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DECIDING ON HOW TO DECIDE BEST

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ABSTRACT

Regarding all methods of decision making as parts of the same subject, one is astounded to find that the field speaks to the practitioners about multicriteria decision making with a multi-forked tongue, different methods give different and conflicting outcomes even for very simple decisions. In this paper, five well-known decision theories are described and used to work out a simple decision to choose the best of three cars. The outcomes turn out not to be the same for all the methods, which is troubling because one would expect unique answers in decision making. Several "meta" criteria that are essential for making a decision theory reliable are suggested. These criteria stand on their own and do not require yet another decision theory to make a choice as to which decision approach is best.

1. INTRODUCTION

We have a substantial concern that theories of multicriteria decision making do not all yield the same best outcome. In fact, some of them can recommend as best, the worst alternative of another approach. We begin by listing many of the multi-criteria methods mentioned in the literature, but then confine our exposition to five well-known ones. For these, we give a summary of their theoretical under-pinnings and then work out the same example for all of them, and note the best recommended outcomes. We found that they do not always agree and believe that in different hands, still other outcomes might have resulted. The question is whether there are some fundamental criteria that a decision theory must meet in order for its outcome to be "right" or "true" in some scientific and mathematical sense. We give several suggested criteria at the end of the paper. We ask that we might be forgiven by the versed practitioners of these methods, in case we have not included all the latest refinements known about them. We have tried to be as simple and direct as we could, to make the ideas accessible to a wide readership.

When the feasible set of alternatives of a decision consists of a finite number of elements that are explicitly known in the beginning of the solution process, we have an important class of problems called multicriteria evaluation problems. Sometimes these problems are referred to as discrete multicriteria problems or selection problems.

When the number of alternatives of a decision is uncountably infinite, the alternatives are not specified directly, but defined in terms of decision variables as usually done in single optimization problems like linear programming. The problem is called a continuous decision problem in which the alternatives are only implicitly known. This kind of problem is referred as a multicriteria design problem or a continuous multicriteria problem. Here is a listing of many decision techniques found in the literature [1, 2, 10, 17, 19, 28, 33].

(Multicriteria) Evaluation Methods: The outranking approach (Software: ELECTRE) by Roy [23, 24]; other methods developed by some French-Belgian School researchers are: ORESTE by Roubens [22] and Pastijn and Leysen [21], PROMETHEE by Brans, Mareschal, and Vincke [3]. Multiattribute utility theory (MAUT) by Keeney and Raiffa [11], The analytic hierarchy process (AHP) (software: Expert Choice)by Saaty [27], The regime method by Hinloopen, Nijkamp and Rietveld [9], The convex cone approach by Korhonen, Wallenius and Zionts [16], The hierarchical interactive approach by Korhonen [12], The visual reference direction approach (software: VIMDA) by Korhonen [13], The aspiration-level interactive method (AIM) by Lotfi, Stewart, and Zionts [18].

The second group is very large, and is almost impossible to list. We mention some of the better known approaches developed for multiple objective linear programming.

Multiple Criteria Design Methods: Goal programming of Charnes & Cooper [5], The method of Geoffrion, Dyer, and Feinberg [8], The method of Zionts-Wallenius [38], The reference point method of Wierzbicki [34], The reference direction method of Korhonen and Laakso [14], Pareto race of Korhonen & Wallenius [15], Interactive weighted Tchebycheff procedure of Steuer and Choo [29, 30].

Among these methods, we deal with the five methods that is widely used: The Analytic Hierarchy Process (AHP), Bayesian Analysis (BA), Data Envelopment Analysis (DEA), Multiattribute Utility Theory (MAUT), and Outranking Methods of which we discuss ELECTRE.

All these theories assume independence among the alternatives of the decision. Some beg the question of independence by exercising behind-the-scene freedom to examine and condition thinking about the alternatives so they no longer can be considered independent. What is needed here is a general theory for dependence and feedback to enable one to deal with such complexity without compromising one's basic assumptions. However, alternatives may be independent in their function, but dependent in their structure. For example, pieces of gold may be independent with respect to their value, which is high because they are gold, but having mountains of gold depreciates the value of any single piece and makes its value depend on how much gold is out there. It turns out that depending on how one looks at it such considerations can lead to different outcomes in a decision. One cannot use uniqueness as criterion because uniqueness requires an awareness of other alternatives, thus violating the assumption of independence which is essential.

2. FIVE WELL-KNOWN METHODS

We now briefly describe each method and apply it to the decision to choose a best car from the set A_1, A_2, A_3 and six criteria or attributes used to judge the cars: purchasing price, performance, economy, value depreciation, maintenance cost, overall appeal [37]. Information about the criteria and their ranges and about the alternatives is provided in Tables 1 and 2. The information in Table 1 gives ranges of values of the criteria, but says nothing about their intrinsic importance.

Table 1. Six criteria and their ranges

Criteria		Ranges
C_1	Purchasing Price	2000 to 5000 (cash or financed price in dollars)
C_2	Performance	100 to 150 (horsepower)
C_3	Economy	20 to 30 (miles per gallon)
C_4	Value Depreciation	20 to 60 (percent of the purchase price recoverable 5 years from now)
C_5	Maintenance Cost	1500 to 2200 (dollars per year)
C_6	Overall Appeal	1 to 5 (where 1 is ugly and 5 is beautiful)

Table 2. Estimated values of alternatives for the criteria

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	3000	120	30	40	1600	3
A_2	3500	140	21	30	2000	4
A_3	3600	130	25	50	1800	5

2.1 The Analytic Hierarchy Process (AHP)

The AHP, developed by Saaty [25, 26] in the early 1970's, is a general theory of measurement. It is used to derive ratio scales from both discrete and continuous paired comparisons in multilevel hierarchic structures. These comparisons may be taken from actual measurements or from a fundamental scale of absolute numbers, that reflects the relative strength of preferences, applied to homogeneous clusters of elements. The use of pivots from cluster to cluster inherently extends the scale through paired comparisons far beyond the 1 to 9 range. It has found its widest applications in multicriteria decision making, in planning and resource allocation, and in conflict resolution [31, 36].

