

## Three-Dimensional Trajectory of a Fluid Particle in Air with Wind Effects and Air Resistance

D-R Lee\*

공기 저항과 바람의 영향을 고려한 대기에서의 유체입자의 3차원 궤적

이 동 렬\*

**Key words** : Three-dimensional trajectory of a particle, particle projectile, Trajectory of a liquid jet, nozzle and vertical pump design

### Abstract

Three-dimensional trajectory of fluid particle is simulated by a particle motion, which is able to examine the influences of changes in the several parameters. To calculate the trajectory of a particle, the Runge-Kutta method was utilized. The use of a projectile of particles for the trajectory of liquid jet has been shown to be useful to estimate the influence of different operating parameters such as best particle diameter, density of liquid body, initial take-off velocity, wind velocity, cross wind velocity, take-off angle, and base angle for a released flow from the nozzle. The results give the trajectories of various types of particle of body and at different elevations, base angles, wind velocities, and densities of liquid body. The trajectories in a vacuum show that air resistance decreases both the distance and the maximum height of a projectile, and also explain that the termination time is also reduced in air. In addition, the maximum distance in the x direction was obtained with take-off angles from 30 degrees to 45 degrees in still air and the projectile of particles was highly effected by wind and cross wind. Clearly, a particle has to be so positioned as to take the optimum possible advantage of the wind if the maximum distance is requested. The wind astern increased the maximum distance of x direction compared with the wind ahead. Finally, it is possible to optimize the design of pump by using these results.

\* 대구가톨릭대학교 공과대학 기계자동차공학부(원고접수일 : 2000년 10월)

## 1. INTRODUCTION

Particles of material discharging in air could make significant industrial applications in fire fighting, turbines, jet cutting, etc. Therefore it is not surprising that a number of research work have been performed to determine the optimum shape of fire fighting jet nozzle by observing a jet for various parameters ; distance, elevation, and wind velocity.

A comprehensive series of similar tests to the present work were carried out by Rouse et al.<sup>1)</sup> in place of the US Coast Guard. A plenty of work have been accomplished for a correlation of the break-up distance and turbulent jets by virtue of the Weber number proposed by Phinney<sup>2)</sup> who made use of data by Grant and Middleman<sup>3)</sup> besides his own experiments. But the radius of the jets is small (maximum, 19 mm) and moving speed is low. Hence it is inappropriate to use these correlation such as those used by Rouse (maximum, 76.2 mm diameter) to this case. This is not surprising when the particle surface drag force comes to be decreasingly important as the jet diameter increase in all these studies.

More additional tests by Hoyt and Taylor<sup>4)</sup> on different nozzle form were performed leading to a conclusion that nozzle shape does not effect on throw-distance significantly. Rouse et al.<sup>1)</sup> and Arato et al.<sup>5)</sup> had got to a similar conclusion, but all these authors agree that it is the most important to get rid of swirl, to get a constant velocity profile, and to decrease turbulence at the nozzle entrance in order to obtain maximum jet throw distance.

The motion of a liquid-jet flow should be examined by means of the particle trajectories and Runge-Kutta method is applied for the position of the particle. This problem above, however, is unable to obtain a solution without

applying a numerical method suggested by Gerald<sup>6)</sup>.

The objective of the current research is to examine the three-dimensional motions and trajectories of a particle in the air with wind effects, and air resistances by a computer simulation of trajectories of liquid-jet, which consider the influences of changes in initial speed of liquid body, take-off angle, nozzle diameter, elevation, head and tail wind, density of liquid body, base angle and cross wind to be investigated.

The limited information on air resistance, namely aerodynamic drag of jet immersed in atmosphere can be used by empirical formulas authorized by Stokes formula in White<sup>7)</sup> for the purpose of calculating the forces due to the viscosity. In this problem it is not usually possible to get the accurate maximum throw distance and maximum height such as trajectory, for it is not easy to predict the exact trajectories with only information of Stokes formula.

Generally speaking, the effect of air resistance can be neglected by using the Bernoulli equation in White<sup>8)</sup>, but in this practical problem the jet flow is too scattered and dispersed to know the drag coefficient of liquid body, so this problem has to be treated as a particle dynamics trajectory in Knori<sup>9)</sup> and Johnson<sup>10)</sup>. The calculation for the forces due to viscosity is surely able to be made in the air by using the Stokes formula. On the other hand, there should be a small discrepancy between the computer simulation result and the experimental data.

