

Improvement of Transient Step Response Using Feedforward Compensator in Nonminimum Phase Systems

Byung-Moon Kwon, Hee-Seob Ryu, and Oh-Kyu Kwon

Abstract: This paper proposes a simple feedforward compensator in order to decrease the amount of undershoots and overshoots on the step response in nonminimum phase systems. This compensator makes the step type input be a ramp input with saturation for $0 \leq t < \alpha$. It is shown in this paper that the compensated system has small amount of undershoot and overshoot at the price of rise time compared to the system without compensator. Also, provided the system is properly stable, the influence of the design parameter α on the step response of the nonminimum phase system is investigated in the case of Type A, and Type B undershoot, which gives a guideline for the compensator design.

Keywords: feedforward compensator, transient response, nonminimum phase system, undershoot, overshoot

I. Introduction

In most industrial control plants, it is difficult to track a rapidly varying reference input like a step change. Also the step type input usually generates a bad transient response characteristic of the plant, and it may result in a violation of the constraints which can cause the instability of the system[1][2]. However, if the reference input is changed smoothly unlike the rapidly varying reference input, the transient response can be improved[2][3]. The vibration of flexible structures can also be greatly reduced by using smoothly varying reference input which generated by input shaper[4][5]. The type of smooth varying reference trajectory is often used in servo systems, e.g., industrial robots[3].

Reference models are used in many control problems for improvement of transient response characteristics, e.g., overshoots, saturation, etc. They provide desired trajectories that the plant should follow. Hence the plant is to track the output of reference model not the reference input[2]. Amount of undershoots and overshoots also can be reduced by tracking the smooth reference model output not the rapidly varying reference input. Hence the reference models are very important in the model reference control, but it would be a big problem to select the proper reference model for a given plant[1][2][3].

In this paper, instead of using a reference model, a simple feedforward compensator is to be proposed, which makes the step type input be a ramp input with saturation for $0 \leq t < \alpha$. The reference input generated by the proposed compensator can improve the transient response like undershoots, overshoots and settling time. But it is noted that the compensator does not affect the stability of the system because it is of feedforward type and its output is smaller than the step reference input. Using this compensator, it will be shown that the compensated system has small amount of undershoots and overshoots at the price of rise time. But the settling time of the system may be shortened by the compensator. Also, assuming the system is properly stable, the influence of the design parameter α on the step response of

the compensated system will be investigated in the case of Type A and Type B undershoot, which gives a guideline for the compensator design.

The layout of this paper is organized as follows: In Section 2, the type of undershoots will be defined for preliminaries. In Section 3, a simple feedforward compensator is to be proposed which decreases amount of undershoots and overshoots at the price of rise time. And the effects of changing α in the compensator are also analyzed. In Section 4, performances of the proposed compensator are exemplified via computer simulation to show that the ramp input with saturation $0 \leq t < \alpha$ can improve the transient response compared to the step changes in the reference input. The concluding remarks are given in Section 5.

II. Preliminaries

In this paper, we consider an SISO(single-input, single-output) system with RHP(right half plane) zeros. Let us introduce the classification of the undershoot which is caused by RHP zeros[6][7].

Definition 1: Let $y(t)$ ($t \geq 0$) be a sufficiently smooth scalar time response of a dynamical system and assume that $y(0) = 0$ and the following conditions:

a) There is a finite positive integer η that satisfies

$$y(0_+) = \dots = \frac{d^{\eta-1}y(t)}{dt^{\eta-1}} \Big|_{t=0_+} = 0, \quad (1)$$

and

$$\frac{d^\eta y(t)}{dt^\eta} \Big|_{t=0_+} \neq 0. \quad (2)$$

b) The steady-state value of $y(t)$ exists and is not zero, i.e.,

$$y(\infty) = \lim_{t \rightarrow \infty} y(t) \neq 0. \quad (3)$$

Then $y(t)$ is said to have a Type A undershoot if

$$y^{(\eta)}(0_+)y(\infty) < 0. \quad (4)$$

If Eq. (4) is not true, and there is an open interval (a, b) such that

$$y(t)y(\infty) < 0, \quad \forall t \in (a, b), \quad (5)$$

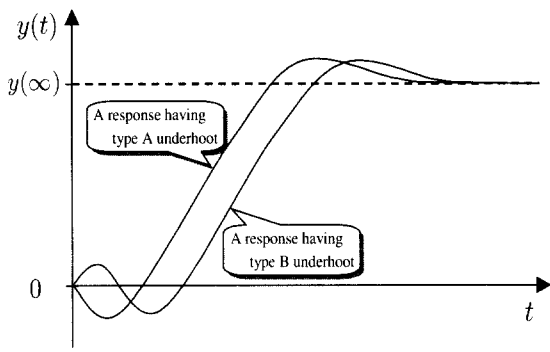


Fig. 1. Classification of the undershoot.

then $y(t)$ is said to have a Type B undershoot.