The AHP is a systematic procedure for representing the elements of any problem. It organizes the basic rationality by breaking down a problem into its smaller constituent parts and then calls for only simple pairwise comparison judgments to develop priorities in each hierarchic level. It provides a comprehensive framework to cope with the intuitive, the rational, and the irrational and emotional at the same time [4]. It is a method used to integrate perceptions and purposes into an overall synthesis. The AHP does not require that judgments be consistent or transitive. The degree of consistency (or inconsistency) of the judgments is calculated at each stage of the AHP process. The steps followed in the AHP are:

Step 1. Define the problem and structure the hierarchy of that problem from the top goal through intermediate levels of criteria, subcriteria, and actors to the lowest level of alternatives. Most problems involve four separate hierarchies: benefits, costs, opportunities, and risks.

Step 2. Construct a set of pairwise comparison matrices for each level in a hierarchy, and make all the pairwise comparisons. Use the fundamental scale of absolute numbers from 1 to 9 to indicate the relative dominance with respect to given property of one alternative over another used as the unit of the paired comparison in a cluster of homogeneous elements. For alternatives that are spread farther apart than indicated by the scale clustering using a pivot element is applied and thus the scale is essentially extended as far out as needed to make the comparisons. Consistency in the judgments is computed through the principal eigenvalue of each matrix and is required to be in the order of 10% or less when compared with the corresponding random value. A measure of overall hierarchic consistency is also provided.

Step 3. Hierarchic composition is used to weight the eigenvectors in a level by the eigenvector weights of the corresponding criteria and the sum is then taken over all weighted eigenvector entries in the next lower level of the hierarchy. The resulting priorities are thus determined with respect to the overall goal of the hierarchy which has the value one.

Step 4. The consistency of the entire hierarchy is determined by multiplying each consistency index by the priority of the corresponding criterion and adding. The result is then divided by the same type of expression using the random consistency index corresponding to the dimensions of each matrix weighted by the priorities of the corresponding criterion.

Step 5. For the sake of rank preservation the AHP uses a performance or distributive mode and a benchmarking or ideal mode. In the latter, the entries of the eigenvector are all divided by the largest among them.

The AHP also involves a method to rate a large number of alternatives one at a time with respect to a criterion by choosing the appropriate intensity from a range of intensities (for example excellent, very good, good, average, below average, poor) for that criterion. One needs to pairwise compare the intensities relative merits with respect that criterion. The final rank of each alternative is obtained as the sum of products of the priority of the criterion and the intensity assigned to that alternative. Here too, there are two modes, one to preserve rank, and one to allow rank to change. This approach, known as absolute measurement, is not illustrated in this paper.

Now, we apply the AHP to the car example. Before proceeding, we note that when there are several criteria measured in units of some known scale such as dollars, the AHP requires that these criteria be brought together into a cluster and the values of the alternatives are summed for all these criteria and then normalized by the total value under the larger cluster.

To solve the car example by the AHP, we first combine the measureable criteria and then normalize the measurement of the alternatives under those criteria for which we have well defined scale measurements of the alternatives. We obtain Table 3 and its decimal representation in Table 4. Tables 5 gives the pairwise comparisons of the alternatives based on the ordinal values assigned with respect to criterion C_6.

Table 3. Combination of the criteria with same scale

	C_1 and C_5	C_2	C_3	C_4	C_6
A_1	4600/15500	120/390	30/76	40/120	3
A_2	5500/15500	140/390	21/76	30/120	4
A_3	5400/15500	130/390	25/76	50/120	5

Table 4. Relative importance weights of the alternatives

	C_1 and C_5	C_2	C_3	C_4	C_6
A_1	0.352	0.308	0.395	0.333	3
A_2	0.322	0.359	0.276	0.250	4
A_3	0.326	0.333	0.329	0.417	5

Table 5. Pairwise comparison matrix for the alternatives with respect to the criterion, 'overall appeal'

C_6	A_1	A_2	A_3	Priority vector
A_1	1	1/2	1/3	0.163
A_2	2	1	1/2	0.297
A_3	3	2	1	0.540

(Inconsistency Ratio = 0.01)

Next, we establish priorities for the criteria through pairwise comparisons using the authors' judgments. The resulting priorities are used to weight the normalized values of the alternatives as they are given above. The matrices of pairwise comparisons of the criteria are shown in Table 6, along with the resulting vectors of priorities. The AHP does not use the information in Table 1.

Table 6. Pairwise comparison matrix for the criteria with respect to goal

	C_1 and C_5	C_2	C_3	C_4	C_6	Priority
C_1 and C_5	1	2	2	3	4	0.378
C_2	1/2	1	1	2	3	0.217
C_3	1/2	1	1	2	2	0.201
C_4	1/3	1/2	1/2	1	2	0.123
C_6	1/4	1/3	1/2	1/2	1	0.081

(Inconsistency Ratio=0.01)

After applying the principle of composition of priorities, we obtain the following ranking: $A_2 = 0.435 = [(0.378 \times 0.352) + (0.217 \times 0.308) + (0.201 \times 0.395) + (0.123 \times 0.333) + (0.081 \times 0.163)]$, $A_2 = 0.271$, and $A_3 = 0.293$. Car A_1 is the most preferred alternative.

2.2 Bayesian Analysis

Bayesian analysis is a popular statistical decision making process that provides a paradigm for updating information in the form of possibilities [20]. It is based on the premise that decisions involving uncertainty can only be made with the aid of information about the uncertain environment of the decision. Bayesian theory updates information by using Bayes' theorem, a statement in conditional probabilities relating causes (states of nature) to outcomes. Outcomes are results of experiments used to uncover the causes. Here, $P(A|B) = P(A \cap B)/P(B)$. $P(A|B)$ is the probability of an event A, given the occurrence of a second event B, $P(B)$ is the unconditional probability of the second event B, and $P(A \cap B)$ is the probability of joint occurrence of event A and event B.

Bayesian theory revises initial or prior probabilities of causes, known from a large sample of a population, into posterior probabilities by using the outcome of an experiment or test with a certain probability of success. Prior probabilities are obtained either subjectively or empirically by sampling the frequency of occurrence of a cause in a population. Posterior probabilities are based on the prior probabilities and on both the outcome of the experiment and the observed reliability of that experiment. That is, it is the ratio of a joint probability to a marginal probability.

Bayesian analysis is a way to determine the impact of information, in the form of probabilities, on decision-making outcomes. Usually, probabilities are only known post hoc, if then, which makes this approach well suited for analyzing decisions that have already been made, but is of questionable value for problems other than those involving what may happen to an individual, given what has happened in a population. The steps followed in Bayesian Analysis are:

Step 1. Separate the criteria into two categories: benefits and costs, those that work to the advantage of the decision and those that work against it. Obtain measurement for the alternatives with respect to each criterion. These measurements are given as positive numbers for the benefits criteria and negative numbers for the costs criteria.

