International Journal of Management Science
Vol 7, No 2, November 2001

NONCONVEX BULK TRANSPORTATION PROBLEM

S. R. Arora
Department of Mathematics, Hans Raj College,
University of Delhi, Delhi-110007 (India)

Anu Ahuja
Department of Mathematics, Jesus and Mary College,
Chanakyapuri, New Delhi-110021 (India)

(Received April 2000 ; Revised Dec. 2000 and Sep. 2001 ; Accepted Nov. 2001)

ABSTRACT

In the present paper, we present a method to solve a Fractional Bulk Transportation Problem (FBTP) in which the numerator is quadratic in nature and the denominator is linear. A related (FBTP) is formed whose feasible solutions are ranked to reach an optimal solution of the given problem. The method to find these feasible solutions makes use of parametric programming wherein a series of Ordinary Bulk Transportation Problems arc solved by the usual methods.

1. INTRODUCTION

Bulk Transportation Problem (BTP) is an important class of Transportation Problems. It is linked with the supply of material from different sources to various warehouses with the condition that each warehouse will draw its total requirement from only one source. This additional constraint named as bulk or zero-one constraint arises often in practical situations. Applications of (BTP) have been adopted in various areas like assigning software development tasks to programmers, the vehicle routing problems [5], the scheduling and facility location problem [6] and the fixed-charge plant location models in which customer requirements must be satisfied by single plant [11, 12]. Bahman et al. [1], Hochbaum [4] have developed algorithms for solving quadratic transportation problem having the 0-1 character. The fractional programming problems have been studied extensively by many researchers. Gutenberg [7] tackled the problem of optimizing productivity of the material in industry plant, Mjelde [10] maximized the ratio of the return and the cost in resource allocation problems, Kydland [8] on the other hand maximized the profit per unit time in a cargo-loading problem, etc. The linear and nonlinear models of fractional programming problems have been studied in [2, 8]. In this paper, we have considered a fractional bulk transportation problem in which the numerator is quadratic in nature and denominator is linear. This problem has not been studied so far. An attempt is made to develop a solution methodology for this problem. A Related Linear Fractional Bulk Transportation Problem is formed whose basic feasible solutions are ranked to reach an optimal solution of the given problem. Ranking of the basic feasible solutions of the related problem tightens the interval containing the optimal value of the given problem and then helps to find a solution to the given problem. A method based on Dinkelbach's approach [3] is used to find these feasible solutions wherein a series of ordinary Bulk Transportation Problems are solved by the usual methods [13, 14].

2. THEORETICAL DEVELOPMENT

The mathematical model of the (FBTP) in which the numerator is quadratic and denominator is linear is given by

$$(P\text{-}1): \quad \underset{x \in S}{\text{Minimize}} \quad P(x) = \frac{p_1(x)}{p_2(x)}$$

$$= \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} + \left(\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}\, x_{ij}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij}\, x_{ij}\right)}{\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}\, x_{ij}}$$

where

$$S = \left\{ x = \{x_{ij}\} \,\middle|\, 0 < \sum_{j=1}^{n} b_j\, x_{ij} \le a_i \; ; i \in I \; ; \sum_{i=1}^{m} x_{ij} = 1, \; j \in J \; ; \; x_{ij} = 0 \text{ or } 1, (i,j) \in I \times J \right\}$$

$I = \{1, 2, \cdots m\}$, the index set of the sources

$J = \{1, 2, \cdots n\}$, the index set of the destinations, $m \le n$,

$b_j > 0$, denotes the requirement of the destination j,

$a_j > 0$, denotes the availability at the source i,

c_{ij}, d_{ij}, e_{ij} and f_{ij} are all non-negative and are independent of the quantity transported from the i^{th} source to the j^{th} destination.

$$x_{ij} = \begin{cases} 1, & \text{if the requirement of the } j^{th} \text{ destination is met by the } i^{th} \text{ source} \\ 0, & \text{otherwise} \end{cases}$$

Here we assume that $P_2(x) > 0, \forall\ x \in S$.

