⊙ 研究論文

Numerical Investigations of Enhancement of a Convective Fin Efficiency by Convection-Radiation Conjugate Heat Transfer

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대류-복사 복합 열전달을 고려한 대류 핀효율의 향상에 관한 수치적 연구

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Key words : Fin efficiency, Conjugate heat transfer, Fin emissivity, Radiation exchange, Radiating fin

Abstract

In almost all real situations, there will be a radiant interchange between adjacent fins with the base surface as well as with the external environment. In the problem of this study, a rectangular fin is attached to a base. Our concern is whether the convective fin efficiency can be increased by the radiation heat exchange between the fin and the base surface and how much. If the fin temperature toward the tip increases by the effect of radiation, the convective heat transfer increases due to the temperature difference between the ambient temperature and the surface temperature of the fin. If this is true, the efficiency of the fin due to the radiation will increase. Attention is directed toward several parameters which have major roles on getting value of the fin efficiencies in several different values of parameters. Many different cases are, therefore, to be examined to have maximum fin efficiency by varying the values of each parameter.

1. Introduction

This radiation-convection fin problem conjugates all three forms of heat transfer : radiation, conduction, and convection. The heat conducted into the fin is balanced against the heat leaving the fin through convection and radiation, and the heat entering the fin by radiation. In order to get a grip on all these flows the potential field, temperature, must obviously be solved for. Unfortunately, as will be seen later, the equation for the temperature

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field derived from the heat balance is a nonlinear differential equation and it is forced to be analyzed by numerical methods as the theoretical methods are inadequate. The point in analyzing this fin solicited by the fin over a logical range of the other parameters in the problem (Mills[1]), Eckert and Drake[2], Jakob[3].

The results of the computer model generated to solve the problem seems to indicate that a choice of emissivity as close to one as possible gives the most efficient fin. The only question posed against this result is when values of N_C, the convection parameter are very large, and Ta^+, the normalized convective fluid temperature is very small in which case the solution fails to converge and no result. Also, the contribution of radiation to heat transfer is usually large, and the magnitude of the normalized temperature distribution never gets very small except in the above exceptional case (N_C large, T_a^+ small). A reduction of the convective fin efficiency of the system with a increase in emissivity can be due to a slight reduction in the magnitude of the normalized temperature profile which reduces the total net convective heat flow while increasing the radiative heat flow due to its roughly linear dependence on emissivity and increase in the total efficiency (Sparrow and Cess[4]).

Due to the large possible variation of the four independent parameters arising from our differential equation, N_R the radiation parameter $\left(\frac{\sigma T_0^3 l^2}{KA}\right)^{\frac{1}{2}}$, N_C the convection parameter $\left(\frac{hp\ l^2}{KA}\right)^{\frac{1}{2}}$, T_a^+ the convective medium temperature normalized by the base temperature, and ε_f the fin emissivity hereafter symbolized EF whose effects are investigated.

A complete analysis produces a solution space too large to analyze in this paper, and only a judicious sampling of that space is used here. An iterating solution technique is used because it doesn' t converge rapidly. More is discussed in the conclusion.

2. Analysis and Derivation of Working Equations

This is an analytic project which calculates the fin efficiency both with convection only and with convection and radiation. Therefore it can be examined whether the convective fin efficiency can be increased by the radiation exchange between the fin and the base surface and by how much. The effect of this radiation will increase the fin temperature toward tip, therefore increasing the convective heat transfer and the fin efficiency.

Fortunately the problem has been simplified with several restrictions on the general case. The radiating bodies are gray and all radiation is diffuse. It may be, therefore, helpful to use the relation $\alpha=\varepsilon$, absorptivity is equal to emissivity, and the standard shape factor, radiosity, irradiation formulation of the radiation problem. In addition the emissivity of the flat surface from which the fin protrudes is 1.0 (it is a blackbody), the radiation exchange at infinity above the fin is an absolute zero surface and thus is not included in the analytical equations. The geometry is restricted such that the dimension H/l is restricted to the value 2.0, and the dimension L is infinity, the surface temperature of the black body from which the fin protrudes is a uniform T, and the tip of the fin is adiabatic.