Step 2. Standardize the data so that the relative ranges of the variable inputs could be expressed on an absolute scale whose largest value is 1 for the benefits and smallest value is -1 for the costs. In many problems where the alternatives are measured in dollars under all the criteria, normalization is not used.

Step 3. Assign prior probability weights to the criteria. These probabilities must sum to 1. However, these probabilities are determined with respect to an overall goal, but represented as in medical cases.

Step 4. List the possible research outcomes (for details, see the example below) and calculate their marginal probabilities.

Step 5. Assume that each of the research outcomes has been obtained. For each research outcome:

a. Revise the prior probabilities.

b. Compute the expected payoff of each course of action under consideration and select the act with the largest expected payoff.

c. Multiply the expected payoff of the best course of action by the marginal probability of the research outcomes.

Step 6. Sum the products of Step 5c (see example) to get the expected payoff of the strategy that includes ordering research before taking final action.

Step 7. Subtract the cost of the research from Step 6 to get the expected net payoff of the strategy.

Step 8. Compare the expected net payoff of the strategy that includes research with the expected payoff of the strategy of choosing among the alternatives without research.

Step 9. Choose the strategy that maximizes the expected net payoff.

Note that when dollars are used, their results are an expected dollar value. Thus one can compare the expected gain from step to step and determine whether it is worth buying additional information to perform the next step.

Applying Steps 1 and 2 to the data given in Table 2 above we obtain Table 7.

Table 7. Standardized data

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.83	0.86	1.00	0.80	-0.80	0.60
A_2	-0.97	1.00	0.70	0.60	-1.00	0.80
A_3	-1.00	0.93	0.83	1.00	-0.90	1.00

We use the weights obtained from the AHP for the marginal probabilities of the criteria instead of guessing them: $C_1 = .334$, $C_2 = .196$, $C_3 = .185$, $C_4 = .108$, $C_5 = .108$, and $C_6 = .069$. In this example, because price and maintenance are the costs that have negative impacts, their expected values are subtracted from the sum of the benefits to arrive at the final benefit/cost ratio.

The benefit/cost for each of the alternatives can now be assessed in relative terms. From this information, the best alternative is car A_3 with a value of 0.121 = (.334 × -1.00) + (.196 × .93) + (.185 × 0.84) + (.108 × 1.00) + (.108 × -.90) + (.069 × 1.00) obtained by multiplying the numbers in each row of Table 7 by the weight of the corresponding criterion and adding. One similarly obtains the following values for A_1 and A_2 in the same way: $A_1 = 0.118$, $A_2 = 0.014$. This is done by prior judgment estimates for the criteria weights. By prior analysis, we find that we should choose alternative A_3 if we have to make the decision immediately without additional information. In Bayesian analysis, prior judgments must be revised by incorporating new information. In this example, we begin our consideration as to whether we should buy the research as new information by structuring the problem in the form of the decision tree shown in Figure 1 to compute the expected gain from carrying out the next posterior decision.

We identify three categories of possible research outcomes represented as follows: serious problem (T_1; brakes, engine, gas tank etc.), moderate problem (T_2; seat belt, airbag, etc), and no problem (T_3; fine). We reason that if the price of car A_1 is $3000, the probability of observing T_1 in the research is 0.6, the probability of observing T_2 is 0.3, and the probability of observing T_3 is 0.1. These probabilities may be obtained from sampling research on the cars that charges money for the probability information.

Table 8. Conditional probabilities of research results

	Research Results		
	T_1	T_2	T_3
C_1	0.6	0.3	0.1
C_2	0.2	0.6	0.2
C_3	0.1	0.2	0.7
C_4	0.5	0.3	0.2
C_5	0.2	0.5	0.3
C_6	0.1	0.3	0.6

Similarly, we assign conditional probabilities to the possible research outcomes for the other possible states (Table 8). We then multiply the conditional probabilities by the prior probabilities to arrive at the joint probabilities (Table 9). For example, our prior probability of 0.334 for C_1 times the conditional probability of 0.6 for T_1 given C_1 results in a joint probability for C_1 and T_1 of 0.200, and so on.

Next, we revise our prior probabilities for the possible outcomes of implementation of car A_1, using Bayes' theorem. For example, here are six of our calculations in the case of T_1;

$$P(C_1|T_1)=\frac{P(C_1\cap T_1)}{P(T_1)}=\frac{0.200}{0.341}=0.586 \quad P(C_2|T_1)=\frac{P(C_2\cap T_1)}{P(T_1)}=\frac{0.039}{0.341}=0.114$$

$$P(C_3|T_1)=\frac{P(C_3\cap T_1)}{P(T_1)}=\frac{0.019}{0.341}=0.056 \quad P(C_4|T_1)=\frac{P(C_4\cap T_1)}{P(T_1)}=\frac{0.054}{0.341}=0.158$$

$$P(C_5|T_1)=\frac{P(C_5\cap T_1)}{P(T_1)}=\frac{0.022}{0.341}=0.065 \quad P(C_6|T_1)=\frac{P(C_6\cap T_1)}{P(T_1)}=\frac{0.007}{0.341}=0.021$$

Table 9. Joint probabilities of states and research results

	Research Results			Marginal Probability
	T_1	T_2	T_3	
C_1	0.200	0.100	0.033	0.334
C_2	0.039	0.118	0.039	0.196
C_3	0.019	0.037	0.130	0.185
C_4	0.054	0.032	0.022	0.108
C_5	0.022	0.054	0.032	0.108
C_6	0.007	0.021	0.041	0.069
M.P	0.341	0.362	0.297	1.000

We made similar calculations under the assumption that each of the other two possible outcomes is observed in the research and entered the revised probabilities in the decision tree (Figure 1). By multiplying the possible outcomes of car A_1 by the revised probabilities, we found that the expected benefit of buying car A_1 would be 0, 0.140, and 0.514 if T_1, T_2, and T_3 were observed respectively in the research. The marginal probabilities of the possible research outcomes are obtained by summing their joint probabilities (Table 9).

To arrive at the expected benefit of the research strategy, we multiply the expected benefit of the action we would take under each of the possible research outcomes by the probability of observing the outcome and sum the products: $(0 \times 0.341) + (0.140 \times 0.362) + (0.514 \times 0.297) = 0.203$ (Table 10). We now see that alternative A_3 is the best car under the prior decision, but alternative A_1 is the best car under the posterior decision. This terminates the Bayesian decision approach.

