the conventional method obtaining outputs of system and communication channel as a convolution integration of the input with the impulse response of the system and the communication channel. The convolution integration can be interpreted as weighted superpositions of linear time shifts on the input where the weight is the impulse response of the system and the communication channel. Application examples in analysis and detection demonstrate the advantages of our new results.

Key Words : Spreading function, Weyl symbol, Generalized warping function, Time-frequency analysis.

1. Introduction

The conventional narrowband Weyl symbol (NWS) and narrowband spreading function (NSF) have been successfully used for analyzing LTV systems producing constant time shifts and constant frequency shifts or nonstationary random processes undergoing constant time shifts and constant frequency shifts [6,11].

The NWS and NSF are defined as

$$NWS_L(t,f)=\int K_L(t+\frac{\tau}{2},t-\frac{\tau}{2})e^{-j2\pi\tau f}d\tau$$

$$NSF_L(\tau,\nu)=\int\int NWS_L(t,f)e^{-j2\pi(t\nu-f\tau)}dtdf$$

where $K_L(t,\tau)$ is the kernel of the operator L defined on $L_2(R)$ [2]. The NSF provides important information on the amount of time lags and frequency lags induced by a system. The quadratic form of a random process x(t), $\int (L\ x)(t)x^*(t)dt$, can be expressed as the 2-D inner product of the NWS of the operator L and the Wigner distribution of the process $WD_X(t,f)$ or the 2-D inner product of the NSF of L and the narrowband ambiguity function of the process $AF_X(\tau,\nu)$,

$$\int(Lx)(t)x^*(t)dt=\int\int NWS_L(t,f)WD_X(t,f)dtdf$$
$$=\int\int NSF_L(\tau,\nu)\ AF_L^*(\tau,\nu)d\tau d\nu$$

where $WD_X(t,f)=\int x(t+\tau/2)\ x^*(t-\tau/2)\ e^{j2\pi\tau f}\ d\tau$ and $AF_X(\tau,\nu)=\int x(t+\tau/2)\ x^*(t-\tau/2)\ e^{j2\pi t\nu}\ dt$ [3].

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When a wideband LTV system $\boldsymbol{B}$ defined on $L_2(\mathbf{R}^+)$ in frequency domain produces constant time shifts, scale changes and hyperbolic time shifts in the output, the NWS and NSF are not adequate tools for analyzing such a wideband system. Shenoy and Parks were the first people who introduced the concept of the affine Weyl symbol (AWS) and wideband spreading function (WSF) to express the quadratic form as the 2-D inner product of the AWS of the operator $\boldsymbol{B}$ and the affine Wigner distribution of the process $AWD_X(p,f)$ or the 2-D inner product of the WSF of $\boldsymbol{B}$ and the wideband ambiguity function of the process $WAF_X(\tau,\alpha)$ [10,12] where AWD_X and WAF_X are defined as

$$AWD_X(p,f)=f\int q_X(f,\alpha)\lambda(\alpha)e^{j2\pi p\alpha}d\alpha,p>0,$$

$$WAF_X(\tau,\alpha)=\int q_X(f,\alpha)\lambda(\alpha)e^{j2\pi f\tau}df$$

where $q_X(f,\alpha)=X(f\lambda(\alpha)e^{\alpha/2})X^*(f\lambda(\alpha)e^{-\alpha/2})$ and $\lambda(\alpha)=\alpha/(2\sinh(\alpha/2))$.

In [4,5], we defined the P_0-Weyl symbol (P_0WS) based on the Bertrand P_0-distribution as an affine version of the Weyl symbol. The AWS and the P_0WS have the same relationship as the affine Wigner distribution and the Bertrand unitary P_0-distribution do. These AWS and P_0WS are adequate tools for analyzing systems producing time shifts, scale changes and hyperbolic time shifts in the output. However, compared to the study on the narrowband Weyl correspondence and its generalization [5,6,11], the study on the wideband Weyl correspondence and its generalization have not been well developed. For example, if a system produces dispersive (non-constant) time shifts such as hyperbolic time shifts in the output, neither the AWS (or P0WS) nor the WSF is proper tools for analyzing such a system.