The results also give the information that for optimum initial speed of liquid body, an optimum nozzle diameter and take-off angle exist for maximum throw distance which is important to the design of the whole system. The optimum take-off angle is between 30

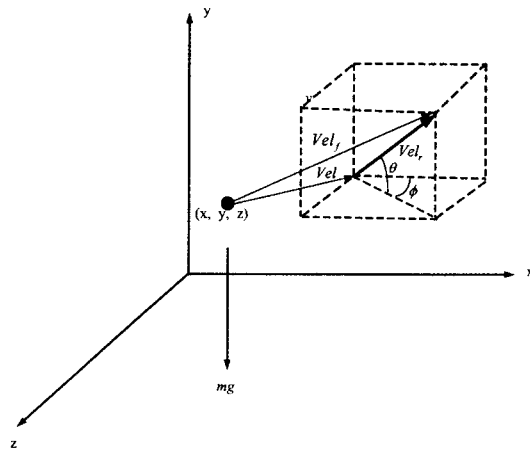
degrees and 40 degrees. Wind effects which include three dimensional motion of liquid body are turned out to be very important. The accurate prediction of the liquid jets are essential to design water hose, fire hydrant, vertical pump, and precombustion injection of liquid fuel spray. It is necessary to project lots of liquid jet over the maximum pre-dictable distance. The problem was initiated to take a proper size of jet at high speed or a larger size of jet at a lower speed of initial liquid body. Little work has been done on the larger jets and also very little experimental work has been carried out to compare the simulated computer data of trajectories. The comparison of the results by computer simulation is unable to be made with the accurate data in practice.

For a given quantity of release, it is remarkably interesting to have an idea of the distance performed by the liquid jet and how this is influenced by such parameters as elevation, base angles, nozzle diameter, initial speed of liquid body, density of liquid body, and wind velocity (Kanury<sup>11</sup>, Birkhoff<sup>20</sup>)

**2. MATHEMATICAL MODEL AND NUMERICAL METHOD**

**2.1 Three-dimensional Motion of a Liquid jet as a Particle in a Fluid**

The motion of a liquid body as a particle through the air in the  $x-y-z$  plane can be shown in Fig. 1. In order to analyze this motion, several velocity components should be assumed.  $Vel$  is relative velocity vector of the body to the stationary coordinate system.  $Vel_f$  is fluid velocity in motion at the location of a body.  $Velr (= Vel_f - Vel)$  is the velocity of fluid relative to that of the body, which determines the fluiddynamic forces, where



**Figure 1 : Three-dimensional motion of a body through a fluid**

$$Vel = [(u_f - u)^2 + (v_f - v)^2 + (w_f - w)^2]^{1/2}$$

If the resultant fluid dynamic force acting on the body is  $f$  with components  $f_x, f_y,$  and  $f_z,$  which excludes the force related with the added mass  $m'$ , the equation of Newton's law of motion on the liquid body of mass  $m$  are

$$\begin{aligned} (m + m') \frac{d^2x}{dt^2} &= f_x \\ (m + m') \frac{d^2y}{dt^2} &= -(m - m_f)g + f_y \\ (m + m') \frac{d^2z}{dt^2} &= f_z \end{aligned} \tag{3.1}$$

where  $x, y, z$  are the coordinates of the projectile and  $m_f$  is the mass of fluid displaced by the body and  $m'$  is added mass of the body. Because  $f$  is a function of position, velocity, and time, the simultaneous ordinary differential equations (3.1) can be described as shown below, with each second-order equation replaced by three simultaneous equations of the first order.

$$\frac{dx}{dt} = u, \quad \frac{du}{dt} = F_1(x, t, z, u, v, w, t)$$

$$\begin{aligned} \frac{dy}{dt} &= v, \quad \frac{dv}{dt} = F_2(x, t, z, u, v, w, t) \\ \frac{dz}{dz} &= w, \quad \frac{dw}{dt} = F_3(x, t, z, u, v, w, t) \end{aligned} \quad (3.2)$$

The forms of the functions  $F_1$ ,  $F_2$ , and  $F_3$  change in different situations. Assuming that the position  $(x_0, y_0, z_0)$  and velocity  $(u_0, v_0, w_0)$  are given at an initial instant  $t_0$ , the trajectory of the body for  $t > t_0$  can be found as functions of time. The initial-value problem can be solved numerically by Runge-Kutta method.

### 2.2 Study of a Particle Trajectory

It is very renowned that a projectile draws out a parabola releasing upward in a direction not perpendicular to the earth's surface in small-scale motions. This assumption above, however, comes from trajectories in a vacuum. It is necessary to trace back to the previous equations of motion in Eq. (3.1) in order to consider the influences of air resistance, wind effect, and cross wind effect on the motion of a particle of liquid body from the nozzle. If the body does not rotate and roll in the fluid,  $f$  becomes the viscous drag force in the direction of the relative velocity  $Vel_r$ , which elevates an angle  $\theta$  (take-off angle of the projectile) with x-axis and an angle with z-axis. As shown in Fig. 1, it is obvious

$$\begin{aligned} \sin\theta &= \frac{v_f - v}{vel_r} & \cos\theta &= \frac{u_f - u}{vel_r \cos\phi} \\ \sin\phi &= \frac{w_f - w}{vel_r \cos\theta} & \cos\phi &= \frac{u_f - u}{vel_r \cos\theta} \end{aligned} \quad (3.3)$$

where  $u = Vel_0 \cos\phi$   
 $v = Vel_0 \sin\theta$   
 $w = Vel_0 \cos\theta \sin\phi$