Table 10. Final results of Bayesian analysis

Car	Prior Decision	Posterior Decision	
A_1	0.118	(T_1) 0.000, (T_2) 0.140, (T_3) 0.514	0.203
A_2	0.014	(T_1) 0.000, (T_2) 0.089, (T_3) 0.376	0.144
A_3	0.121	(T_1) 0.000, (T_2) 0.125, (T_3) 0.489	0.191

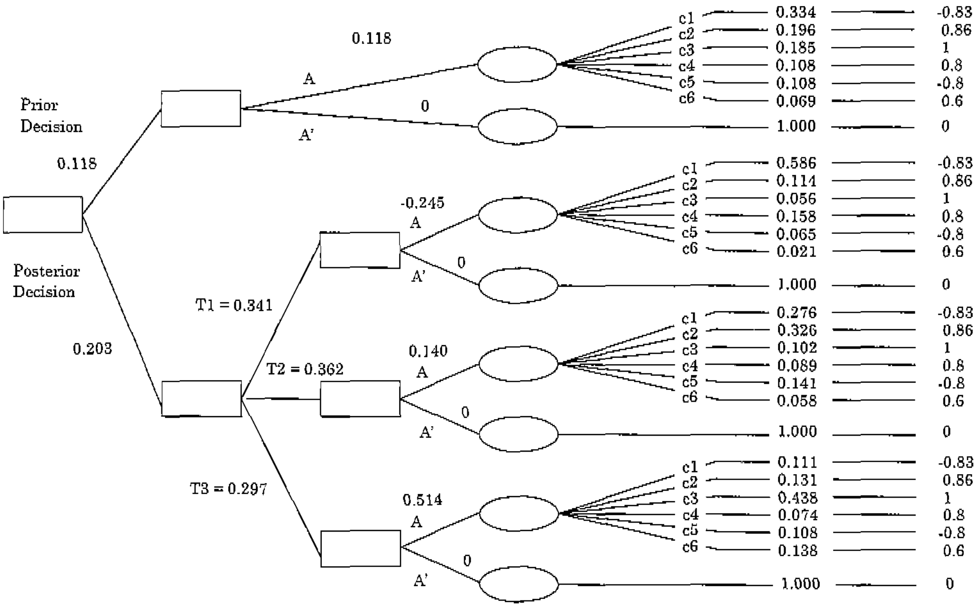

Figure 1. Decision Tree for Car A

2.3 Data Envelopment Analysis (DEA)

DEA, originated by Charnes, Cooper, and Rhodes [6], is a method for analyzing the relative efficiency of each alternative. DEA explicitly considers inputs and outputs associated with each alternative. The basic elements of a DEA analysis are the alternatives, and input and output criteria. The assumption is that an increase in input should produce increase in output. It is desirable to minimize inputs because they require resources which incur costs. The approach used to develop the envelopment or efficient frontier as the ground for ranking the alternatives is based on an additive model that involves using a pair of dual linear programming models as shown below. In particular, the following formulation is used: given m inputs, p outputs, and n alternatives, let x_{ij} correspond to the value of the i th input variable $(i = 1, 2, \cdots, m)$ for the j th alternative $(j = 1, 2, \cdots, n)$ and let y_{kj} correspond to the value of the

k th output variable $(k = 1, 2, \cdots, p)$ for the j th alternative.

Primal Problem for the j th alternative:

$$\text{Minimize} \quad -\sum_{i=1}^{p} s_i - \sum_{i=1}^{m} \varepsilon_i$$

$$\text{Subject to} \quad \sum_{l=1}^{n} \lambda_l\, y_{il} - s_i = y_{ij} \qquad i = 1, 2, \cdots, p$$

$$-\sum_{l=1}^{n} \lambda_l\, x_{il} - \varepsilon_i = -x_{ij} \qquad i = 1, 2, \cdots, m$$

$$\sum_{i=1}^{n} \lambda_i = 1$$

$$\lambda_i,\ \varepsilon_i,\ s_i \geq 0 \quad \textit{for all } i$$

Dual Problem for the j th alternative:

$$\text{Maximize} \quad \sum_{i=1}^{p} \mu_i y_{ij} - \sum_{i=1}^{m} \upsilon_i x_{ij} + \omega_j$$

$$\text{Subject to} \quad \sum_{i=1}^{p} \mu_i y_{ik} - \sum_{i=1}^{m} \upsilon_i x_{ik} + \omega_j \leq 0 \qquad k = 1, 2, \cdots, n$$

$$-\mu_i \leq -1 \qquad i = 1, 2, \cdots, p$$

$$-\upsilon_i \leq -1 \qquad i = 1, 2, \cdots, m$$

$$\omega_j \quad \textit{free}$$

Note that only one of the preceding problems needs to be solved for the j th alternative.

In general, DEA only solves the dual problem in which the variables correspond to prices for the inputs and outputs. Efficiency is defined to be the ratio of the sum of the outputs to the sum of the inputs. It is assumed that no alternative can be more than 100 percent efficient. Thus, we have constraints of the form:

$$\frac{\sum_{i=1}^{p} \mu_i y_{ik}}{\sum_{i=1}^{m} \upsilon_i x_{ik}} \leq 1.0 \qquad k = 1, 2, \cdots, n$$

By multiplying both sides of the inequality by the denominator, we obtain linear expressions that become part of the dual problem. In the end, one obtains a table in which the initial values of the objective function for all the alternatives are 0, and the final values are obtained by solving the dual linear programming problems. The best alternative has the smallest final objective function absolute value which means it is close to the efficient frontier.

Alternatives	Initial value of the objective function	Final value of the objective function
A_1	0	z_1
A_2	0	z_2
⋮	⋮	⋮
A_n	0	z_n

This method requires that one use three times the number of input and output criteria to obtain non-zero outcomes and make better discrimination of efficiency. We regret that using many alternatives would have made this paper considerably longer and more complicated. To compare the outcome of the several methods would have required that we use the same number of alternatives in all our examples and that would have made the paper prohibitively long. In any case, the example only illustrates how the method works. The outcome should not be taken literally. Of course, many important decision problems deal with only a small number of alternatives.

The information in Table II is divided into two categories, the inputs are cost criteria C_1 and C_5 and outputs are benefit criteria C_2, C_3, C_4, and C_6. The inputs are price and maintenance; the outputs are performance, economy, depreciation, and overall appeal. The inputs and outputs for the three cars are summarized in Table 11.