In this paper, we investigate the theory related to the wideband Weyl correspondence and its generalization. In Section II, we review the AWS, P0WS and WSF. We provide the relationship between the AWS and P0WS and investigate important properties of the P0WS. In Section III, we generalize the P0WS and WSF to provide appropriate analysis tools for systems producing dispersive time shifts. We investigate relationships between the generalized formulation and the conventional formulation of the P0WS and WSF. We also derive special cases of the generalized formulation of the P0WS and WSF. In Section IV, we demonstrate the advantages of new results by providing application examples.

2. Affine Formulation of The Weyl Symbol

Shenoy and Parks introduced the concept of an affine version of the Weyl correspondence [12]. They used the affine Weyl symbol and wideband spreading function in the quadratic form of a random process [10,12], i.e.

$$\int (BX)(f)X^*(f)df = \int\int AWS_B(p,f)AWD_X(p,f)dpdf$$
$$= \int\int WSF_B(\tau,\alpha)\ WAF_X^*(\tau,\alpha)d\tau d\alpha.$$

The AWS and WSF can be derived as

$$AWS_B(p,f) = f\int \Gamma_B(f\lambda(\alpha)e^{\frac{\alpha}{2}}, f\lambda(\alpha)e^{\frac{-\alpha}{2}})\lambda(\alpha)e^{-j2\pi p\alpha}d\alpha, \quad (1)$$

$$WSF_B(\tau,\alpha) = \int f\Gamma_B(f\lambda(\alpha)e^{\frac{\alpha}{2}}, f\lambda(\alpha)e^{\frac{-\alpha}{2}})\lambda(\alpha)e^{j2\pi f\tau}df. \quad (2)$$

The AWS is the 2-D Fourier transform of the WSF [12]

$$AWS_B(p,f) = \int\int WSF_B(\tau,\alpha)e^{-j2\pi(\tau f+p\alpha)}d\tau d\alpha.$$

Shenoy and Parks also showed the system output can be written as a weighted superposition of constant time-shifted and scale changed versions of the input signal [12], i.e.

$$(BX)(f) = \int\int WSF_B(\tau,\alpha)e^{-j2\pi f\tau'(\alpha)}\frac{1}{\sqrt{e^{\alpha}}}X(\frac{f}{e^{\alpha}})d\tau d\alpha$$

where $\tau'(\alpha)=\tau e^{-\alpha/2}/\lambda(\alpha)$. Hence, the weighting function, WSF_B, provides information on the amount of time shifts and scale changes induced by the system.

We define a wideband version of the Weyl symbol (P0WS) [5] based on the Bertrand unitary P0-distribution [1] as

$$P_0WS_B(t,f) = f\int \Gamma_B(f\lambda(\alpha)e^{\frac{\alpha}{2}}, f\lambda(\alpha)e^{\frac{-\alpha}{2}})\lambda(\alpha)e^{j2\pi tf\alpha}d\alpha. \quad (3)$$

The relationship between the AWS in (1) and the P0WS in (3) is

$$P_0WS_B(t,f) = AWS_B(p,f)|_{p=-tf}.$$

The relationship between the P0WS and the WSF is [5]

$$P_0WS_B(t,f) = \int\int WSF_B(\tau,\alpha)e^{-j2\pi(\tau f-tf\alpha)}d\tau d\alpha. \quad (4)$$

The P0WS preserves constant time shifts, scale changes and hyperbolic time shifts on a random process

$$Y(f) = X(f)e^{-j2\pi\tau f} \Rightarrow P_0WS_{R_Y}(t,f) = P_0WS_{R_X}(t-\tau,f),$$

$$Y(f) = \frac{1}{\sqrt{a}}X(\frac{f}{a}) \Rightarrow P_0WS_{R_Y}(t,f) = P_0WS_{R_X}(at,\frac{f}{a}),$$

$$Y(f) = e^{-j2\pi\zeta\ln(f)}X(f) \Rightarrow P_0WS_{R_Y}(t,f) = P_0WS_{R_X}(t-\frac{\zeta}{f},f)$$

where $\mathbf{R}_X$ and $\mathbf{R}_Y$ are the correlation operators of X(f) and Y(f), respectively. The kernel of $\mathbf{R}_X$ is defined as $KR_X(f_1,f_2)=\mathbf{E}[X(f_1)X^*(f_2)]$ with the expectation operator $\mathbf{E}[\cdot]$.