Remark 1: Since $x_{ij} = 0$ or 1, $\forall\ (i, j) \in I \times J$, the solution $x = \{x_{ij}\}$ of (P-1) will be an extreme point of a hypercube $I_{m+n}x \le 1$.

Remark 2: (1) A feasible solution of a (BTP) is written as a set of cells (i, j)'s for which $x_{ij} = 1$.

(2) A feasible solution of a (BTP) can exist, if

(i) for every b_j, there exists an a_i, such that $b_j \le a_i$, and

(ii) $\sum_{j=1}^{n} b_j \le \sum_{i=1}^{m} a_i$

A (BTP) related to (P-1) is (P-2) given by

(P-2):
$$\underset{x \in S}{\text{Minimize}} \quad Q(x) = \frac{Q_1(x)}{Q_2(x)} = \frac{Q_1(x)}{P_2(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij} + d_{ij}e_{ij})x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}x_{ij}}$$

Theorem 1. $Q(x) \le P(x), \quad \forall\ x \in S$.

Proof. Since c_{ij}'s, d_{ij}'s and e_{ij}'s are all non-negative, clearly $Q(x) \le P(x)$, $\forall\ x \in S$.

Remark 3: Let x_1 be an optimal solution to problem (P-2). Then $Q(x_1)$ is the lower bound on the optimal value P^* (say) of problem (P-1) because $Q(x) \le P(x), \ \forall\ x \in S$. Therefore, problem ($P$-2) provides the lower bound, and hence is the bounding (BTP) for problem (P-1)

Theorem 2. Let $S^k (k \ge 1)$ be the set of k^{th} best feasible solutions of (P-2) yielding the value Q^k and let

$$P^k = \text{Min}\{P(x) \mid x \in S^k\}$$

If $Q^k \geq \text{Min}\{P^1, P^2, ..., P^{k-1}\} = P^q = P(x^r)$ (say), then P^q is the optimal value of $P(x)$ and x^r is a global optimal solution of (P-1).

Proof. Since, $P^q \geq \text{Min}\{P^1, P^2, ..., P^{k-1}\}$, we have

$$P^i \geq P^q, \forall\, i = 1, 2, ..., k-1 \tag{1}$$

As $\quad p^k \geq Q^k \quad$ (By Theorem 1)

and $\quad Q^k \geq P^q \quad$ (By hypothesis)

$\therefore \quad P^k \geq P^q \quad$ (2)

Also, $\quad P^i \geq Q^i > Q^k, \forall\, i \geq k+1$

$\quad \geq P^q \quad$ (3)

Thus by (1), (2) and (3) $P^i > P^q, \forall\, i = 1, 2, 3, ...$

Hence x^r is a global optimal solution of (P-1).

Theorem 3. If for a $k(\geq 1), Q^k = P^k$

then optimal value of $P(x)$ is $\text{Min}\{P^1, P^2, ..., P^k\}$.

Proof. Let $\text{Min}\{P^1, P^2, ..., P^k\} = P^l = P(x^t)$, (say) $\Rightarrow P^i \geq P^l, \forall\, i = 1, 2, ..., k$.

Also, $\quad P^i \geq Q^i > Q^k, i \geq k+1$

$\quad = P^k$

$\quad = P^l$

$\therefore \quad P^i \geq P^l, \forall\, i.$

$\therefore$ p^l is the optimal value of (P-1) and x^t is its global optimal solution.

Theorem 4. If $Q^k < \text{Min}\{P^1, P^2, ..., P^k\}$, then $Q^k < P^* \leq \text{Min}\{P^1, P^2, ..., P^k\}$ where P^* is the optimal value of $P(x)$ in problem (P-1).