Table 11. Input and output data for the three cars

Cars	Inputs		Outputs			
	1	2	1	2	3	4
A_1	3000	1600	120	30	40	3
A_2	3500	2000	140	21	30	4
A_3	3600	1800	130	15	50	5

Using the dual linear programming formulation, we must solve the following three problems, one for each car.

Dual Problem for Car A_1*:*

$$\text{Maximize} \quad 120\mu_1 + 30\mu_2 + 40\mu_3 + 3\mu_4 - 3000\nu_1 - 1600\nu_2 + \omega_1$$

$$\text{Subject to} \quad 120\mu_1 + 30\mu_2 + 40\mu_3 + 3\mu_4 - 3000\nu_1 - 1600\nu_2 + \omega_1 \leq 0$$

$$140\mu_1 + 21\mu_2 + 30\mu_3 + 4\mu_4 - 3500v_1 - 2000v_2 + \omega_1 \leq 0$$
$$130\mu_1 + 25\mu_2 + 50\mu_3 + 5\mu_4 - 3600v_1 - 1800v_2 + \omega_1 \leq 0$$
$$\mu_i \geq 1, \quad v_j \geq 1, \quad i = 1,2,3,4 \quad j = 1,2$$

Dual Problem for Car A_2:

$$\text{Maximize} \quad 140\mu_1 + 21\mu_2 + 30\mu_3 + 4\mu_4 - 3500v_1 - 2000v_2 + \omega_2$$
$$\text{Subject to} \quad 120\mu_1 + 30\mu_2 + 40\mu_3 + 3\mu_4 - 3000v_1 - 1600v_2 + \omega_2 \leq 0$$
$$140\mu_1 + 21\mu_2 + 30\mu_3 + 4\mu_4 - 3500v_1 - 2000v_2 + \omega_2 \leq 0$$
$$130\mu_1 + 25\mu_2 + 50\mu_3 + 5\mu_4 - 3600v_1 - 1800v_2 + \omega_2 \leq 0$$
$$\mu_i \geq 1, \quad v_j \geq 1, \quad i = 1,2,3,4 \quad j = 1,2$$

Dual Problem for Car A_3:

$$\text{Maximize} \quad 130\mu_1 + 25\mu_2 + 50\mu_3 + 5\mu_4 - 3600v_1 - 1800v_2 + \omega_3$$
$$\text{Subject to} \quad 120\mu_1 + 30\mu_2 + 40\mu_3 + 3\mu_4 - 3000v_1 - 1600v_2 + \omega_3 \leq 0$$
$$140\mu_1 + 21\mu_2 + 30\mu_3 + 4\mu_4 - 3500v_1 - 2000v_2 + \omega_3 \leq 0$$
$$130\mu_1 + 25\mu_2 + 50\mu_3 + 5\mu_4 - 3600v_1 - 1800v_2 + \omega_3 \leq 0$$
$$\mu_i \geq 1, \quad v_j \geq 1, \quad i = 1, 2, 3, 4 \quad j = 1,2$$

The 'problem Solver' in Microsoft EXCEL is used to solve these problems. For illustrative purposes, the selected output is shown below. Specifically, the objective function values for all the cars are shown in Table 12.

Table 12. Objective function values for the three cars

Alternatives	Initial value of the objective function	Final value of the objective function
A_1	0	0
A_2	0	0
A_3	0	0

As seen in Table 12, the values of the objective function at the optimal solution for the three cars A_1, A_2, and A_3 are 0, 0, and 0, respectively. Thus we see that in this example all cars A_1, A_2, and A_3 are efficient (lie on the efficient frontier). That is, each car neither dominates nor is dominated by the other cars. As a result, the value 0 is a measure of the distance of cars A_1, A_2, and A_3 from the efficient frontier. DEA does not determine the priority weight for the alternatives, but only computes an efficiency measure for them. Here, the method fails to show dominance among the alternatives and hence any alternative may be considered a best choice.

2.4 Multiple Attribute Utility Theory

MAUT, developed by Keeney and Raiffa [7, 11, 35], attempts to maximize a decision maker's utility or value (preference) represented by a function that maps an object measured on an absolute scale into the decision maker's utility or value relations. It is based on the following fundamental axiom: any decision-maker attempts unconsciously to maximize a real valued function $u = u(x_1, x_2, \cdots, x_n)$ of the criteria $x_1, x_2, \cdots x_n$. The role of the researcher is to try to estimate this function by asking the decision-maker some well-chosen questions. It is assumed that utility functions are monotonic and that, sometimes decision makers are risk averse. A utility function may be monotonically increasing (that is, if x_k is greater than x_j, x_k is always preferred to x_j) or monotonically decreasing (that is, if x_k is less than x_j, x_k is always preferred to x_j).

In order to estimate a utility function, several points on the function curve are determined by the decision maker. Consider the assessment of the component utility function, $u_i(x_i)$, for a particular criterion x_i. The first step is to choose two values of the given criterion i, x_i^L and x_i^H, that correspond to lowest (worst) and highest (best) values of the i^{th} criterion, respectively. Thus, utility theory requires that nonmeasurable criteria be measured on same numerical indicator such as the number of people, the number of hours, the amount of money, and even an ordinal rating from 1 to 5. For example, preference is then assumed to be a linear function of these measurements.

The following utilities are assigned to these two values: $u_i(x_i^L) = 0$ and $u_i(x_i^H) = 1$. The decision maker is then told that he has a probability $(1-p)$ of getting x_i^L and a probability p of getting x_i^H. This part is called the *gamble*. He is then asked, "what least amount x (sure value you get no matter what) would you accept for certain instead of taking the gamble?" This part of trading off the gamble against the sure value offered is called a *lottery*. A very conservative risk averse individual would take the sure value, whereas a risk prone individual may prefer the gamble. Once x (the sure equivalent) has been specified by the decision maker, the utility of x is set equal to p (that is, $u_i(x) = p$). The probabilities are offered in a trial and error process. The assumption is that the decision maker would make his or her choice at that probability. The equation the gambler must consider is $(1-p)u_i(x_i^L) + pu_i(x_i^H) = (1)u_i(x)$. Because $u_i(x_i^L) = 0$ and $u_i(x_i^H) = 1$, he or she must focus on the equation $p = u_i(x)$ with two unknowns, one on each side. The decision maker would then experiment with values of x whose utility would yield a probability that makes the gamble worthwhile. It would be the value where he is indifferent between the sure amount and taking the gamble. In the case of a monotonic increasing utility function, the probability would be larger, the larger x is. However, if the monotonicity of the function is damped (because of the law of diminishing value of returns), increase in p is gradually diminished, and there is no incentive to use larger and larger values of x. At some point, the decision maker would have no desire to take more risk. That is what the lottery process attempts to capture for each particular decision maker. If x is greater than the sure value, he goes for the gamble with the indifference value p. If less, he takes the sure value. Lotteries and gambles are themselves a mathematically risky process because of the different subjectivity involved in different situations, and gambles would be difficult for the decision maker for things not easily convertible to monetary value.