Thus

$$\begin{aligned} f_x &= f \cos\phi \cos\theta \\ f_y &= f \sin\theta \\ f_z &= f \cos\theta \sin\phi \end{aligned} \quad (3.4)$$

Now substitute  $f$ ,  $\cos\phi$ ,  $\sin\theta$ ,  $\sin\phi$  into the equations above to simplify

$$\begin{aligned} f_x &= \frac{1}{8} \pi \rho_f d^2 C_d Vel_r^2 \cos\theta \frac{(u_f - u)}{Vel_r \cos\theta} \\ &= \frac{1}{8} \pi \rho_f d^2 C_d Vel_r (u_f - u) \end{aligned} \quad (3.5)$$

$$\begin{aligned} f_y &= \frac{1}{8} \pi \rho_f d^2 C_d Vel_r^2 \frac{(v_f - v)}{Vel_r} \\ &= \frac{1}{8} \pi \rho_f d^2 C_d Vel_r (v_f - v) \end{aligned} \quad (3.6)$$

$$\begin{aligned} f_z &= \frac{1}{8} \pi \rho_f d^2 C_d Vel_r^2 \cos\theta \frac{(w_f - w)}{Vel_r \cos\theta} \\ &= \frac{1}{8} \pi \rho_f d^2 C_d Vel_r (w_f - w) \end{aligned} \quad (3.7)$$

where  $C_d$  is the drag coefficient of the particle of liquid-jet (Goldstein<sup>13</sup>, White<sup>7</sup>, Knori<sup>9</sup>) at the speed  $Vel_r$  pictured in Fig. 1 and the directions of  $f_x$ ,  $f_y$ , and  $f_z$  are also decided by the signs of  $(u_f - u)$ ,  $(v_f - v)$ , and  $(w_f - w)$ , respectively. Then substitute Eq. (3.5), (3.6), and (3.7) into Eq.(3.1) with  $m$ ,  $m_f$  in terms of densities  $\rho$  and  $\rho_f$ , where  $m'$  is equal to  $m_f/2$ . After substitutions, (3.1) becomes

$$\begin{aligned} \left(m + \frac{m_f}{2}\right) \frac{d^2x}{dt^2} &= \frac{1}{8} \pi \rho_f d^2 C_d Vel_r (u_f - u) \\ \left(m + \frac{m_f}{2}\right) \frac{d^2y}{dt^2} &= \frac{1}{8} \pi \rho_f d^2 C_d Vel_r (v_f - v) - (m - m_f)g \\ \left(m + \frac{m_f}{2}\right) \frac{d^2w}{dt^2} &= \frac{1}{8} \pi \rho_f d^2 C_d Vel_r (w_f - w) \end{aligned} \quad (3.8)$$

Next step is to solve for  $\frac{d^2x}{dt^2}$ ,  $\frac{d^2y}{dt^2}$ ,  $\frac{d^2z}{dt^2}$  and

$$\frac{d^2x}{dt^2} = \frac{\frac{3\bar{\rho}}{4d} C_d Vel_r (u_f - u)}{\left(1 + \frac{1}{2}\bar{\rho}\right)} \quad (3.9)$$

$$\frac{d^2y}{dt^2} = \frac{[-(1 - \rho)g + \frac{3\bar{\rho}}{4d} C_d Vel_r (v_f - v)]}{\left(1 + \frac{1}{2}\bar{\rho}\right)} \quad (3.10)$$

$$\frac{d^2z}{dt^2} = \frac{\frac{3\bar{\rho}}{4d} C_d Vel_r (w_f - w)}{(1 + \frac{1}{2}\bar{\rho})} \tag{3.11}$$

where,  $m = \frac{1}{6}\pi\rho d^3$ ,  $m_f = \frac{1}{6}\pi\rho_f d^3$

$\rho$  = density of liquid body

$\rho_f$  = density of fluid,  $\bar{\rho} = \rho_f/\rho$

$C_d$  = drag coefficient of particle of body

$d$  = diameter of particle of body

In a numerical calculations several diameters of liquids moving in air should be considered. Initially the particle is released from the origin of the three dimensional coordinate system with a initial speed of body at an elevation of  $\theta_0$  when  $t=0$ . Elevation is the take-off angle between x-axis and initial speed of projectile, and  $\phi$  angle is the base angle between z-axis and initial speed of projectile. In addition, y altitude on a xz base plane can be also calculated in a distance of 0.1 m at both x and z axis.

The calculation should be stopped if the projectile falls back to its original level or below its original, namely, when  $y \leq 0$  for a negative  $v$ .