Proof. As $P(x) \geq Q(x)\ \forall\, x \in S$, we have

$$P^w \geq Q^w.$$

Since for $w \geq k+1, Q^w > Q^k$, we get

$$P^w > Q^k, \forall\, w \geq k+1$$

Therefore, $\text{Min}\{P^{k+1}, P^{k+2}, ..., P^N\} > Q^k$ (4)

Also, by hypothesis $\text{Min}\{P^1, P^2, ..., P^k\} > Q^k$ (5)

By (4) and (5), we get $P^* > Q^k$ (6)

Thus $Q^k > P^* \leq \text{Min}\{P^1, P^2, ..., P^k\}$.

Remark 4: Problem (P-2) is a linear fractional programming problem whose optimal solution lies at the extreme point of the hypercube $I_{m+n}x \leq 1$. These extreme points are finite in number and our algorithm moves from one extreme point to another till such an extreme point is obtained for which $Q^k \geq P^q = P(x^r)$. Hence the algorithm will converge in a finite number of steps. (P-2) is a Fractional Bulk Transportation Problem. To solve (P-2), a Dinkelbach's [3] approach is made use of wherein a series of Ordinary Bulk Transportation Problems are solved. Consider the following Ordinary Bulk Transportation Problem.

(P-3): $$F(\lambda) = \underset{x \in S}{\text{Minimize}} \; \{ Q_1(x) - \lambda Q_2(x) \}, \lambda \in \Re.$$

(P-3) is a parameterized form of problem (P-2). The following results due to Dinkelbach provide motivation for a computational approach to solve (P-2) in terms of the parametric programme (P-3).

Result 1: $F(\lambda) = \underset{x \in S}{\text{Minimize}} \; \{ Q_1(x) - \lambda Q_2(x) \}, \lambda \in \Re.$ is a strictly monotonically decreasing function of λ.

Remark 5: $F(\lambda)$ cannot vanish. at more than one point, i.e., $F(\lambda) = 0$ has a unique solution.

Result 2: $F(\lambda)$ is a concave function of λ.

Remark 6: As $F(\lambda)$ is a concave function of λ over $\Re$, it is continuous over $\Re$.

Result 3: For any $\overline{x} \in S, F(\overline{\lambda}) \leq 0$, where $\overline{\lambda} = \dfrac{Q_1(\overline{x})}{Q_2(\overline{x})}$

Result 4: $x^o \in S$ is an optimal solution of (P-2) iff x^o is an optimal solution of

(P-3) yielding value zero, where $\lambda = \dfrac{Q_1(x^o)}{Q_2(x^o)}$.

3. METHOD TO SOLVE (P-2)

Initial Step: Solve $F(\lambda_1) = \underset{x \in S}{\text{Minimize}}\ Q_1(x)$ for $\lambda_1 = 0$.

Compute $\lambda_2 = \dfrac{Q_1(x^1)}{Q_2(x^1)}$, where x^1 is an optimal solution of $F(\lambda_1)$.

General Step: Solve $F(\lambda_r) = \underset{x \in S}{\text{Minimize}}\ \{ Q_1(x) - \lambda_r Q_2(x) \}, r \geq 2,$

where $\lambda_r = \dfrac{Q_1(x^{r-1})}{Q_2(x^{r-1})}$, x^{r-1} being the optimal solution of $F(\lambda_{r-1})$

If $F(\lambda_r) = 0$, then x^r is an optimal solution of (P-3) and hence an optimal solution of (P -2); otherwise repeat the general step for $r = r + 1$.

On the basis of the theory developed above we give below an algorithm to solve (P -1).

4. ALGORITHM TO SOLVE (P-1)

Step-1. Form the Related Bulk Transportation Problem (P-2)

Step-2. Using the Dinkelbach's, approach from the corresponding parameteric programming problem (P-3)

Step-3 (a) Let $X^0 = \{(i_1^o, 1), (i_2^o, 2), ..., (i_n^o, n)\}$, where $i_p^o \in I$, $p = 1, 2, ..., n$ be the optimal solution of (P-2) with corresponding objective function value Q'.