The undershoot type is shown in Fig. 1 when the steady-state value is positive. Many papers have discussed such phenomena in SISO systems and have shown that the Type A undershoot occurs if and only if the number of the zeros having positive real parts is odd[6][7][8].

As a matter of fact, it has been realized that the nonminimum phase system, compare to the minimum phase system, has more various fundamental limitations imposed by RHP zeros[9][10][11]. One of these limitations is the step response extrema including undershoot and overshoot phenomena. In this paper, the reference input is restricted to the step function since the problem is not severe to the ramp or parabolic input. Furthermore we assume that the system $G(s)$ in Fig. 2 is stable and has a positive steady-state value, i.e., the feedback controller $K(s)$ has been already designed in order to guarantee the stability.

III. Design of feedforward compensator

In this chapter, we will propose a feedforward compensator in order to improve the transient response of the system. The main idea is that small input gives rise to small amount of undershoots and overshoots at the price of rise time.

Let us consider the feedforward compensator $C(s)$ as follows:

$$C(s) = \frac{1}{\alpha} \frac{1}{s} (1 - e^{-\alpha s}), \quad (6)$$

where α is a positive real value which is selected by designer. It is noted that this compensator has only one zero at $+\infty$ and one pole at $-\infty$. It also has similar properties to low pass filter in frequency domain. According to the value of α , the proposed compensator makes the step type input be a ramp input with saturation for $0 \leq t < \alpha$ as shown in Fig. 3. When a step type function is applied to this compensator, its output has the shape

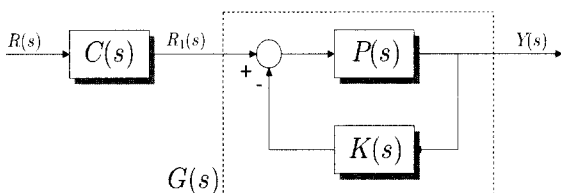


Fig. 2. Feedback system with feedforward compensator.

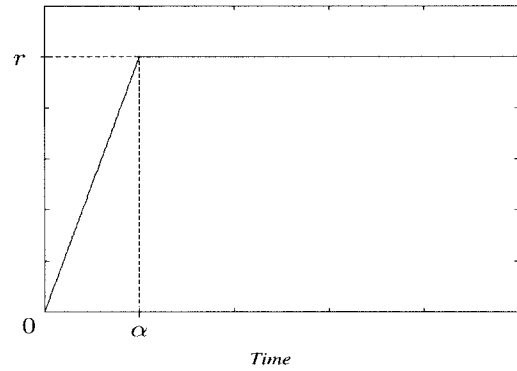


Fig. 3. Compensator response to the step input.

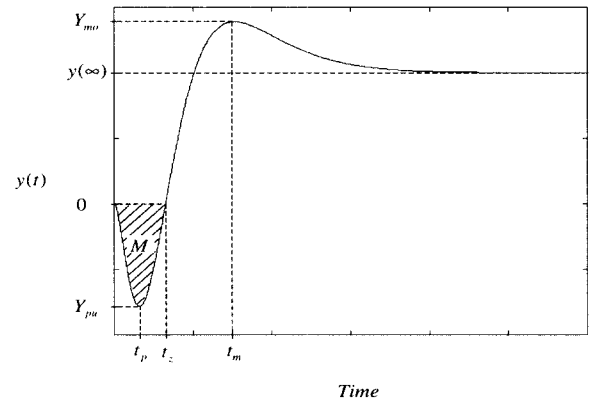


Fig. 4. Definition of parameters in the system with Type A undershoot.

of Fig. 3. We will investigate the characteristic of this compensator concentrated on the transient response characteristic of the nonminimum phase system to the step type reference input.