To simplify life, some reasonable looking functions are used in practice. Assuming a risk averse decision maker, one usually uses either $u(x) = a + b\log(x + c)$ for a monotonically increasing utility function or $u(x) = a + b\log(c - x)$ for a monotonically decreasing utility function.

Utility independence is one of the central concepts in MAUT. Various utility-independence conditions imply specific forms of utility functions. However, only additive and multiplicative forms are generally used in practice. An additive utility function can be represented by $u(x_1, \cdots, x_m) = k_1u_1(x_1) + \cdots + k_mu_m(x_m)$, where $u(x_1, \cdots, x_m)$ ranges from 0 to 1 and the component utility functions $u_i(x_i)$ also range from 0 to 1, and the scaling constants k_i are positive and sum to one. A multiplicative utility function has the form

$$1 + ku(x_1, \cdots, x_m) = \prod_{i=1}^{m}[1 + k\,k_iu_i(x_i)]$$

where the functions $u_i(x_i)$ are restricted as in the additive case. However, the scaling constants k_i may be greater or less than one, and the constant k is chosen to satisfy the equation: $1 + k = \prod_{i=1}^{m}[1 + k\,k_i]$

To assess the scaling constants k_i, the decision maker first chooses a criterion against which the other criteria will be compared. Suppose that the i th criterion is chosen. In order to compare another criterion j with the i th criterion, the decision maker is presented with the following two alternatives: The first alternative has the i th criterion at its best value and all of the remaining criteria at their worst values. The second alternative has all of the criteria at their worst values. The level of the j th criterion in the second alternative is improved until the decision maker iss indifferent between the two alternatives. The steps followed in MAUT are as follows:

Step 1. Identify the relevant criteria (attributes).
Step 2. Assign quantifiable variables to each of the attributes and specify their restrictions.
Step 3. Select and construct utility functions for the individual attributes.
Step 4. Synthesize the individual utility functions into a single additive or multiplicative utility function.
Step 5. Evaluate the alternatives using the function obtained in Step 4.
Step 6. Choose the alternative with the largest utility value.

We now apply the foregoing steps to our car decision example. We use the logarithmic form given above for the utility functions.

$$C_1:\ u_1(x_1) = -1.878 + 0.442\ln(3670 - x_1),\ \ C_2:\ u_2(x_2) = -34.79 + 7.143\ln(x_2 + 10)$$
$$C_3:\ u_3(x_3) = -35.81 + 8.4\ln(x_3 + 50),\ \ C_4:\ u_4(x_4) = -34.79 + 7.143\ln(x_4 + 100)$$
$$C_5:\ u_5(x_5) = -82.9 + 10\ln(6000 - x_5),\ \ C_6:\ u_6(5) = 1,\ u_6(3) = 0.5, u_6(4) = 0.7$$

Here, for example, we determine the parameters for C_1 (Price is monotonically decreasing utility function) by using Table I to construct a lottery (1 / 2, 2000; 1 / 2, 5000) and ask the question: what value of Price, x_1, makes this lottery indifferent to the sure value x_1? The answer is $x_1 = 3500$. Finally, we construct the function $u_1(x_1) = a + b\ln(c - x_1)$ passing through the three points (u_1(2000), 2000), (u_1(5000), 5000), and (u_1(3500), 3500). The result is the function: $u_1(x_1) = -1.878 + 0.442\ln(3670 - x_1)$. Applying the same procedure to the remaining five criteria with the help of Table I, we obtain the remaining equations. We next use Table II to substitute values of each alternative and criterion in the relevant equation to obtain the utilities of the alternatives for each criterion as shown in Table 13.

Table 13. Utilities of alternatives

	A_1	A_2	A_3
C_1	0.72	0.54	0.50
C_2	0.40	0.80	0.60
C_3	1.00	0.02	0.40
C_4	0.40	0.20	0.70
C_5	0.91	0.40	0.68
C_6	0.50	0.70	1.00

Next, we must evaluate the scaling constants $k_i (i = 1, 2, \cdots, 6)$. The criteria's ordinal preference is as follows: Price (k_1) > Performance (k_2) > Economy (k_3) > Depreciation (k_4) > Maintenance (k_5) > Appeal (k_6). Thus, for example, k_1 is assessed by comparing the lottery:

$$\begin{bmatrix} p & (1-p) \\ (3000, 140, 30, 50, 1600, 5) & (3600, 120, 21, 30, 2000, 3) \end{bmatrix}$$

with the sure value

$$\begin{bmatrix} 1 \\ (3000, 120, 21, 30, 2000, 3) \end{bmatrix}$$

We explained above a lottery with respect to a single variable x. Here we have a similar value for a function of several variables: $(1-p)u(x_1{}^L, \cdots, x_m{}^L)$ $+pu(x_1{}^H, \cdots, x_m{}^H) = u(x_1, \cdots x_m)$. That is, $p = u(x_1, \cdots x_m)$. We must use a bundle of variables, instead of a single variable, in our lottery. We take $p = k_1$. Suppose that $k_1 = 0.25$, what bundle makes the decision maker indifferent between taking a lottery giving the best possible consequences with probability 0.25 and the worst possible consequences with probability 0.75, and the sure value giving the worst possible consequence for all attributes except x_1 which is taken at its best value while the other variables are taken at their worst value? Once k_1 is known, the other scaling constants are determined from the following equations successively.

$$\begin{aligned} k_1u_1(3300) &= k_2u_2(140) \\ k_2u_2(140) &= k_3u_3(30) \\ k_3u_3(30) &= k_4u_4(50) \\ k_4u_4(50) &= k_5u_5(1600) \\ k_5u_5(1800) &= k_6u_6(5) \end{aligned}$$

The outcome is: $k_1 = 0.25$, $k_2 = 0.19$, $k_3 = 0.15$, $k_4 = 0.21$, $k_5 = 0.16$, and $k_6 = 0.1$.