(b) Let P^1 be the objective function value of (P-1) at x^o.

If $P^1 = Q^1$, stop; x^o is the optimal solution of (P-1). Otherwise generate n mutually exclusive and exhaustive nodes $K_1^1, K_2^1, ..., K_n^1$ defined as

$$K_p^1 = \left\{(i_1^o, 1), (i_2^o, 2), ..., (i_{p-1}^o, p-1), \overline{(i_p^o, p)}\right\}$$

where (i_j^o, j) denotes the cell constrained to be included in the solution and $\overline{(i_j^o, j)}$ denotes the cells constrained to be excluded from the solution.

(c) For each node K_p^1 construct and solve the related linear fractional (BTP) (R_p^1), $p = 1, 2, ..., n$. Each R_p^1 is of the form of (P-2) and hence can be solved as given in Step-2.

(d) Let x_p^1 be the optimal solution of problem R_p^1 with the corresponding objective function value Z_p^1, $p = 1, 2, ..., n$.

Step-4. Find $\underset{p=1,2,...,n}{\text{Minimize}} \left[Z_p^1 \middle| R_p^1 \text{ is feasible}\right] = Z_{u_1}^1$ (say) corresponding to the solution $x_{u_1}^1$.

$$x_{u_1}^1 = \left\{ (i_1^o, 1), (i_2^o, 1), ..., (i_{u_1-1}^o, u-1), \overline{(i_{u_1}^o, u_1)} \right\}$$

If $Q^1 = Z_{u_1}^1$, then $x_{u_1}^1$ is the alternate optimal solution of (P-2).

If $z_{u_1}^1 > Q^1$ then $x_{u_1}^1$ is second best feasible solution of (P-2). We call this solution as x^1.

Step-5. At the second best solution x^1, let the objective function value of (P-2) be Q^2 and that of (P-1) be P^2.

There are three possibilities:

(i) $Q^2 = P^2$, then x^1 is the optimal solution of (P-1).

(ii) $Q^2 > P^1$, then x^o is the optimal solution of (P-1).

(iii) $Q^2 < \text{Min}(P^1, P^2)$, then we proceed to find the third best solution of (P-2) by branching R_u^1.

General Step Set $t = r$ and proceed to find the $(r+1)^{th}$ best feasible solution of (P-2) by branching $R_{u_{r-1}}^{r-1}$ with the corresponding optimal solution x^{r-1}.

Generate new nodes K_p^r, $p = 1, 2, ..., n$. For each node construct and solve the fractional bulk transportation problem R_p^{r-1}, $p = 1, 2, ..., n$. Let X_p^r, $p = 1, 2, ..., n$ be their optimal solutions, with the objective function value Z_p^r.

Find $\text{Min}\{Z_p^r \mid R_p^r \text{ is feasible}\} = Z_{u_r}^r$ (say) corresponding to the solution $x_{u_r}^r$.

Let Q^r be the objective function value of (P-2) at $x_{u_r}^r$. If $Q^{r-1} = Z_{u_r}^r$, then $x_{u_r}^r$ is the alternative optimal solution of (P-2), otherwise $x_{u_r}^r$ is the alternate optimal solution of (P-2). We call this solution as x^r.

Let p^r be the objective function value of (P-1) at x^r. There are three possibilities.

(i) $Q^r = P^r$, then x^r is the optimal solution of (P-1).

(ii) $Q^r \geq \text{Min}\{P^1, P^2, ..., P^{r-1}\} = P^t = P(x^t)$ (say), then x^t is the optimal solution of problem (P-1).

(iii) If $Q^r \leq \text{Min}\{P^1, P^2, ..., P^r\}$, then repeat this step to find the next best solution.

Remark 7. The algorithm is bound to converge in a finite number of steps because at each stage, the value of Q^r increases and $\text{Min}\{P^1, P^2, ..., P^{r-1}\}$ either remains the same or decreases.