A rectangular fin in Fig.1 is attached to a base surface of dimension $H\times L$. The fin has a length l, a thickness 2b, and emissivity ε_f. The edge of the fin (2b) are insulated both to impose a tip adiabatic boundary condition and to keep the conduction problem one dimensional. There is a constant convection heat transfer coefficient h

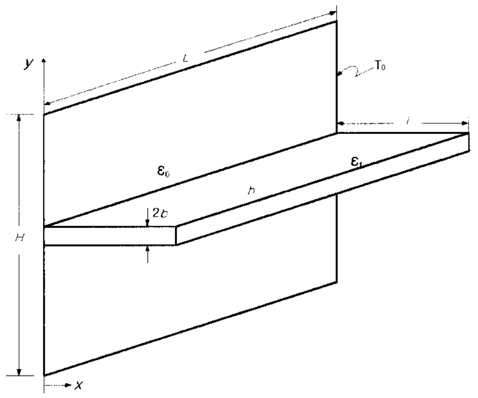

Fig. 1 Schematic diagram of the rectangular fin.

and a constant fluid temperature Ta. The emissivity of the base surface is $\varepsilon_0(=1)$. To get the maximum fin efficiency(η_f), values of the parameters over a practical range of Nc^2 and other parameters (ε_f, T_a^+, N_R) must be chosen.

Here, the condition that $\frac{H}{l}=2$ is given.

This is a conjugated problem with radiation and convection and the purpose of this problem is to investigate the influence of design factors such as ε_f, T_a^+, N_R, N_C for the efficiency of the fin. The equation is how much the efficiency of the fin varies due to the radiation and what factor is involved in radiation process. In conclusion, it is necessary to find out how to design this kind of fin to maximize the fin efficiency. In this problem, let $L=\infty$ and consider that the radiation takes place between gray and diffuse surfaces. Let $H/l=2.0$, choose values of ε_0 and ε_f to maximize the fin efficiency over a practical range of N_c^2, and other parameters (Ruperti et al[5]).

2.1 Differential equations

Considering a control volume as shown in Fig.2, the energy balance equation can be led to a governing differential equation ;

$$kA\frac{d^2T}{dx^2}+L\cdot(G_x-J_x)=hp(T-T_a) \qquad (1)$$

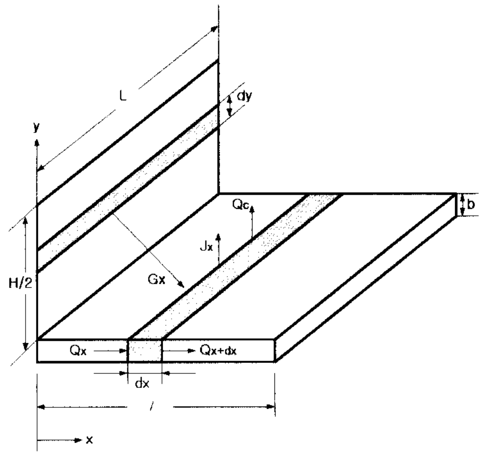

Fig. 2 Sketch of the control volume of the fins.

with boundary conditions

$T(0)=T_0,\ T'(l)=0$

where, $J_x=\varepsilon_f\,\sigma T^4+\rho G_x$

$\alpha=\varepsilon_f$ and $\alpha+\rho=1$

$$J_x=\varepsilon_f\,\sigma T^4+(1-\varepsilon_f)\,G_x \qquad (2)$$

$$G_x=\int_0^{\frac{H}{2}} e_{by}\,F_{dx,dy}\,dy \qquad (3)$$

Defining the dimensionless parameters ;

$$T^+=\frac{T}{T_0},\ T_a^+=\frac{T_a}{T_0},$$

$$x^+=\frac{x}{l},\ y^+=\frac{y}{l}$$

$$G_x^+=\frac{G_x}{\sigma T_0^4},\ J_x^+=\frac{J_x}{\sigma T_0^4}$$

Substituting above parameters into equation (1),(2) and (3), the following equation can be obtained

$$\frac{d^2T^+}{dx^{+2}}=\frac{hp\,l^2}{kA}(T^+-T_a^+)+\frac{\sigma T_0^3\,l^2\,L}{kA}(J_x^+-G_x^+) \qquad (4)$$