3. Generalized Formulation of The Affine Weyl Symbol

3.1 Generalized P0-Weyl Symbol

The generalized P0WS, GP0WS, is defined as

$$GP_0WS_Y(t,f) = |\xi(f)|\int \Gamma_Y(\Xi(f,\alpha),\Xi(f,-\alpha))\cdot \frac{\lambda(\alpha)e^{j2\pi t\alpha\xi(f)/\varphi(f)}}{|\varphi(\Xi(f,\alpha))\varphi(\Xi(f,-\alpha))|^{1/2}}d\alpha, \quad (5)$$

where $\Gamma_Y(f_1,f_2)$ is the kernel of $\boldsymbol{Y}$ defined on $L_2([c,d])$, $\Xi(f,\alpha)=\xi^{-1}(\xi(f)\lambda(\alpha)e^{\alpha/2})$, $\xi^{-1}(\xi(b))=b$, and $\varphi(f)=\xi'(f)$. Here, [c,d] is determined by the domain of the one to one warping function $\xi(b)$. The GP0WSY preserves the generalized time shifts on a random process X(f), i.e

$$Y(f) = (D_\zeta X)(f) \Rightarrow GP_0WS_{R_Y}(t,f) = GP_0WS_{R_X}(t-\zeta\varphi(f),f)$$

where $(\boldsymbol{D}_\zeta X)(f)=e^{j2\pi\zeta\xi(f)}X(f)$ is the generalized time-shifted signal.

The generalized wideband spreading function, $GWSF_Y(\zeta,\alpha)$, is defined as

$$GWSF_Y(\zeta,\alpha) = \int_c^d \Gamma_Y(\Xi(\xi^{-1}(b),\alpha),\Xi(\xi^{-1}(b),-\alpha))\cdot \frac{b\lambda(\alpha)e^{j2\pi b\zeta}}{|\varphi(\Xi(\xi^{-1}(b),\alpha))\varphi(\Xi(\xi^{-1}(b),-\alpha))|^{1/2}}db. \quad (6)$$

The GWSF provides information on the amount of generalized time shifts induced by an LTV system. Hence, the system output can now be expressed as a weighted superposition of generalized time-shifted and generalized warped scale-changed versions of the input signal, i.e.

$$(YX)(f) = \int\int GWSF_Y(\zeta,\alpha)(W_\xi^{-1}C_{e^\alpha}W_\xi D_{\frac{\zeta e^{-\alpha/2}}{\lambda(\alpha)}}X)(f)d\zeta d\alpha \quad (7)$$

where the generalized warping is $(\boldsymbol{W}_\xi X)(f) = X(\xi^{-1}(f))/|\varphi(\xi^{-1}(f))|^{1/2}$, the scale operator $(\boldsymbol{C}_a X)(f)=X(f/a)/|a|^{1/2}$ and $\boldsymbol{W}_\xi^{-1}C_\alpha W_\xi$ is the generalized warped scale operator[8].

The GP0WSY and GWSFY have the following relationship

Table 1. Examples of various P0-Weyl symbols, operator outputs, and wideband spreading functions.

Cases	TF symbol	WSF	Operator output
Conventional	G(f)	$g(\tau)\delta(\tau)$	G(f)X(f)
	ρ H(tf)	$\sqrt{e^{a}}H(e^{a})(\frac{1}{2}\delta(\tau)-\frac{j}{2\pi\tau})$	$\int\frac{1}{\nu}H(\frac{f}{\nu})X(\nu)d\nu, f>0$
Exponential	$\frac{1}{\varphi(f)}G(f)$	$\rho_G^{(\xi_{exp})}(\zeta)\delta(\alpha)$	$\sqrt{\frac{1}{e^f}}G(f)X(f)$
	h(t)	$H(\alpha)(\frac{1}{2}\delta(\zeta)-\frac{j}{2\pi\zeta})$	$\int H(f-\nu)X(\nu)\,d\nu$
Power	$G(f)/\sqrt{\varphi_\kappa(f)}$	$\rho_G^{(\xi_\kappa)}(\zeta)\delta(\alpha)$	$G(f)X(f)/\sqrt{\varphi_\kappa(f)}$
	$\sqrt{\kappa}\rho_H^{(\xi_\kappa)}(tf)$	$\sqrt{\frac{1}{\kappa}\xi_\kappa^{-1}(e^{a})}H(\xi_\kappa^{-1}(e^{a}))(\frac{1}{2}\delta(\zeta)-\frac{j}{2\pi\zeta})$	$\int\frac{\sqrt{\kappa}}{\nu}H(\frac{f}{\nu})X(\nu)d\nu$
Power exponential	$\frac{1}{\sqrt{\kappa e^{\kappa f}}}G(f)$	$\rho_G^{(\xi_{exp}^{(\kappa)})}(\zeta)\delta(\alpha)$	$\frac{1}{\sqrt{\kappa e^{\kappa f}}}G(f)X(f)$
	h(t)	$\frac{1}{\kappa}H(\frac{\alpha}{\kappa})(\frac{1}{2}\delta(\zeta)-\frac{j}{2\pi\zeta})$	$\int H(f-\nu)X(\nu)d\nu$