### 3. RESULTS

A number of calculations have been made on a particle projectile of water and other materials for different wind velocities, cross wind velocities, diameters of particle of liquid bodies, initial speed of bodies, take-off angles, and base angles. Classification of particles of a projectile

Investigated case	effect
Case 1	only air resistance
Case 2	air resistance + wind
Case 3	air resistance + cross wind
Case 4	air resistance + wind + cross wind
Case 5	no air resistance + Bernoulli water jet
Case 6	different particle densities

in terms of three different sizes of diameter with six cases of different effects are listed below, which is a breakdown of the investigated effectson particle trajectory.

The important parameters in the computer code are illustrated below. The initial take-off velocity,  $Vel_0$  is 10 m/s, the take-off angle,  $\theta$  are 15, 30, 45, 60, 75, 90 degrees, the base angle on xz plane,  $\phi$ , is only zero degree, the wind velocity on x direction,  $u_f$  are 0, 10, -10, 20, -20 m/s, the velocity on y direction,  $v_f$  is 0 m/s, the cross wind velocity on z direction,  $w_f$  are 0, 10, 20 m/s and the diameter of the particle of liquid jet body,  $d$  are 0.005, 0.003, 0.001 m for each case of different liquid densities.

#### 3.1 case 1 (only air resistance)

Figure 2 on case 1 shows trajectories of water projectiles with no wind velocity for three different sizes of body (diameter  $d=0.005$ , 0.003, 0.001 m) with zero base angle  $\phi$  and wind velocity. Figure 2 shows that the largest

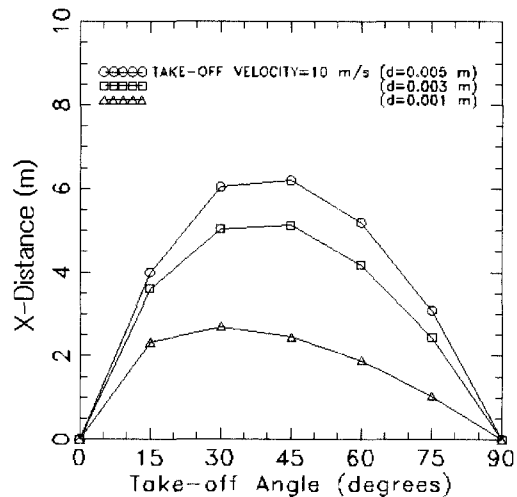


Figure 2 : x-distance vs. take-off angle for three particle sizes at take-off velocity = 10 m/s, base angle = 0 degree, wind and cross velocity = 0 m/s, density of water = 1000 kg/m<sup>3</sup>

diameter,  $d=0.005\text{ m}$  can give the longest  $x$  distance of a projectile because the larger particle of a projectile can take less air resistance for the same density in the fluid. The result show sthat the optimum take-off angle is between 30 and 45 degrees according to the different operating parameters.

**3.2 case 2 (air resistance + wind)**

Figure 3 on case 2 shows trajectories of water projectiles with air resistance and wind velocity on  $x$  component for one size of body (diameter  $d=0.003\text{ m}$ ). The particle remains in the  $xy$ -plane with no wind velocity on  $z$  component and base angle  $\phi=0$  degree.

As a matter of a fact, case 2 also illustrates two-dimensional trajectory due to the fact that both base angle and cross wind velocity are zero like case 1.

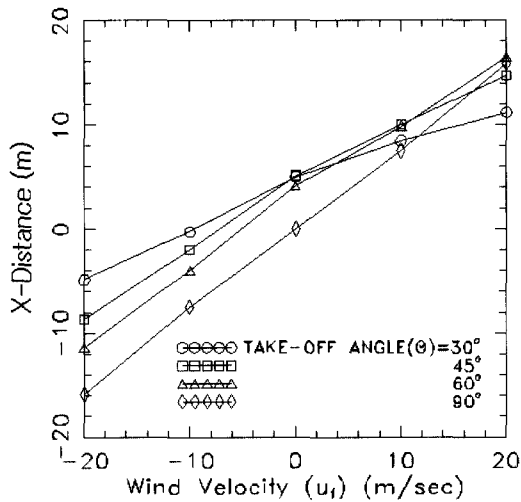
Figure 3 also reveals that  $x$  maximum distance of a projectile is proportional to the wind velocity on  $x$  component, which means that

the larger  $u_f$  and take-off velocity give the longer  $x$  maximum distance of a projectile on the same diameter,  $d=0.003\text{ m}$  when compared with case 1. For one take-off velocity chosen, the optimum take-off angle can be variable for wind velocity between 45 and 75 degrees, so there is nolinear relationship among  $x$  maximum distance,  $u_f$  and take-off angle.