In the system with Type A undershoot, let Y_{pu} , t_p , Y_{mo} and t_m be the peak undershoot, the peak undershoot time, the maximum overshoot and the maximum overshoot time of the step response for the system without the compensator (6), respectively. And let Y_{pc} , t_{pc} , Y_{mc} and t_{mc} be the peak undershoot, the peak undershoot time, the maximum overshoot and the maximum overshoot time of the step response for the compensated system, respectively. Also let us define $t_z (> 0)$ as a time point such that the step response crosses over the zero level, and define M as the area of step response from 0 to t_z , which can be written by

$$M = \int_0^{t_z} [-y(t)] dt. \quad (7)$$

The graphical representation of these parameters is given by Fig. 4. The compensating effects of the feedforward compensator depend on the design parameter α , which are demonstrated in Theorem II as follows:

Theorem 1: Effects of adjusting α in the compensator (6) on the system with Type A undershoot are as follows:

In the case of undershoot:

1. For $\alpha = 0$,

$$t_{pc} = t_p, \quad (8)$$

$$Y_{pc} = Y_{pu}. \quad (9)$$

2. For $0 < \alpha < t_z$,

$$\max [\alpha, t_p] < t_{pc} < \min [t_z, t_p + \alpha], \quad (10)$$

$$\frac{1}{\alpha} \int_0^\alpha |y(t)| dt < |Y_{pc}| < |Y_{pu}|. \quad (11)$$

3. For $t_z \leq \alpha$,

$$t_{pc} = t_z. \quad (12)$$

$$|Y_{pc}| = \frac{1}{\alpha} \int_0^{t_z} [-y(t)] dt < |Y_{pu}|. \quad (13)$$

In the case of overshoot:

1. For $\alpha = 0$,

$$t_{mc} = t_m, \quad (14)$$

$$Y_{mc} = Y_{mo}. \quad (15)$$

2. For $0 < \alpha$,

$$t_m < t_{mc} < t_m + \alpha, \quad (16)$$

$$Y_{mc} < Y_{mo}. \quad (17)$$

Proof: The unit step response of the system $G(s)$ with the feedforward compensator is given as follows:

$$Y_c(s) = \frac{1}{\alpha} \frac{1}{s} (1 - e^{-\alpha s}) Y(s). \quad (18)$$

At first, if the design parameter $\alpha = 0$, it can be seen that $Y_c(s) = Y(s)$, and so Eqs. (8), (9), (14) and (15) are holding. The Y_{pc} , t_{pc} , Y_{mc} and t_{mc} can be derived from the derivative of the response $Y_c(s)$, which can be written by

$$sY_c(s) = \frac{1}{\alpha} (1 - e^{-\alpha s}) Y(s). \quad (19)$$

Eq. (19) gives the time-domain expression as follows:

$$\dot{y}_c(t) = \frac{1}{\alpha} [y(t)1(t) - y(t - \alpha)1(t - \alpha)], \quad (20)$$

where $1(t)$ is the unit step function as follows:

$$1(t) = \begin{cases} 0, & \text{for } t < 0, \\ 1, & \text{for } t \geq 0. \end{cases} \quad (21)$$

According to the time periods, it is also rewritten as follows:

$$\dot{y}_c(t) = \begin{cases} \frac{1}{\alpha} y(t), & \text{for } 0 \leq t < \alpha, \\ \frac{1}{\alpha} [y(t) - y(t - \alpha)], & \text{for } t \geq \alpha. \end{cases} \quad (22)$$

The peak undershoot time t_{pc} and the maximum overshoot time t_{mc} become the first and second time points that $\dot{y}_c(t) = 0$ in transient response to the step input, respectively. For $0 \leq t < \alpha$, it can be seen that the maximum overshoot time t_{mc} does not appear since there is no second time point such that $\dot{y}_c(t) = 0$. In the case of undershoot, if the design parameter α is smaller than t_z , the peak undershoot time t_{pc} does not appear since $\dot{y}_c(t)$ is not zero in this time period. And if the design parameter α is equal to or larger than t_z , the peak undershoot

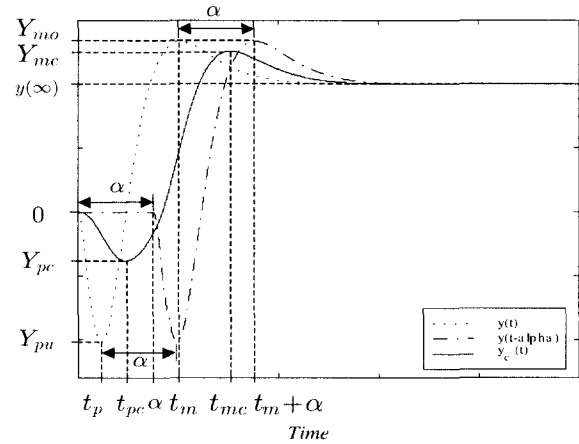


Fig. 5. Compensated system analysis in the case of $\alpha > t_z$.

time t_{pc} always becomes t_z since $\dot{y}_c(t) = 0$ at time $t = t_z$.