Letting, $N_c^2=\frac{hp\,l^2}{kA}$

$N_R^2=\frac{\sigma T_0^3 l^2 L}{kA}$

Equation (4) becomes

$$\frac{d^2 T^+}{dx^{+2}}=N_c^2 (T^+ - T_a^+)+N_R^2(J_x^+ - G_x^+)$$

with boundary conditions

$$T^+(0)=1$$

$$T^{+'}(1)=0$$

$$J_x^+ = \varepsilon_f T^{+4}+(1-\varepsilon_f) G_x^+ \quad (5)$$

$$G_x^+ = \int_0^{H/2l} e_{by} F_{dx,dy}\, dy^+ \quad (6)$$

In addition, fin efficiency is defined as follows ;

$$\eta_f=\frac{Q_f}{Q \text{ if } T(x)=T_0}$$

Using dimensionless parameters, the fin efficiencies for the two different cases (with and without radiation) can be written as following

$$\eta_{f,conv}=\frac{\int_0^1 hp(T^+ - T_a^+)dx^+}{1-T_a^+} \quad (7)$$

$$\eta_{f,conv+rad}=\frac{\int_0^1(T^+ - T_a^+)dx^+ + N^2\int_0^1(J_x^+ - G_x^+)dx^+}{1-T_a^+} \quad (8)$$

where $N^2=\dfrac{\sigma T_0^3 L}{hp}$

2.2 Derivation of the Numerical Equations

A) Temperature considering a control volume

Using the dimensionless parameters defined above

$$T_{i-1}^+ - (2+N_c^2 \cdot \triangle x^{+2})\, T_i^+ + T_{i+1}^+ = -N_c^2 \cdot \triangle x^{+2} \cdot T_a^+ + N_R^2 \cdot \triangle x^{+2}(J_x^+ - G_x^+) \quad (9)$$

where

$$J_x^+ - G_x^+ = \varepsilon_f T_i^{+4} - \varepsilon_f G_x^+ = \varepsilon_f (T_i^{+4} - G_x^+)$$

$$=\varepsilon_f\left[T_i^{+4} - \frac{1}{2}\left(1-\frac{i\triangle x}{(1+(i\triangle x)^2)^{\frac{1}{2}}}\right)\triangle x\right]$$

where i=1, 2, 3, ⋯ , n−1

At the boundary,

$$x^+ = 0 : T_0^+ = 1$$

$$x^+ = x_n^+ : kA\frac{T_{n-1}-T_n}{\triangle x}+\frac{\triangle x}{2}L \cdot G_x - \frac{\triangle x}{2}L \cdot J_x = hp \cdot \frac{\triangle x}{2}(T_n - T_a) \quad (10)$$

Using dimensionless parameters

$$2\, T_{n-1}^+ - (2+N_c^2 \cdot \triangle x^{+2})\, T_n^+ = -N_c^2 \cdot \triangle x^{+2}\, T_a^+ + N_R^2 \cdot \triangle x^{+2} \cdot L(J_x^+ - G_x^+) \quad (11)$$

where

$$J_x^+ - G_x^+ = \varepsilon_f(T_n^{+4} - G_{xn}^+)$$

$$=\varepsilon_f\left[T_n^{+4} - \frac{1}{2}\left(1-\frac{\left(i-\frac{1}{4}\right)\cdot \triangle x}{\left[1+\left[\left(i-\frac{1}{4}\right)\triangle x\right]^2\right]^{\frac{1}{2}}}\cdot\frac{1}{2}\triangle x\right]$$