Table 2. Various wideband Weyl symbols and wideband spreading functions for a given warping function ξ(b). Here, Y is defined based on the domain of ξ(b).

$\xi(b)$	Time-freq. representation	Wideband spreading function
1 to 1	$GP_0WS_Y(t,f)=P_0WS_{W_\xi YW_\xi^{-1}}(\frac{t}{\varphi(f)},\xi(f))$	$GWSF_Y(\zeta,\alpha)=WSF_{W_\xi YW_\xi^{-1}}(\zeta,\alpha)$
b	$P_0WS_Y(t,f)$ in(3)	$WSF_Y(\tau,\alpha)$ in(2)
$\xi_{\exp}(b)$	$EP_0WS_Y(t,f)=P_0WS_{W_\xi YW_\xi^{-1}}(te^{-f},e^{f})$	$EWSF_Y(\zeta,\alpha)=WSF_{W_\xi YW_\xi^{-1}}(\zeta,\alpha)$
$\xi_\kappa(b)$	$P_0WS_Y^{(\kappa)}(t,f)=P_0WS_{W_\xi YW_\xi^{-1}}(\frac{t}{\varphi_\kappa(f)},\xi_\kappa(f))$	$PWSF_Y^{(\kappa)}(\zeta,\alpha)=WSF_{W_\xi YW_\xi^{-1}}(\zeta,\alpha)$
$\xi_{\exp}^{(\kappa)}(b)$	$EP_0WS_Y^{(\kappa)}(t,f)=P_0WS_{W_\xi YW_\xi^{-1}}(\frac{t}{\kappa e^{\kappa f}},e^{\kappa f})$	$EWSF_Y^{(\kappa)}(\zeta,\alpha)=WSF_{W_\xi YW_\xi^{-1}}(\zeta,\alpha)$

$$GWSF_Y(\zeta,\alpha)=GT_{f\to\zeta}[\frac{\xi(f)}{\sqrt{\varphi(f)}}F_{t\to\alpha(f)}[GP_0WS_Y(t,f)]], \quad (8)$$

where $\alpha(f)=\alpha\,\xi(f)/\varphi(f)$ and the generalized transform dependent on the warping function $\xi(b)$ is

$$GT_{f\to c}[X(f)]=\rho^{(\xi)}{}_X(c)=\int_c^d X(f)\sqrt{\varphi(f)}e^{j2\pi c\xi(f)}df. \quad (9)$$

When $\xi(b)=b$, the generalized transform reduces to the Fourier transform and the relationship in (8) simplifies to the relationship between the P0WS and WSF in (4). In Table 1, we provides special examples of the GP_0WS, its corresponding GWSF and operator output. For example, if the operator output is the product of the input signal X(f) and a function G(f) (column 5, row 5), then the GWSF is the generalized transform of G(f).

The GP_0WS and GWSF have the warping relationship with the P_0WS and WSF in (3) and (2), respectively

$$GP_0WS_Y(t,f)=P_0WS_{W_\xi YW_\xi^{-1}}(\frac{t}{\varphi(f)},\xi(f)) \quad (10)$$

$$GWSF_Y(\zeta,\alpha)=WSF_{W_\xi YW_\xi^{-1}}(\zeta,\alpha). \quad (11)$$

Depending on the warping function ξ(b), we can derive a specific P_0WS and WSF which are proper for analyzing systems producing a certain type of dispersive time shifts. For example, if ξ(b)=b, we obtain the conventional P_0WS and WSF from (5) and (6). In the following, we will show other special cases of the GP_0WS and GWSF (See Table 2.).