Remark 8: In the algorithm we need to solve a series of ordinary bulk transportation problems of the type (P-3). For the set in which a cell (i', j') is constrained to be included in the optimal solution of (P-3), we know that (i', j') will be the only cell in column j'. Consequently, column j' is dropped, $a_{i'}$, is modified to $a_{i'} - b_{j'}$, and a smaller transportation problem is solved. This reduction in $a_{i'}$, further simplifies the problem, since the cells (i', j) for which b_j is greater than $a_{i'} - b_j$ cannot be basic in any feasible solution and for such cells the cost is taken as ∞. Thus the number of cells under consideration is also reduced.

5. MUMERICAL EXAMPLE

Consider the following FBTP

$$(P\text{-}1): \quad \underset{x \in S}{\text{Minimize}} \quad P(x) = \frac{P_1(x)}{P_2(x)}$$

$$= \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} + \left(\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}\, x_{ij}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij}\, x_{ij}\right)}{\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}\, x_{ij}}$$

where
$$S = \left\{ x = \{x_{ij}\} \,\middle|\, 0 < \sum_{j=1}^{n} b_j\, x_{ij} \le a_i \;;\; i \in I \;;\; \sum_{i=1}^{m} x_{ij} = 1, \right.$$
$$\left. j \in J \;;\; x_{ij} = 0 \text{ or } 1, (i, j) \in I \times J \right\}$$

$I = \{1, 2, 3\}$, $J = \{1, 2, 3, 4\}$, $a_1 = 10$, $a_2 = 15$, $a_3 = 18$; $b_1 = 10$, $b_2 = 10$, $b_3 = 5$, $b_4 = 12$. c_{ij}'s, d_{ij}'s, e_{ij}'s and f_{ij}'s are given in Table 1 below:

Table 1.

	c_{ij}	d_{ij}	e_{ij}									
c_{ij} ←	5	5	0	7	0	2	3	4	0	2	0	1
	3			4			1			1		
	4	0	0	4	3	0	5	1	1	2	1	0
	2			1			3			1		
	5	1	2	3	1	2	4	0	1	5	1	0
f_{ij} ←	2			1			1			2		

The related linear fractional BTP is $(P\text{-}2)$ given by

$$(P\text{-}2): \quad \underset{x \in S}{\text{Minimize}} \quad Q(x) = \frac{Q_1(x)}{Q_2(x)} = \frac{Q_1(x)}{P_2(x)}$$

$$= \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} (c_{ij} + d_{ij}\, e_{ij})\, x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}\, x_{ij}}$$

$$= \frac{5x_{11}+7x_{12}+3x_{13}+2x_{14}+4x_{21}+4x_{22}+6x_{23}+2x_{24}+7x_{31}+5x_{32}+4x_{33}+5x_{34}}{3x_{11}+4x_{12}+x_{13}+x_{14}+2x_{21}+x_{22}+3x_{23}+x_{24}+2x_{31}+x_{32}+x_{33}+2x_{34}}$$

To solve (P-2) we consider the following BTP

(P-3): $F(\lambda) = \underset{x\in S}{\text{Minimize}}\{Q_1(x) - \lambda Q_2(x)\}$,

Let $\lambda = \lambda_1 = 0$.

Then $F(\lambda_1) = F(0) = \underset{x\in S}{\text{Minimize}}\, Q_1(x)$

$F(\lambda_1)$ is an ordinary BTP and is solved by any of the methods [11, 12].
The optimal solution of $F(\lambda_1)$ is given by

$$x^{\lambda_1} = \{(1,2),(2,1),(3,3),(3,4)\}$$

Which is the starting solution to solve (P-3).

$$\text{Evaluate } \lambda_2 = \frac{Q_1(x^{\lambda_1})}{Q_2(x^{\lambda_1})} = \frac{20}{9}.$$