B) Fin Efficiency

considering the whole fin

$$Q_{f,conv}=\int_0^l hp(T-T_a)dx$$

$$Q_{f,conv+rad}==\int_0^l hp(T-T_a)dx+\int_0^l(J_x-G_x)dx$$

Using dimensionless parameters, the fin efficiencies for the two cases can be written as following ;

$$\eta_{f,conv}=\frac{\int_0^1(T^+ - T_a^+)dx^+}{1-T_a^+}=\frac{1}{2}\cdot\triangle x^+ + \frac{\sum_{i=1}^{n-1}(T_i^+ - T_a^+)\triangle x^+ + \frac{1}{2}\triangle x^+(T_n^+ - T_a^+)}{1-T_a^+} \quad (12)$$

$$\eta_{f,conv+rad} = \frac{\int_0^1(T^+ - T_a^+)dx^+ + N^2\int_0^1(J_x^+ - G_x^+)dx^+}{1-T_a^+}=\frac{1}{2}\cdot\triangle x^+ + \frac{\sum_{i=1}^{n-1}(T_i^+ - T_a^+)\triangle x^+ + \frac{1}{2}\triangle x^+(T_n^+ - T_a^+)}{1-T_a^+} + \frac{N^2\left[\frac{1}{2}\triangle x^+\varepsilon_f(1-G_x^+)+\sum_{i=1}^{n-1}\varepsilon_f\left(T_i^{+4}-G_x^+\right)\triangle x^+ + \frac{1}{2}\triangle x^+\varepsilon_f\left(T_n^{+4}-G_x^+\right)\right]}{1-T_a^+} \quad (13)$$

where $i=1,2,3,\cdots\cdots, n-1$

C) Derivation of G_x^+

According to Figure 3, the geometric relations are as follows ;

$$\cos\phi_1=\frac{x}{r},\ \cos\phi_2=\frac{y}{r}$$

$$r=(x^2+y^2+z^2)^{\frac{1}{2}}$$

$$F_{dx,dy}=\frac{\cos\phi_1\cos\phi_2}{\pi r^2}dA_2=\frac{xydzdy}{\pi(x^2+y^2+z^2)^2}$$

$$F_{x,dy}=\int_0^z\int_0^{H/2}\frac{xydzdy}{\pi(x^2+y^2+z^2)^2}=\frac{1}{2}\left[1-\frac{x}{\left(x^2+\frac{H^2}{4}\right)^{\frac{1}{2}}}\right]$$

$$G_x=\frac{1}{A_2}\int_{A1}e_{\ by}\cdot F_{1,dz}\,dA_1=\frac{2\times L}{H\times L}\int_0^l e_{\ by}\cdot\frac{1}{2}\left[1-\frac{x}{(H^2/4+x^2)^{\frac{1}{2}}}\right]dx$$

Letting H/l=2.0

$$x^+=\frac{x}{l},\ G_x^+=\frac{G_x}{\sigma T_0^{\ 4}}$$

$$G_x^+=\frac{1}{2}\int_0^1\left[1-\frac{x^+}{(1+x^{+2})^{\frac{1}{2}}}\right]dx^+ \qquad (14)$$

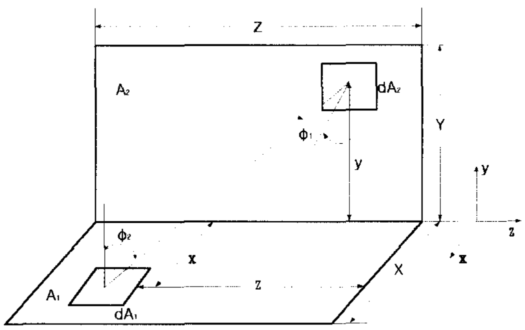

Fig. 3 Sketch showing area elements in radiation shape factor.

where the total radiation approaching the whole fin surface is $G_x^+=0.2929$

3. Calculation Method of Numerical Solution

In order to calculate the fin efficiency for pure convection and convection with radiation in two cases, first it is necessary to calculate the fin temperature distribution for different parameters. The iteration method was applied to make a computer program to solve equations (10), (11) and (12). Since there are four parameters (N_c, N_R, T_a^+ and ε_f) which are chosen in the engineering design, a lot of circulation loops were made in the program to get fin efficiency increase or decrease trend for different parameters. The main processes of the iteration method is as follows ;

Step 1 : Assume initial temperature to be equal to zero.

Step 2 : Calculate T_1^+ from the first equation.

Step 3 : Put T_1^+ from second equation into third equation and calculate T_2^+

Step 4 : Repeat above calculation until the absolute error is less than designated value.