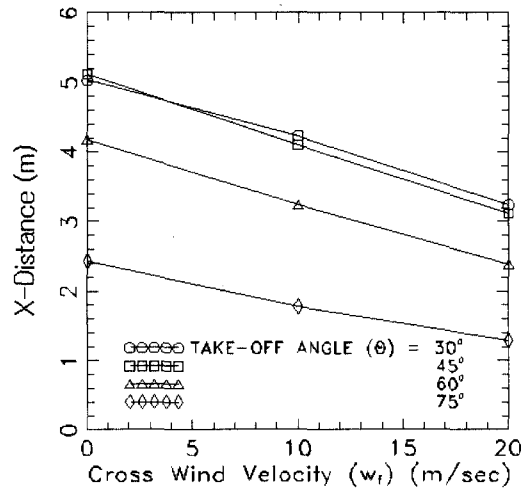
In addition, Fig. 3 shows that for a little while the flow is moving forward, and after a moment the flow changes into backward motion because of negative  $u_f$ . In this case, the higher the elevation of the take-off is, the easier the flow changes from forward to backward.

**3.3 case 3 (air resistance + cross wind)**

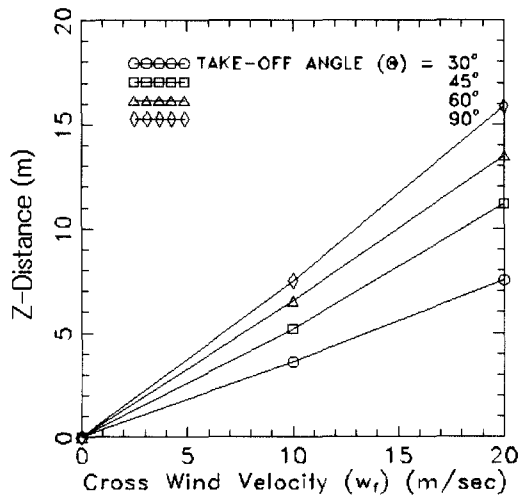
Figures 4 and 5 on case 3 reveal trajectories of water projectiles with air resistance and only crosswind velocity for one size of body (diameter  $d=0.003\text{ m}$ ). The particle remains in the  $xyz$ -plane with crosswind velocity,  $w_f$  and base angle  $\phi=0$  degree. Therefore, case 3 ought



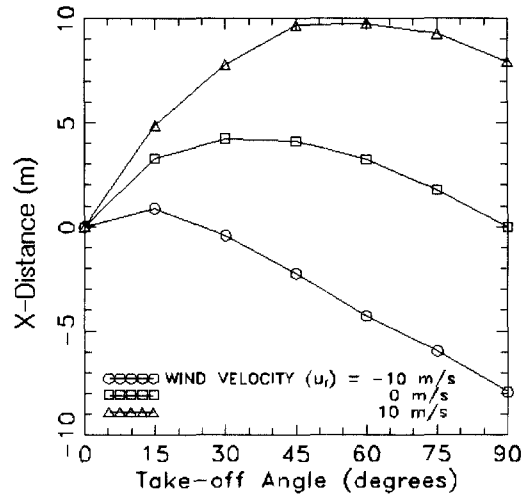
**Figure 3 :**  $x$ -distance vs. wind velocity for different take-off angles at take-off velocity=10 m/s, base angle=0 degree, diameter=0.003 m, cross wind velocity=0 m/s, density of water=1000 kg/m<sup>3</sup>



**Figure 4 :**  $x$ -distance vs. cross wind velocity for different take-off angles at take-off velocity=10 m/s, base angle=0 degree, diameter=0.003 m, wind velocity=0 m/s, density of water=1000 kg/m<sup>3</sup>



**Figure 5 :** z-distance vs. cross wind velocity for different take-off angles at take-off velocity=10 m/s, base angle=0 degree, diameter=0.003 m, wind velocity=0 m/s, density of water=1000 kg/m<sup>3</sup>



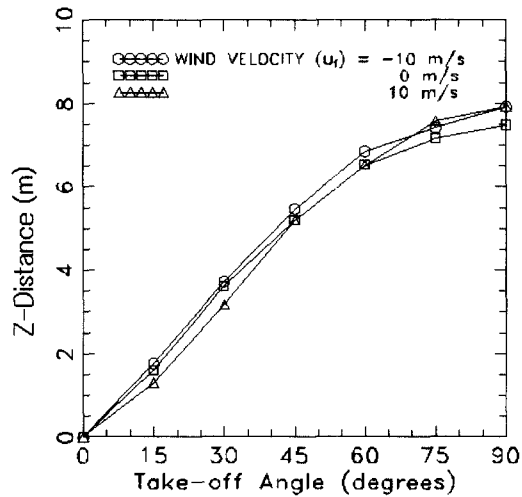
**Figure 6 :** x-distance vs. wind velocity for different take-off angles at take-off velocity=10 m/s, base angle=0 degree, diameter=0.003 m, cross wind velocity=10 m/s, density of water=1000 kg/m<sup>3</sup>

to have a three-dimensional trajectory on the ground that  $w_f$  exists as a nonzero value even if the base angle is zero. Figure 4 reveals that the optimum take-off angle for  $x$  maximum distance of a projectile is around 30 degrees.