For $t \geq \alpha$, it can be also seen that if $\alpha < t_z$, then t_{pc} and t_{mc} are given as the first and second time points such that $y(t) = y(t - \alpha)$, respectively, and that if $\alpha \geq t_z$, then t_{pc} does not appear and t_{mc} is the time point such that $y(t) = y(t - \alpha)$. Hence the peak undershoot time t_{pc} always appears between t_p and t_z , and the maximum overshoot time t_{mc} is always between t_m and $t_m + \alpha$. Thus, the peak undershoot time t_{pc} can be written by Eqs. (10) and (12), and the maximum overshoot time t_{mc} can be also written by Eq. (16). It is shown that $|Y_{pc}|$ and Y_{mc} are always smaller than $|Y_{pu}|$ and Y_{mo} , respectively, since the output of feedforward compensator is smaller than step input. For $0 \leq t < \alpha$, the compensated step response $y_c(t)$ is given by $1/\alpha$ times of integral of $y(t)$. Using the peak undershoot time t_{pc} , Y_{pc} is also written by Eqs. (11) and (13), which completes the proof. ■

Theorem 1 can be used to analysis of the compensated system. For a given step response of the system with Type A undershoot, the step response of the system with feedforward compensator can be predicted using the Theorem 1. For instance, Fig. 5 shows in the case of $\alpha > t_z$ that Y_{pc} and Y_{mc} are smaller than Y_{pu} and Y_{mo} , respectively. Also, t_{pc} is equal to t_z since $\alpha > t_z$, and t_{mc} is the time point that $y(t) = y(t - \alpha)$.

In addition to Theorem 1, it is useful result that the area M has the bounds as follows:

Theorem 2: Let us define $y(t)$ as the step response of the system with Type A undershoot and at least one RHP real zero at $s = z_1$. Then the area of undershoot has the lower bound as follows:

$$\left[\int_{t_z}^{t_s} y(t) dt + \frac{y(\infty)}{z_1} \right] e^{-z_1 t_s} < M < \int_{t_z}^{t_s} y(t) dt + \frac{y(\infty)}{z_1} e^{-z_1 (t_s - t_z)} \quad (23)$$

with

$$M = \int_0^{t_z} [-y(t)] dt, \quad (24)$$

where $t_z (> 0)$ is a time point such that $y(t)$ crosses over the zero level, t_s is a minimum time point such that $y(t)$ has a zero steady-state error from $t = t_s$ to $t = +\infty$, and $y(\infty)$ is a steady-state value of the system.

Proof: It is clear that the Laplace transform of $y(t)$ has the open RHP as its region of convergence, and is explicitly written by

$$\frac{G(s)}{s} = \int_0^\infty e^{-st} y(t) dt, \quad (25)$$

where $G(s)$ is the system with one RHP real zero at $s = z_1$. The zero z_1 is obviously in the convergence region of Eq. (25). Consequently, after evaluation Eq. (25) at $s = z_1$ we have that

$$\begin{aligned} 0 &= \int_0^\infty e^{-z_1 t} y(t) dt \\ &= \int_0^{t_z} e^{-z_1 t} y(t) dt + \int_{t_z}^{t_s} e^{-z_1 t} y(t) dt \\ &\quad + \int_{t_s}^\infty e^{-z_1 t} y(t) dt \quad (26) \\ &= \int_0^{t_z} e^{-z_1 t} y(t) dt + \int_{t_z}^{t_s} e^{-z_1 t} y(t) dt \\ &\quad + \frac{y(\infty)}{z_1} e^{-z_1 t_s}, \end{aligned}$$

since $y(t) = y(\infty)$ for $t_s \leq t$. Now from Eq. (26) it follows that

$$\begin{aligned} M &> \int_0^{t_z} e^{-z_1 t} [-y(t)] dt \\ &= \int_0^{t_s} e^{-z_1 t} y(t) dt + \frac{y(\infty)}{z_1} e^{-z_1 t_s} \quad (27) \\ &> \left[\int_{t_z}^{t_s} y(t) dt + \frac{y(\infty)}{z_1} \right] e^{-z_1 t_s}, \end{aligned}$$

since $e^{-z_1 t} \leq 1$ for $0 \leq t \leq t_z$ and $e^{-z_1 t_s} \leq e^{-z_1 t}$ for $t_z \leq t \leq t_s$. Furthermore, the upper bound of M can be shown by

$$\begin{aligned} e^{-z_1 t_z} M &< \int_0^{t_z} e^{-z_1 t} [-y(t)] dt \\ &= \int_{t_z}^{t_s} e^{-z_1 t} y(t) dt + \frac{y(\infty)}{z_1} e^{-z_1 t_s} \quad (28) \\ &< e^{-z_1 t_z} \int_{t_z}^{t_s} y(t) dt + \frac{y(\infty)}{z_1} e^{-z_1 t_s}, \end{aligned}$$

since $e^{-z_1 t_z} \leq e^{-z_1 t}$ for $0 \leq t \leq t_z$ and $e^{-z_1 t} \leq e^{-z_1 t_z}$ for $t_z \leq t \leq t_s$, which completes the proof. ■

It is noted that Theorem 2 is holding for compensated step response $y_c(t)$ since $C(s)G(s)/s = 0$ at $s = z_1$.