4. Results and Discussion

The computation results (Nc=1.0 to 5.0, N_R= 0.05 to 1.0, T_a^+=0.1 to 1.0 and ε_f=0.1 to 1.0) are shown in Figs.4~12. Some particular cases are selected to tabulate the following tables and plot figures to compare the fin efficiency of increasing trend with N_C, N_R, T_a^+ and ε_f change. Figure 4 shows the dimensionless temperature of the rectangular fin.

(1). Fin efficiency changes with N_c.

From Table 1 and Fig.5, it is possible to

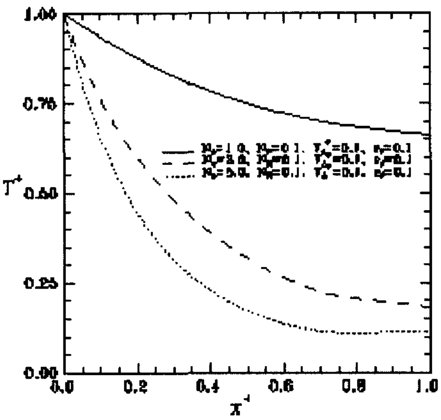

Fig. 4 Dimensionless temperature distribution of the fin.

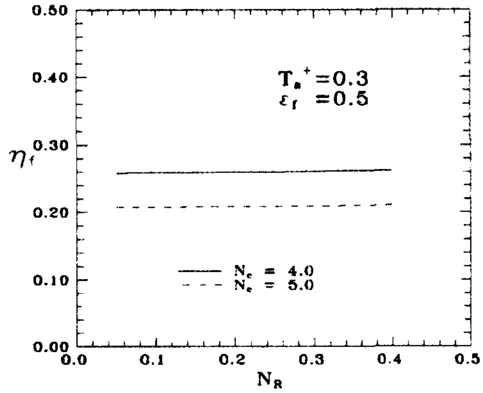

Fig. 7 Variation of the fin efficiency with Nc at different T_a^+.

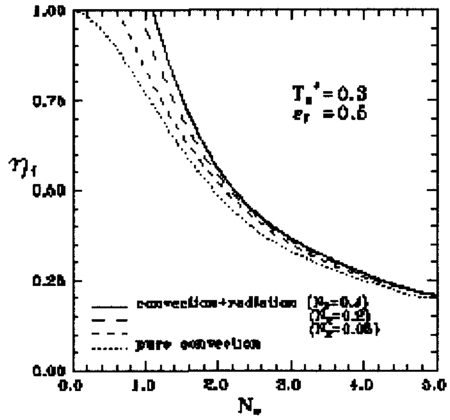

Fig. 5 Variation of the fin efficiency with Nc at different N_R.

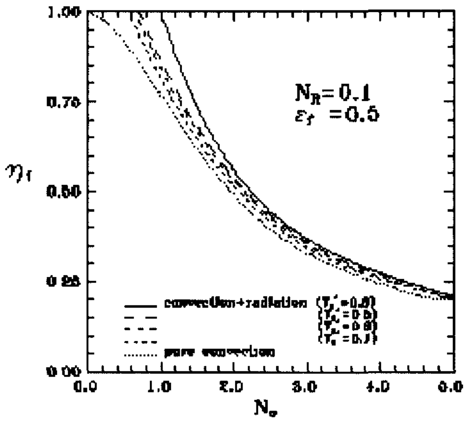

Fig. 8 Variation of the fin efficiency with Nc at different ε_f.

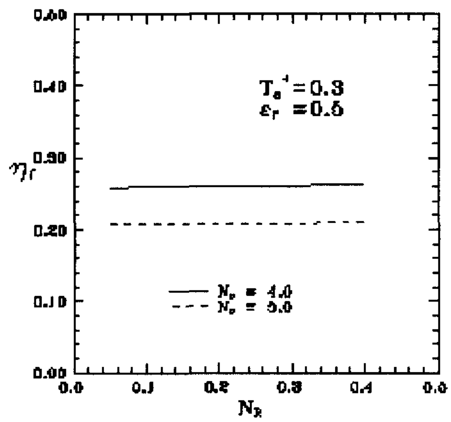

Fig. 6 Variation of the fin efficiency with N_R at different N_c.