Figure 4 explains that  $x$  maximum distance of a projectile is inversely proportional to the cross wind velocity and Fig. 5 shows that the maximum distance of a projectile is proportional to the cross wind velocity on the same  $x$  wind velocity,  $u_f=0$  m/s. When compared with previous cases, the larger  $u_f$  and the smaller  $w_f$  give the longer  $x$  maximum distance of a projectile.

**3.4 case 4 (air resistance + wind + cross wind)**

Figures 6 and 7 on case 4 show trajectories of water projectiles with air resistance, wind velocity, and cross wind velocity for one size of body (diameter  $d=0.003$  m). The particle remains in the  $xyz$ -plane with cross wind,  $w_f$



**Figure 7 :** z-distance vs. wind velocity for different take-off angles at take-off velocity=10 m/s, base angle=0 degree, diameter=0.003 m, cross wind velocity=10 m/s, density of water=1000 kg/m<sup>3</sup>

and base angle  $\phi=0$  degree, which means three-dimensional trajectory. Figures 6 and 7 also reveal the composite trajectory on  $x$  and  $z$

distances for three different  $x$  wind velocities and one crosswind velocity on the same diameter,  $d=0.003\text{ m}$ . In addition, figs. 6 and 7 show that the optimum take-off angle can be changeable according to the wind velocity and take-off velocity.

When compared with previous cases, the bigger  $u_f$  gives the longer  $x$  maximum distance of a projectile for a given take-off angle and velocity, but the  $z$  maximum distance of a projectile can be changeable for different wind velocity on  $x$  component. Furthermore, the bigger take-off angle give the longer  $z$  maximum distance of a projectile.

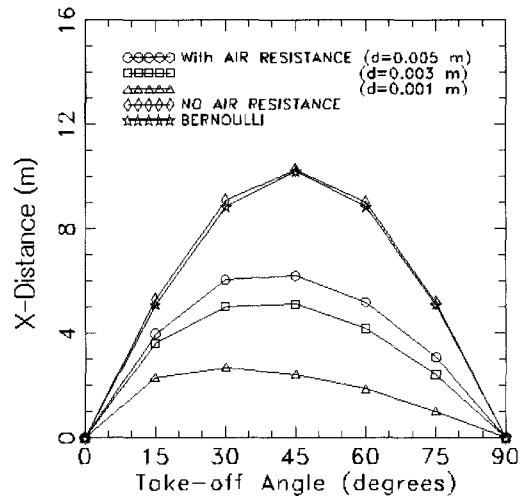
Conclusively, the bigger  $u_f$  and  $w_f$  give the longer  $x$  and  $z$  respective maximum distance of a projectile.

**3.5 case 5 (no air resistance + Bernoulli water jet)**

Figure 8 on case 5 shows trajectory water projectiles with air resistance, with no air resistance, and in a vacuum by Bernoulli equation.

Figure 8 also reveals that  $x$  or  $z$  distance whether without air resistance or with Bernoulli water jet are the same respectively regardless of the diameter of liquid body, but initial take-off velocity and take-off angle of body effect much on the  $x$  and  $z$  distances of a projectile. The optimum angle for  $x$  maximum distance of a projectile is between 30 and 45 degrees. Therefore, the bigger take-off velocity and the larger particle size give the longer  $x$  distance of a projectile for the same density.

Finally, case 5 leads to several important conclusions. First, the trajectory with all possible effects, such as air resistance and wind effects, is very different from a trajectory in a vacuum by Bernoulli equation and different from a trajectory with no drag coefficient. This is



**Figure 8 :** Comparison with idealized cases of no air resistance and Bernoulli jet for different take-off angles at take-off velocity=10 m/s, base angle=0 degree, diameter=0.005, 0.003, 0.001 m, wind velocity=0 m/s, density of water=1000 kg/m<sup>3</sup>

because of very great air resistance from drag force by the viscosity of air. In addition, the trajectory with no drag coefficient is negligibly different from the trajectory in a vacuum by Bernoulli water jet, because the former should be calculated by the finite difference method and the latter should be a continuous flow trajectory by Bernoulli equation constructed. Due to this fact, there should be very little difference between trajectory with no drag coefficient and trajectory in a vacuum, but ultimately the former should be the same as the latter.

**3.6 case 6 (different particle densities)**

Figures 9 through 11 show trajectories of different particle projectiles on different particle densities of materials with air resistance and no wind velocity for three different sizes of body (diameter  $d=0.005, 0.003, 0.001\text{ m}$ ). The particle remains in the  $xy$ -plane with base angle  $\phi=0$  degree. In this case, three



actual materials should be used to compare the effects and phenomena such as gasoline, polypropylene, and water. In addition, a different initial take-off velocity and six different take-off angles were applied for each particle size and each density. Case 6 used different densities to compare this with case 1 about possible effects. Three different densities of materials are as follows ; Gasoline-density  $700 \text{ kg/m}^3$ , Polypropylene-density  $900 \text{ kg/m}^3$ , Water-density  $1000 \text{ kg/m}^3$ .