Since $\sum_{i=1}^{6} k_i = 1.06 \approx 1$, we can use the additive utility function approach from which we obtain for the utilities of the alternatives: $u(A_1) = 0.686$, $u(A_2) = 0.466$, $u(A_3) = 0.655$. Thus, the best alternative that provides the maximum utility is car A_1. It is followed by car A_3, and then by car A_2. If the scaling constants were much greater or much less than 1, we would have used a multiplicative utility function.

Multiattribute value theory (MAVT) is a new version of multiattribute utility theory. Generically, it assigns values from a ratio scale that fall in a range (for example, 0 to 100) to the criteria and similarly assigns values to the alternatives from appropriate ranges chosen for each criterion. Unlike MAUT, it has come to recognize that criteria weights are important in decision making. However, its alternatives are still measured on interval scales.

2.5 Outranking Method (ELECTRE)

Outranking method (ELECTRE), developed by Roy [23, 24, 32], is based on MAUT principles with the motivation to improve efficiency without affecting the outcome while considering less information. It is a procedure that sequentially reduces the number of alternatives the decision-maker is faced with in a set of non-dominated alternatives.

The concept of an outranking relation S is introduced as a binary relation defined on the set of alternatives A. Given two alternatives A_i and A_j, A_i outranks A_j, or $A_i S A_j$, if given all that is known about the two alternatives, there are enough arguments to decide that A_i is at least as good as A_j. The goal of outranking methods is to find all alternatives that dominate other alternatives while they cannot be dominated by any other alternative. To find the best alternative, outranking also requires knowledge of the weights of the criteria. Each criterion $C_j \in C$ is assigned a subjective weight w_j, and every pair of alternatives A_i and A_j is assigned a concordance index $c(A_i, A_j)$ given by:

$$c(A_i, A_j) = \frac{1}{\sum_{k=1}^{n} w_k} \sum_{\{k: g_k(A_i) \geq g_k(A_j)\}} w_k$$

where the sum of the criteria weights in the numerator is taken only for those criteria where the values of A_i dominate the values of A_j in Table II. A discordance index $d(A_i, A_j)$ is also calculated and is given by:

$$d(A_i, A_j) = \begin{cases} 0 \;\; if \; g_k(A_i) \geq g_k(A_j) \;\; for\ all\ k, \\ \frac{1}{\delta} \max\{g_k(A_i) - g_k(A_j)\}, \;\; otherwise. \end{cases}$$

where, $\delta = \max\{g_k(A_i) - g_k(A_j)\}$. Here, the criteria weights are not used but only normalized values in each row of Table II. These are the $g_k(A_i)$. Once the two indices are known, an outranking relation S is defined by:

$$A_i S A_j \text{ if and only if } \begin{cases} c(A_i, A_j) \geq \hat{c}, \\ d(A_i, A_j) \leq \hat{d}, \end{cases}$$

where $\hat{c}$ and $\hat{d}$ are thresholds set by the decision maker. A problem with this discordance index is the requirement that criteria levels be quantifiable. If that is not the case, then a discordance set D_j is defined for each criterion j for all the ordered pairs (x_j, y_j) such that if $g_j(A) = x_j$ and $g_j(B) = x_j$ then the outranking of B by A is refused. The outranking relation is defined by:

$$A_i S A_j \text{ if and only if } \begin{cases} c(A_i, A_j) \geq \hat{c}, \\ (g_j(A_i), g_j(A_j)) \notin D_j, \forall j. \end{cases}$$

Given the outranking relation it is now possible to find the set of alternatives $\mathrm{N} \subset \mathrm{A}$ for which:

$$\forall B \in \mathrm{A} - \mathrm{N} \quad \exists A \in \mathrm{N} \textit{ such that } ASB$$
$$\forall A, B \in \mathrm{N}, ASB.$$

The outranking relation determines the set of non-dominated alternatives. The alternatives in N form the kernel of the graph defined by the alternatives (vertices) and the outranking relation (edges). Thus, if alternative A_i outranks alternative A_j, then a directed arc exists from A_i to A_j: $A_i \rightarrow A_j$. The steps followed in the Outranking Method are as follows:

Step 1. Obtain the values of the criteria.
Step 2. Construct the outranking relations by following the concordance and discordance definitions, and construct a graph representing the dominance relations among the alternatives.
Step 3. Obtain a minimum dominating subset by using the minimum concordance and maximum discordance indices (see example below). If a kernel exists, it is chosen as the minimum dominating subset.
Step 4. If the subset has a single element or is small enough to apply value judgment, select the final decision. Otherwise, Steps 2 through 4 are repeated until a single element or small subset exists.

We now use the information in Tables 1 and 2 to apply this method. Instead of assigning the criteria subjective weights, we took the liberty to borrow their derived weights from the paired comparison approach of the AHP. They were $C_1 = .334$, $C_2 = .196$, $C_3 = .185$, $C_4 = .108$, $C_5 = .108$, and $C_6 = .069$. Normalizing the rows of Table II, we have Table 14.

Table 14. Relative importance of alternatives

	A_1	A_2	A_3
C_1	0.351	0.327	0.322
C_2	0.308	0.359	0.333
C_3	0.395	0.276	0.329
C_4	0.333	0.250	0.417
C_5	0.352	0.315	0.333
C_6	0.250	0.333	0.417

For the concordance indices we have (if there are ties between the alternatives, they would receive one half the weight): c(1, 2) = .334 + .185 + .108 + .108 = .735, c(1, 3) = .334 + .185 + .108 = .627, c(2, 1) = .196 + .069 = .265, c(2, 3) = .334 + .196 = .530, c(3, 1) = .196 + .108 + .069 = .373, c(3, 2) = .185 + .108 + .108 + .069 = .470. The complete set of indices is represented by the concordance matrix shown in Table 15.

Table 15. The concordance matrix

	A_1	A_2	A_3
A_1	-	0.735	0.627
A_2	0.265	-	0.530
A_3	0.373	0.470	-

For the discordance indices we have: d(1, 2): C2 = .359 - .308 = .051, C6 = .333 - .250 = .083, d(1, 3): C2 = .333 - .308 = .025, C4 = .417 - .333 = .084, C6 = .417 - .250 = .167, d(2, 1): C1 = .351 - .327 = .024, C3 = .395 - .276 = .119, C4 = .333 - .250 = .083, C5 = .352 - .315 = .037, d(2, 3): C3 = .329 - .276 = .053, C4 = .417 - .250 = .167, C5 = .333 - .315 = .018, C6 = .417 - .333 = .084, d(3,1): C1 = .351 - .322 = .029, C3 = .395 - .329 = .066, C5 = .352 - .333 = .019, d(3,2): C1 = .327 - .322 = .005, C2 = .359 - .333 = .026. For d(1, 2), the discordance index would be 0.083 and for d(3, 1), the index would be 0.066. The complete set of indices is represented by the discordance matrix shown in Table 16.