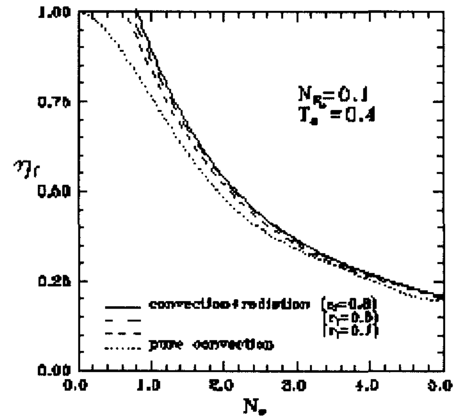

Fig. 9 Variation of the fin efficiency with Nc at different N_R.

show that fin efficiency increases as N_c goes to decrease and fin efficiency with radiation is greater than pure convection.

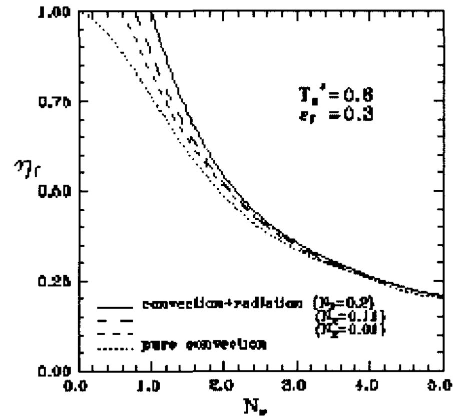

Fig. 10 Variation of the fin efficiency with N_c at different ε_f.

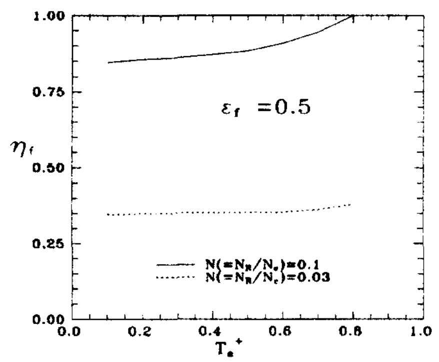

Fig. 11 Variation of the fin efficiency with $T_a^{'}$ at different N.

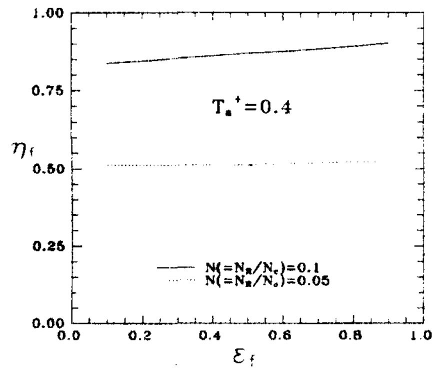

Fig. 12 Variation of the fin efficiency with ε_f at different N.

(2). Fin efficiency changes with N_R.

From Table 2 and Fig. 6, it is possible to see that fin efficiency increases as N_R increases. From Fig 4~12, it is also possible to find that as N_c is greater than 3.0, fin efficiency increases slowly as N_R increases.

Table 1 The variation of the fin efficiency with $N_c(N_R=0.05,\ T_a^{'}=0.3,\ \varepsilon_f=0.5)$, FE1 : fin efficiency (convection+radiation), FE2 : fin efficiency (pure convection)).

N_c	1.	2.	3.	4.	5.
FE1	0.8350	0.5116	0.3456	0.2586	0.2076
FE2	0.7623	0.4843	0.3354	0.2548	0.2061
$\frac{FE1-FE2}{FE1}\times 100\%$	8.68	5.34	2.95	1.47	0.72

Table 2 The variation of the fin efficiency with N_R (N_c= 0.05, $T_a^{'}$ = 0.3, ε_f= 0.5), FE1 : fin efficiency(convection+radiation), FE2:fin efficiency (pure convection)).