Important factors, as mentioned above, are optimum take-off angle and velocity for each diameter and density. Figures 9 through 11 show that certain materials which have smaller densities than water have optimum take-off angles for  $x$  maximum distance of a projectile between approximately 15 degrees and 30 degrees, with important regard to operating variables, especially diameter and density. But other materials which have bigger densities than water have optimum take-off angles for  $x$

maximum distance of a projectile with also regard to operating variables. In other words, different particle sizes and densities can give different optimum take-off angles of a projectile. Take-off angles between 15 and 45 degrees, therefore, can be almost optimum take-off angle for a projectile in all of the three cases.

When compared with the composite distance trajectory all together, Fig. 8 shows the composite distance trajectory with three important effects such as with air resistance, no air resistance, and Bernoulli distance according to the different particle sizes,  $d=0.005, 0.003, 0.001 \text{ m}$ . Important notice should be taken that the distance of air resistance case is much shorter than the Bernoulli distance or the distance of no air resistance case ( $C_d=0$  and no buoyancy term), because air resistance from drag force caused by great air viscosity greatly influences the  $x$  and  $z$  maximum distances of a projectile.

In addition, the trajectory with no air

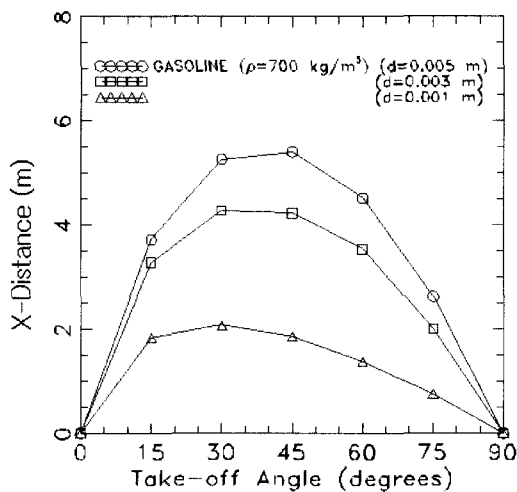


Figure 9 :  $x$ -distance vs. take-off angle for three particle sizes at take-off velocity=10 m/s, base angle=0 degree, wind and cross velocity= 0 m/s, density of gasoline=700 kg/m<sup>3</sup>

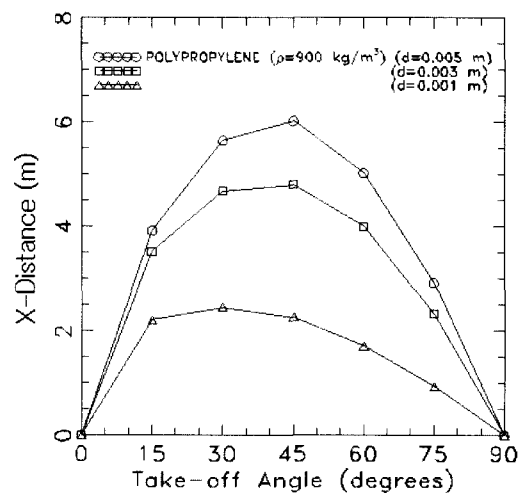


Figure 10 :  $x$ -distance vs. take-off angle for three particle sizes at take-off velocity=10 m/s, base angle=0 degree, wind and cross velocity= 0 m/s, density of polypropylene=900 kg/m<sup>3</sup>

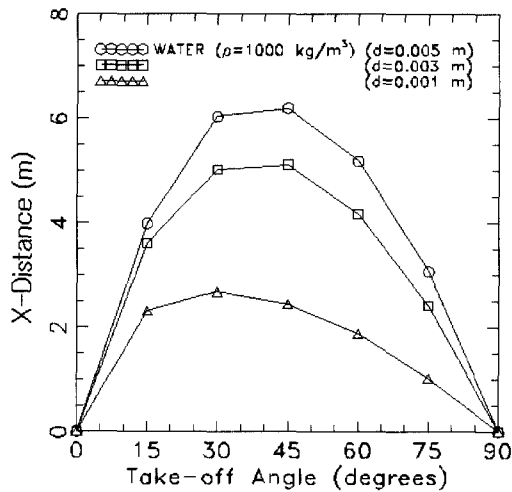


Figure 11 : x-distance vs. take-off angle for three particle sizes at take-off velocity = 10 m/s, base angle = 0 degree, wind and cross velocity = 0 m/s, density of water = 1000 kg/m<sup>3</sup>

resistance is very little different from that in vacuum by Bernoulli trial. However, if the time step is reduced to very small number, ultimately the trajectory of Bernoulli and no air resistance should be the same. It should also be noted that as the take-off velocity and angle increase, the difference between the distance of air and the distance of no air and Bernoulli becomes greater than before. The physical reason is that the distance with air resistance and the bigger take-off angle is more easily controlled and influenced by the effect of air resistance and the distance with air resistance and the bigger take-off velocity is unable to be longer than that with no air and Bernoulli distance, because the projectile of no air and Bernoulli can travel further than that of air resistance owing to no effect of air.