3.2 Exponential P0-Weyl Symbol

For an operator $\boldsymbol{Q}$ defined on $L_2(\mathbf{R})$ in frequency, when the warping function $\xi(b)=e^b$, the GP_0WS and the GWSF simplify to the exponential P_0-Weyl symbol (EP_0WS) and the exponential wideband spreading function (EWSF), respectively. Table 2 shows definitions of the EP_0WS and the EWSF (See row 3). The EP_0WS preserves exponential time shifts and constant frequency shifts on a random process X(f), i.e.

$$Y(f)=(E_\zeta X)(f)\Rightarrow EP_0WS_{R_Y}(t,f)=EP_0WS_{R_X}(t-\zeta e^{f},f),$$

$$Y(f)=(M_\nu X)(f)\Rightarrow EP_0WS_{R_Y}(t,f)=EP_0WS_{R_X}(t,f-\nu)$$

where $(\boldsymbol{M}_\nu X)(f)=X(f-\nu)$ and $(\boldsymbol{E}_\zeta X)(f)=e^{-j2\pi\zeta e^f}X(f)$ are the constant frequency-shifted signal and exponentially time-shifted signal, respectively [7]. The EWSF provides an important interpretation on the system output as a weighted superposition of exponential time-shifted and constant frequency-shifted versions of the input signal where the weighting function is the EWSF

$$(QX)(f)=\int\int EWSF_Q(\zeta,\alpha)(M_\alpha E_{\zeta e^{\alpha/2}/\lambda(\alpha)}X)(f)d\zeta d\alpha.$$

The relationship between the EP_0WS and the EWSF is given as

$$EWSF_Q(\zeta,\alpha)=ET_{f\to\zeta}\{\sqrt{e^f}F_{t\to\alpha}\{EP_0WS_Q(t,f)\}\}$$

where $ET_{f\to\zeta}\{X(f)\}=\int X(f)\,e^{f/2}\,e^{j2\pi\zeta e^f}df$. Table 1 provides special examples of the EP_0WS, its corresponding EWSF and operator output. For example, if the system output is the convolution of the input signal X(f) and a function H(f) (column 5, row 9), then the EP_0WS is the inverse Fourier transform of H(f).

3.3 Power P_0-Weyl Symbol

When the warping function is $\xi(b)=\xi_\kappa(b)=b^\kappa$, b>0, the power P_0WS, $P0WS^{(\kappa)}$, and the power WSF, $PWSF^{(\kappa)}$, are obtained from (10) and (11) (See row 5 in Table 2.). Note that when $\kappa=1$, the $P_0WS^{(\kappa)}$ and the $PWSF^{(\kappa)}$ simplify to the P_0WS and WSF, respectively. The $P_0WS^{(\kappa)}$ preserves power time shifts on a random process

$$Y(f)=(P_\zeta^{(\kappa)}X)(f)\Rightarrow P_0WS_{R_Y}^{(\kappa)}(t,f)=P_0WS_{R_X}^{(\kappa)}(t-\zeta\varphi_\kappa(f),f),$$

where $(P_\zeta^{(\kappa)}X)(f)=e^{-j2\pi\zeta\xi_\kappa(f)}X(f)$ is the power time shift operator and $\varphi_\kappa(f)=\xi_\kappa'(f)$. The system output is expressed as a weighted superposition of the power time-shifted input signal

$$(BX)(f)=\int\int PWSF_B^{(\kappa)}(\zeta,\alpha)(W_{\xi_\kappa}^{-1}C_{\xi_\kappa^{-1}(e^{-\alpha})}W_{\xi_\kappa}\cdot P_{\zeta e^{\alpha/2}/\lambda(\alpha)}^{(\kappa)}X)(f)d\zeta d\alpha$$

where $W_{\xi_\kappa}^{-1} \boldsymbol{C}a\boldsymbol{W}_{\xi_\kappa}$ is the power warped scale operator with the power warping operator $(\boldsymbol{W}_{\xi_\kappa} X)(f)=X(\xi_\kappa^{-1}(f))/|\varphi_\kappa(\xi_\kappa^{-1}(f))|^{1/2}$ [8]. The $P_0WS^{(\kappa)}$ and the $PWSF^{(\kappa)}$ have the relationship