Table 16. The discordance matrix

	A_1	A_2	A_3
A_1	-	0.083	0.167
A_2	0.119	-	0.167
A_3	0.066	0.470	-

Now suppose that the decision-maker has specified a minimum concordance of 0.60 and a maximum discordance of 0.40, that is, $c(A_i, A_j) > 0.60$ and $d(A_i, A_j) < 0.40$. With this specification the graph can now be constructed. The directed paths which appear in the graph are determined by the set of indices that simultaneously satisfy these two requirements. These indices are: (A_1, A_2) and (A_1, A_3).

The resulting graph is shown below in Figure 2. Using the graph, the decision-maker can determine the optimal choice by eliminating of nodes. The direction of the arrow determines which alternative outranks others. As in the figure, alternative A_1 outranks alternatives A_2 and A_3, and hence, it is the best car to choose.

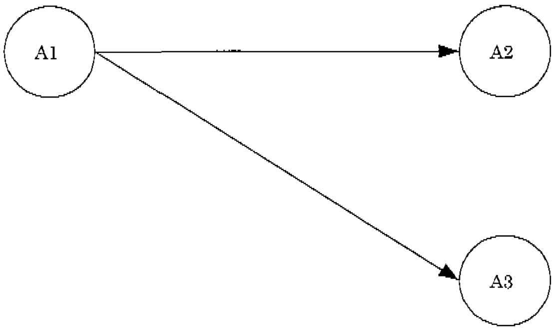

Figure 2. The resulting graph

By this method, however, we cannot say that alternative A_2 outranks or is more preferred to alternative A_3. Further, we cannot say how much alternative A_1 outranks alternatives A_2 and A_3. The method is useful for selecting the best set of alternatives that outranks the others and finding the best alternative in that set.

3. OBSERVATION AND CONCLUSION

In the foregoing example, three of the methods, the AHP, MAUT, and Outranking (ELECTRE) gave alternative A_1 as the best choice. Bayesian Analysis under prior decision gave A_3 as best and A_1 as best, under posterior. All of the three alternatives was considered as best in case of DEA. Thus, there is nothing shown here that assures one that alternative A_1 is really and truly the best one. What do we conclude from all this? In addition, seeing the conflicting outcome of all these best known methods one wonders what the statistics would be had we applied all the methods to many examples. Clearly something must be done to narrow down the practicality of theories of decision making to the level of usefulness of scientific theories in physics, chemistry, astronomy, and biology.

We have often heard people voice the opinion that they themselves were attempting to take the most desirable feature of each method in order to create a new theory that includes all these features. This reminds one of the counterexample often given in systems theory, that one cannot construct the best car by taking the best engine from one car, the best body from a second car, the best brakes from a third car, the best upholstery from a fourth car, the best wheels from a fifth car, and the best paint job from a sixth car. A good theory must stand on its own assumptions and operations and needs to be validated according to its mathematical rigor, its practicality, and general usefulness, in order to stand the test of time. The sophistication and mathematical intricacy of a theory are no guarantee of its validity and truthfulness.

There are several types of numerical scales that may be considered to rank criteria and alternatives in decision analysis. There are ordinal scales, invariant under strictly monotone increasing transformations; interval scales, invariant under positive linear transformations; ratio scales, invariant under positive similarity transformations; and absolute scales, invariant under the identity transformation. If they all lead to the same result it would not matter which is used and the distinction among scales would be superfluous. When there are multiple criteria, however, one cannot simply use any scale since it must be possible to combine the rankings with respect to the different criteria, and not every scale allows the arithmetic operations needed to do the combining. Ordinal numbers, for example, are not serious contenders in this process. In addition, there are situations of interdependence among the alternatives that narrow the choice of scale further. We need to consider other numerical scales and whether arithmetic operations on them results in meaningful outcomes. Note that one cannot multiply numbers from an interval scale because the result is not an interval scale. Thus, $(ax_1+b)(ax_2+b)=a^2x^2+ab(x_1x_2)+b^2$ which does not have the form $ax+b$. One can take the average of interval scale readings but not their sum. Thus, $(ax_1+b)+(ax_2+b)=a(x_1+x_2)+2b$, which does not have the form $ax+b$. However, if we average by dividing by 2, we do get an interval scale value. Similarly, we can multiply interval scale readings by positive numbers whose sum is equal to one and add to get an interval scale result, a weighted average. For a ratio scale, we have $ax_1+ax_2 = a(x_1+x_2) = ax_3$ which belongs to the same ratio scale, and $ax_1= abx_1x_2= cx_1x_2= cx_3$ which belongs to a new ratio scale. However, $ax_1 + bx_2$ does not define a ratio scale and, thus, we cannot add measurement from different ratio scales.

It is clear that we need a higher level of abstraction to pass a new kind of judgment on what should be considered as the best outcome. What we need is a set of high level of criteria that determine the merits of these methods in producing a ranking of the alternatives. But, then we have a new problem, what method do we use to decide which is of these methods is best? One thing seems clear, and it is that the legitimacy of the numbers used in these methods and their manipulations are central in deciding whether what they do is meaningful. We believe that any decision method must meet at least five criteria. First, it should make it possible to deal with a decision problem that is complex and intricate as real life problems present themselves. Second, a method should be able to transcend existing ways of measurement by dealing with intangible criteria such as political merit and artistic ability. The scales derive for intangibles must parallel the derivation and use of measurement of tangibles in scientific practice. Third, since decisions must survive the hazards and risks of the unknown future, a decision theory should be potentially capable of making correct predictions. Otherwise, the best decision of today will sour tomorrow. Fourth, all these methods assume independence of criteria and alternatives in their formulation. The question is which ones among them can be generalized and adapted for dependence and feedback without compromising their existing theory. Real life problems involve all kinds of dependencies that cannot be set aside by always assuming independence. Fifth and finally, a decision theory must intrinsically be amenable to group decision making without assuming that consensus is always the way to combine group decisions. The method should be able to capture the power and knowledge of various individuals involved and factor them in a mathematically precise way into their method. We believe that there is sufficient substance in these observations to make it critical for anyone concerned with decision making to look deeper into these methods.

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