#### 4. CONCLUSIONS

Until now six cases of effects have been

examined. The distances of *x* and *z* directions can be calculated for the given value of take-off velocity and the take-off angle in the form of a three-dimensional table on different cases. Only zero value of base angle,  $\phi$ , should be of importance in order to check the above results with Bernoulli distance with no air resistance next time for convenience. The distances of *x* and *z* directions are the last values of *x* and *z* positions respectively when  $y \leq 0$  for a negative  $v$  (vertical component of liquid body velocity).

The results give the trajectories of various types of particle of body and at different elevations base angles, wind velocities, and densities of liquid body. The motions in a vacuum can be done when  $\rho_f$  is zero value, and trajectories already calculated and drawn show that air resistance decreases both the distance and the maximum height of a projectile, and also explain that the termination time is also reduced in air. In addition, the maximum distance of *x* direction was obtained with take-off angles from 30 to 45 degrees in still air. It is well known that the projectile of particles are highly affected by wind and crosswind. Clearly, a particle has to be so positioned as to take the optimum possible advantage of the wind if the maximum distance is requested. The wind astern increased the maximum distance of *x* direction compared with the wind ahead (Hatton<sup>14</sup>).

Finally, significant conclusions are :

1. For the same particle size of different densities, the body of a bigger density gives a longer *x* maximum distance of a projectile, because the bigger density and the heavier weight takes less air resistance than the smaller density considering the particle size.
2. For the same density of a different particle size, the bigger size gives a longer *x* maximum distance of a projectile, because the bigger size, 0.005 m, takes less air resistance than the

smaller size, 0.001  $m$ , which means the smaller size is more easily controlled by the air motion.

3. For the same density of same particle size, but with different take-off velocities, it is obvious that the bigger initial take-off velocity give us a longer  $x$  maximum distance of a projectile.

4. Considering the first case of the pure water particles produced by the various nozzles, there seems to a clear advantage of a certain design as to the maximum possible distance as it leaves the nozzle and final carry.

The present research was concerned with three dimensional motion and trajectories of particle in the air considering the drag and wind effects. The use of a computer simulation of projectile of particles has been shown to be useful to estimate the results of the particles for different operating parameters such as best particle diameter, density of liquid body, initial take-off velocity, wind velocity, cross wind velocity, take-off angle, and base angle for a released flow from the nozzle. According to the information above it is possible to optimize the designs of both the monitors and pump and the systems of liquid and fire jet. The actual trajectories of the particle may depend more on wind effects than that of the computer simulation.

### NOMENCLATURE

$c_d$	: drag coefficient of particle of liquid body
$d$	: diameter of particle of liquid body
$F$	: total drag force
$f$	: resultant fluid dynamic force on body
$g$	: gravitational acceleration
$m$	: mass of liquid body
$m'$	: added mass
$m_f$	: mass of fluid
$Re$	: Reynolds number
$u$	: X component of relative velocity of projectile

$u_f$	: X component of wind velocity
$v$	: Y component of relative velocity of projectile
$v_f$	: Y component of wind velocity
$Vel$	: velocity of body relative to the stationary coordinate system
$Vel_f$	: fluid velocity in motion
$Vel_0$	: initial take-off velocity of body
$Vel_r$	: fluid velocity relative to that of body
$w$	: Z component of relative velocity of projectile
$w_f$	: Z component of wind velocity
$x$	: X position of projectile
$y$	: Y position of projectile
$z$	: Z position of projectile

### Greek Letters

$\rho$	: density of liquid body
$\rho_f$	: density of fluid
$\rho$	: density ratio ( $=\rho_f/\rho$ )
$\nu$	: kinematic viscosity
$\theta$	: take-off angle of projectile
$\Phi$	: base angle of projectile

### Subscripts

$f$	: relating to indication of fluid
$r$	: relating to indication of relative component
$0$	: relating to indication of initiation

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### 저 자 소 개



#### 이동렬 (李東烈)

63년생, 1986년 연세대학교 기계공학과 졸업. 1989년 Oklahoma state University 기계공학과 졸업(석사), 1995년 State University of New York at Stony Brook 기계공학과 졸업(박사), 1995년~1996년 State University of New York at Stony Brook, Thermal Science Research Laboratory, 1996년~1997년 삼성자동차(주)기술연구소, 1997년~현재 대구가톨릭대학교 기계자동차공학부 조교수