N_c	0.05	0.10	0.15	0.20	0.25
FE1	0.8350	0.8585	0.8975	0.9515	1.0000
FE2	0.7623	0.7623	0.7623	0.7623	0.7623
$\frac{FE1-FE2}{FE1}\times 100\%$	8371	11.21	15.10	19.90	25.30

Table 3 The variation of the fin efficiency with $T_a^{'}$ (N_c=1.0, N_R=0.10, ε_f=0.5), FE1 : fin efficiency (convection+radiation), FE2 : fin efficiency (pure convection)).

$T_a^{'}$	0.1	0.2	0.3	0.4	0.5
FE1	0.8464	0.8681	0.8822	0.9041	1.0000
FE2	0.7623	0.7623	0.7623	0.7623	0.7623
$\frac{FE1-FE2}{FE1}\times 100\%$	9.90	10.60	13.60	15.70	24.50

Table 4 The variation of the Fin efficiency with $T_a^{'}$ (N_c=4.0, N_R=0.10, ε_f=0.50), FE1 : fin efficiency (convection+radiation), FE2 : fin efficiency (pure convection)).

$T_a^{'}$	0.1	0.2	0.3	0.4	0.5
FE1	0.2585	0.2588	0.2590	0.2601	0.2656
FE2	0.2548	0.2548	0.2548	0.2548	0.2548
$\frac{FE1-FE2}{FE1}\times 100\%$	1.50	1.54	1.62	2.03	4.06

Table 5 The variation of the fin efficiency with ε_f ($N_c=0.05$, $N_R=0.3$, $T_a^+=0.5$), FE1 : fin efficiency (convection+radiation), FE2 : fin efficiency (pure convection)).

ε_f	0.1	0.2	0.3	0.4	0.5
FE1	0.8325	0.8518	0.8681	0.8845	0.9001
FE2	0.7623	0.7623	0.7623	0.7623	0.7623
$\frac{FE1-FE2}{FE1}\times 100\%$	8.7	10.5	12.2	13.8	15.4

(3). Fin efficiency changes with T_a^+.

Table 3, 4 and Fig.7 shows that the fin efficiency increases as T_a^+ increases when Nc is less than 3.0. When Nc is greater than 5.0, fin efficiency increases very slowly as T_a^+ increases.

(4). Fin efficiency changes with ε_f.

Table 5. and Fig.8 also shows that fin efficiency increases as the emissivity of the fin surface increases. When N_c is greater than 4.0, fin efficiency changes very slowly as ε_f increases.

From Fig 4~12, it is definite to find that the convective fin efficiency can be increased by the radiation exchange between the fin and the base surface. If N_c is smaller than 2.0, the fin efficiency increases fast with N_R, T_a^+, ε_f. In the engineering design, usually T_a^+ is given, fin geometric shape, size and material should be decided. If the material is chosen, the heat conductivity k is fixed, it is only necessary to decide the fin geometric shape and size.

5. Conclusion

- The convective fin efficiency can be increased by the radiation exchange between the fin and base surface.
- The fin efficiency depends on N_R, N_C, T_a^+, and ε_f.
- The difference of fin temperature distribution between the pure convection and the convection plus radiation is significant.
- The fin efficiency increases evidently with N_R, T_a^+ and ε_f if N_c is smaller than 3.0.

NOMENCLATURE

A : the fin area, defined as $b\times L$
b : 3the thickness of the fin
G_x : total radiation approaching the fin surface
G_x^+ : defined as $G_x/\sigma T_0^4$
H : the height of the fin
h : heat transfer coefficient
J_x : total radiation leaving the fin surface
J_x^+ : defined as $J_x/\sigma T_0^4$
k : thermal conducti½vity
l : the length of the finL the width of the fin ½
N : defined as N_R/N_c
N_c : defined as $\left(\frac{hP}{kA}\right)\ l$
N_R : defined as $\left(\frac{\sigma T_0^3 L}{kA}\right)\ l$
T : temperature
T_0 : temperature at the base of fin
T^+ : dimensionless temperature, T/T_0
T_a^+ : dimensionless convective fluid temperature
x^+ : dimensionless length, defined as x/l
y^+ : dimensionless length, defined as y/l

Greek Letters

ε_f : the emissivity of the fin surface

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