On solving $F(\lambda_2) = \underset{x\in S}{\text{Minimize}}\{Q_1(x) - \lambda_2 Q_2(x)\}$, we find that the optimal solution is given by $x^{\lambda_2} = \{(1,2),(2,1),(2,3),(3,4)\}$ and $F(\lambda_2) = \frac{-22}{9} \neq 0$.

$$\text{Evaluate } \lambda_3 = \frac{Q_1(x^{\lambda_2})}{Q_2(x^{\lambda_2})} = 2.$$

On solving $F(\lambda_3)$, we get its optimal solution as $\{(1,2),(2,1),(2,3),(3,4)\}$ and $F(\lambda_3) = 0$.

Then $x^1 = \{(1,2),(2,1),(2,3),(3,4)\}$ is the optimal solution of (P-2). We call it the best feasible solution of (P-2). Corresponding objective function value of (P-2) is

$$Q^1 = \frac{Q_1(x^1)}{Q_2(x^1)} = 2.$$

Objective function value of (P-1) at x^1 is given by

$$P^1 = \frac{P_1(x^1)}{P_2(x^1)} = 2.45.$$

Here $P^1 \neq Q^1$, therefore we find the second best feasible solution of (P-2).

Form the nodes $K_1^1 = \{\overline{(1,2)}\}$, $K_2^1 = \{(1,2),\overline{(2,1)}\}$, $K_3^1 = \{(1,2),(2,1),\overline{(2,3)}\}$ and $K_4^1 = \{(1,2),(2,1),(2,3),\overline{(3,4)}\}$, where $\overline{(i,j)}$ denotes cell (i,j) is constrained to be excluded from the solution and (i,j) denotes cell (i,j) is constrained to be included in the solution.

Let R_1^1, R_2^1, R_3^1 and R_4^1 be the related problems formed at nodes K_1^1, K_2^1, K_3^1 and K_4^1 respectively.

The optimal solution of R_1^1 is given by $x_1^1 = \{(1,1),(2,2),(2,3),(3,4)\}$ and the corresponding objective function value is given by $Z_1^1 = 2.22$.

The optimal solution of R_2^1 is given by $x_2^1 = \{(1,2),(2,4),(3,1),(3,3)\}$ and the corresponding objective function value is $Z_2^1 = 2.5$.

The optimal solution of R_3^1 is given by $x_3^1 = \{(1,2),(2,1),(3,3),(3,4)\}$ and the corresponding objective function value is $Z_3^1 = 2.22$.

Problem R_4^1 is infeasible.

$$\text{Now, } \text{Min}\{Z_1^1, Z_2^1, Z_3^1\} = Z_1^1 = Z_3^1 = 2.22.$$

Therefore the set of second best feasible solutions of (P-2) is given by $\{x_1^1, x_3^1\}$.

$$Q^2 = 2.22.$$

$$P^2 = \text{Min}\{P(x_1^1), P(x_3^1)\} = \text{Min}\{3.22,\ 2.33\} = 2.33 = P(x_3^1).$$

$$\text{Min}\{P^1, P^2\} = P^2 = 2.33.$$

Here $Q^2 \ngtr P^1$ and $Q^2 < \text{Min}\{P^1, P^2\}$.

Therefore we go on to find the third best feasible solution of (P-2). For that we branch both R_1^1 and R_3^1. We first branch R_1^1. The nodes obtained are as follows:

$$K_1^2 = \{\overline{(1,2)}\,;\,\overline{(1,1)}\}$$

$$K_2^2 = \{\overline{(1,2)}\ ;\ (1,1),\ \overline{(2,2)}\}$$
$$K_3^2 = \{\overline{(1,2)}\ ;\ (1,1),\ (2,2),\overline{(2,3)}\}$$
$$K_4^2 = \{\overline{(1,2)}\ ;\ (1,1),\ (2,2),\ (2,3),\ \overline{(3,4)}\}$$

Let $R_1^2, R_2^2, R_3^2, R_4^2$ be the problems formed at nodes $K_1^2, K_2^2, K_3^2, K_4^2$ respectively.