$$PWSF^{(\kappa)}{}_B(\zeta, \alpha) = PT^{(\kappa)}{}_{f\to\zeta}\{\frac{f\sqrt{\varphi_\kappa(f)}}{\kappa} F_{t\to\frac{f\alpha}{\kappa}}\{P_0WS^{(\kappa)}{}_B(t,f)\}\},$$

where $PT^{(\kappa)}{}_{f\to\zeta}\{X(f)\}=\int X(f)\,|\varphi_\kappa(f)|^{1/2} e^{j2\pi\zeta\xi_\kappa(f)}\,df$.

3.4 Power Exponential P_0-Weyl Symbol

We obtain the κ-th power exponential P_0WS, $EP_0WS^{(\kappa)}$, and the κ-th power exponential WSF, $EWSF^{(\kappa)}$, for an operator $\boldsymbol{Q}$ on $L_2(R)$, by warping the P_0WS and the WSF as shown in row 6 of Table 2. Note that when $\kappa=1$, we obtain the EP_0WS and the EWSF. The $EP_0WS_Q^{(\kappa)}$ preserves power exponential time shifts and constant frequency shifts on a random process

$$Y(f)=(E_\zeta^{(\kappa)}X)(f) \Rightarrow EP_0WS_{R_Y}^{(\kappa)}(t,f) = EP_0WS_{R_X}^{(\kappa)}(t-\kappa\zeta e^{\kappa f}, f)$$
$$Y(f)=(M_\nu X)(f) \Rightarrow EP_0WS_{R_Y}^{(\kappa)}(t,f) = EP_0WS_{R_X}^{(\kappa)}(t, f-\nu)$$

where the power exponential time shift operator is $(E_\zeta^{(\kappa)} X)(f)=X(f)e^{j2\pi\zeta e^{\kappa f}}$ [8]. The system output can be expressed as a weighted superposition of power exponential time-shifted and constant frequency-shifted versions of the input signal, i.e.

$$(QX)(f)=\int\int EWSF_Q^{(\kappa)}(\zeta,\alpha)(M_{\frac{\alpha}{\kappa}} E_{\zeta e^{\kappa f}/\lambda(\alpha)}^{(\kappa)}X)(f)\,d\zeta\,d\alpha.$$

The relation between the $EP_0WS^{(\kappa)}$ and the $EWSF^{(\kappa)}$ is given as

$$EWSF_Q^{(\kappa)}(\zeta,\alpha)=\frac{1}{|\kappa|}\int\int EP_0WS_Q^{(\kappa)}(t,\frac{\ln(b)}{\kappa})e^{-j2\pi(\frac{\alpha}{\kappa}t-b\zeta)}\,dt\,db.$$

4. Application Examples

4.1 Analysis Applications

4.1.1 Narrowband WS vs. P_0WS.

In order to demonstrate the importance of the P_0WS, we analyze deterministic signals with random weights $X(f)=\sum \alpha_m X_m(f)$, $m=1,2,3$. Here, α_m are uncorrelated, zero-mean random weights and $X_m(f)=|f|^{-0.5}e^{j4\pi m \ln(f)}$, $f>0$, $m=1,2,3$, are hyperbolic FM, deterministic signals. Note that each signal term $X_m(f)$ has hyperbolic group delay, $2m/f$. One can show that the P_0WS in (3) of the correlation operator $\boldsymbol{R}_X$ simplifies to

$$P_0WS_{R_X}(t,f)=\sum_m E[|\alpha_m|^2]P_0WS_{R_{X_m}}(t,f)$$
$$=\sum_m E[|\alpha_m|^2]\frac{1}{f}\delta(t-\frac{2m}{f}), \quad f>0. \qquad (12)$$

Figure 1 shows (a) the narrowband WS versus (b) the P_0WS of $\boldsymbol{R}_X$ of a windowed X(f). Both show time-varying transfer functions with hyperbolic TF characteristics. The advantage of the P0WS in (12), is that it is ideally localized along the three group delay curves $t=2m/f$ in the TF plane.

Fig. 1. (a) narrowband Weyl symbol, NWSRx(t, f), and (b) wideband P_0-Weyl symbol, $P_0WSR_X(t, f)$, of a windowed hyperbolic process X(f).