Problems R_1^2 and R_4^2 are infeasible.

The optimal solution of R_2^2 is given by $x_2^2 = \{(1,1)(2,4)(3,2)(3,3)\}$ and the corresponding objective function value is given by $Z_2^2 = 2.67$.

The optimal solution of R_3^2 is given by $x_3^2 = \{(1,1)(2,2)(3,3)(3,4)\}$ and the corresponding objective function value is $Z_3^2 = 2.57$.

$$\text{Min}\{Z_2^2, Z_3^2\} = Z_3^2 = 2.57.$$

The problem formed at nodes obtained by branching problem R_3^1 are infeasible.

Therefore the third best feasible solution of (P-2) is given by $x_3^2 = \{(1,1),(2,2),(3,3),(3,4)\}$.

Objective function value of (P-2) at x_3^2 is $Q^3 = 2.57$.

Objective function value of (P-1) at x_3^2 is $P^3 = 3.84$.

$$\text{Min}\{P^1, P^2\} = P^2 = 2.33. \quad \text{Here } \ Q^3 > P^2 = \text{Min}\{P^1, P^2\}.$$

Hence x_3^1 is the global optimal solution of (P-1).

Acknowledgement: The authors are grateful to the referees for their valuable suggestions.

REFERENCES

[1] Bahman Kalantri and Ansuman Bagchi, "An Algorithm for Quadratic Zero-One Programs," Naval Research Logistics Quarterly, Vol.37 (1990), 527-538.

[2] Charnes, A. and W. W. Cooper, "Programming with Linear Fractional Functionals," Naval Research Logistics Quarterly, Vol.19 (1962), 181-186.

[3] Dinkelbach, W., "On Non Linear Fractional Programming," Management Science, 13 (1967), No.7, 492-498.

[4] Dorit, S. Hochbaum, Rom Shamir and J. C. Shanthikumar, "A Polynomial Algorithm for an Integer Quadratic Non-separable Transportation Problems," Mathematical Programming, Vol.5 (1992), 54-72.

[5] Fisher, N. L. and R. Jai Kumar, "A Generalized Assignment Heuristic for Vehicle Routing," Networks, Vol. 11 (1981), 109-124.

[6] Geoffrion, A. M. and G. W. Graves, "Multi-Commodity Distribution System Design by Benders Decomposition," Management Science, Vol.20 (1974), No.5, 822-844.

[7] Gutenberg, E., Einfuhrung in dre Betrieburitschafts lehra (Gobler, Wierbaden), 1975.

[8] Kydland, F., "Simulation of Linear Operations," Institute of Shipping Research Norwegian School of Economics and Business Administration, Bergen, translated from Sosialoekonomen 23, (1969).

[9] Maio, I. O, DE., Roveda, C. A. "An all Zero-one Algorithm for a Certain Class of Transportation Problems," Operations Research, Vol.19 (1971), 1706, 1418.

[10] Mjelde, K. M., "Allocation of Resources According to a Fractional Objective," European Journal of Operational Research, Vol.2 (1978), 116-124.

[11] Ross, G. T. and Soland, R. M., "A Branch and Bound Algorithm for the Generalized Assignment Problem," Mathematical Programming, Vol.8 (1975), 92-103.

[12] Ross, G. T. and R. M. Soland, "Modeling Facility Location Problems and Generalized Assignment Problems," Management Science, Vol.24 (1977), No.3, 345-357.

[13] Srinivasan, V. and G. L. Thompson, "An Algorithm for Assigning User to sources in a Special Class of Transportation Problems," Operations Research, Vol.21 (1973), 284-295.

[14] Sundra Murthy, M., "A Bulk Transportation Problem," Opsearch, Vol.13 (1976), No.3-4, 143-155.