The disadvantage of the narrowband WS is that it produces spurious components along hyperbolae since it does not match the intrinsic hyperbolic TF characteristics.

4.1.2 P_0WS vs. Power Exponential P_0WS.

In this analysis example, we compare the P_0WS with the $EP_0WS^{(\kappa)}$ to demonstrate the importance of our new generalization. The process has the power exponential TF characteristics, i.e. $X(f)=\sum \alpha_i X_i(f)$ where α_i are uncorrelated, zero-mean random weights and $X_i(f)=|\kappa|^{0.5}e^{\kappa f}e^{j2\pi c_i e^{\kappa f}}$, are power exponential FM, deterministic signals with $c_1=1$, $c_2=2$, $c_3=3.6$ and $\kappa=2.3$. Figure 2 shows (a) the P_0WS and (b) the $EP_0WS^{(\kappa)}$ of $\boldsymbol{R}_X$ of a windowed X(f). The

Fig. 2. (a) P_0-Weyl symbol, $P_0WS_{R_X}(t, f)$, and (b) power exponential P_0-Weyl symbol, $EP_0WS^{(\kappa)}{}_{R_X}(t, f)$, of a windowed power exponential process X(f).

$EP_0WS^{(\kappa)}$ shows ideally localized TF representation along the three group delay curves $t=c_i\kappa e^{\kappa f}$ in the TF plane. The P_0WS shows spurious components inside each group delay curve since it does not match the power exponential TF characteristics.

4.2 Detection Application

Sibul et al. formulated a detector in the time-scale domain using the wideband ambiguity function [9]. For the received signal $r(t)=y(t)+n(t)$ where $y(t)=(\boldsymbol{L}\,x)(t)$ is the output of a channel $\boldsymbol{L}$ for the transmitted signal x(t) and n(t) is the nonstationary zero mean Gaussian random noise, Sibul et al. implemented the estimator-correlator using the WSF and the WAF

$$\Lambda=\int\int WSF_L{}'(\tau,\alpha)WAF_{XR_n}(\tau,\alpha)\,d\tau\,d\alpha$$

where $WSF'_L(\tau, \alpha) = \int\int SC_L(\tau, \alpha, \tau', \alpha') WAF_{XR1}(\tau', \alpha') d\tau' d\alpha'$, $r_1(t) = \int K_{Rr}^{-1}(t, \tau)\, r(\tau)\, d\tau$, when the signal present, $r_2(t) = \int K_{Rr}^{-1}(t, \tau)\, r(\tau)\, d\tau$, when the signal absent, and $R_0(f)$ and $R_1(f)$ are Fourier transforms of $r_0(t)$ and $r_1(t)$, respectively. Here, the scattering function $SC_L(\tau, \alpha, \tau', \alpha') = \mathbf{E}[WSF_L(\tau, \alpha) WSF_L(\tau', \alpha')]$ is assumed to be known [9]. It can be shown that the estimator-correlator is also formulated using the generalized WSF and the generalized WAF

$$\Lambda = \int\int GWSF_L{}'(\zeta, \alpha) GWAF_{XR_0}(\zeta, \alpha)\, d\zeta d\alpha$$

where $GWAF_X(\zeta, \alpha) = WAF_{W\xi X}(\zeta, \alpha)$ and $GWSF'_L(\zeta, \alpha) = \int\int GSC_L(\zeta, \alpha, \zeta', \alpha')\, GWAF_{XR1}(\zeta', \alpha')\, d\zeta' d\alpha'$ with $GSC_L(\zeta, \alpha, \zeta', \alpha') = \mathbf{E}[GWSF_L(\zeta, \alpha) GWSF_L(\zeta', \alpha')]$.

5. Conclusion

In this paper, using warping techniques, we derived the new generalized P_0WS and the generalized WSF. For example, we derived the exponential P_0WS and the exponential WSF, the power P_0WS and the power WSF, and the power exponential P_0WS and the power exponential WSF. These generalized P_0WS and WSF are useful for systems producing generalized time shifts and generalized warped scale changes on the signal whereas the conventional P_0WS and WSF are useful for systems producing constant time shifts and scale changes on the signal. We also investigated the relationship between the AWS and the P_0WS, and the properties of the P_0WS. We demonstrated the advantages of our new results by providing application examples.

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