In the system with Type B undershoot, let Y_{rpu} , t_{rp} , Y_{pu} and t_p be the reverse peak undershoot, the reverse peak undershoot time, the peak undershoot and the peak undershoot time of the step response for the system without the compensator (6), respectively. And let Y_{rpc} , t_{rpc} , Y_{pc} and t_{pc} be the reverse peak undershoot, the reverse peak undershoot time, the peak undershoot and the peak overshoot time of the step response for the system with the compensator, respectively. Also, let us define t_{z_1} and t_{z_2} as a time point such that the step response firstly and secondly crosses over the zero level, respectively, and define M_1 and M_2 as the area of step response from 0 to t_{z_1} and from t_{z_1} to t_{z_2} , respectively, as follows:

$$M_1 = \int_0^{t_{z_1}} y(t) dt, \quad (29)$$

and

$$M_2 = \int_{t_{z_1}}^{t_{z_2}} [-y(t)] dt. \quad (30)$$

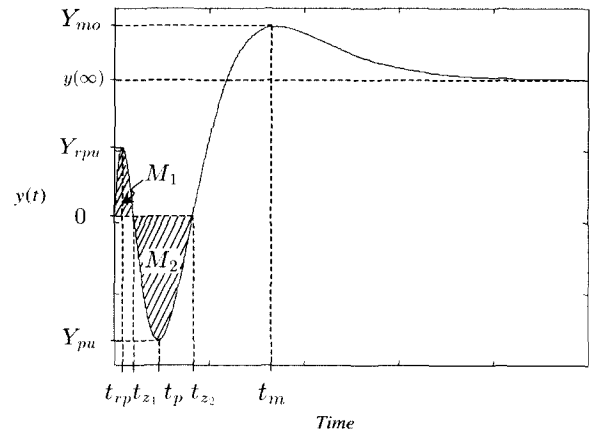


Fig. 6. Definition of parameters on system with Type B undershoot.

The graphical representation of these parameters is given by Fig. 6. Let us define t_i as the value of α such that $y(t) = y(t - \alpha)$ for $t_{z_2} < \alpha < t_m$. Then, similarly to the system with Type A undershoot, compensating effects of the feedforward compensator on the system with Type B undershoot by adjusting the design parameter α are given in Theorem 3 as follows:

Theorem 3: Effects of adjusting α in the compensator (6) on the system with Type B undershoot are as follows:

In the case of undershoot:

1. For $\alpha = 0$,

$$t_{rpc} = t_{rp}, \quad (31)$$

$$Y_{rpc} = Y_{rpu}, \quad (32)$$

$$t_{pc} = t_p, \quad (33)$$

$$Y_{pc} = Y_{pu}. \quad (34)$$

2. For $0 < \alpha < t_{z_1}$,

$$\max[\alpha, t_{rp}] < t_{rpc} < \min[t_{z_1}, t_{rp} + \alpha], \quad (35)$$

$$\frac{1}{\alpha} \int_0^\alpha y(t) dt < Y_{rpc} < Y_{rpu}, \quad (36)$$

$$\max[t_p, t_{rp} + \alpha] < t_{pc} < t_p + \alpha, \quad (37)$$

$$|Y_{pc}| < |Y_{pu}|. \quad (38)$$

3. For $t_{z_1} \leq \alpha < t_{z_2}$,

$$t_{rpc} = t_{z_1}, \quad (39)$$

$$Y_{rpc} = \frac{1}{\alpha} \int_0^{t_{z_1}} y(t) dt < Y_{rpu}, \quad (40)$$

$$\min[t_{z_1} + \alpha, t_{z_2}] < t_{pc} < \max[t_{z_1} + \alpha, t_{z_2}], \quad (41)$$

$$|Y_{pc}| < |Y_{pu}|. \quad (42)$$

4. For $t_{z_2} \leq \alpha < t_i$,
the t_{rpc} and Y_{rpc} are same as the case of III But t_{pc} and Y_{pc} are given by

$$t_{pc} = t_{z_2}, \quad (43)$$

$$|Y_{pc}| = \frac{1}{\alpha} \left| \int_0^{t_{z_2}} y(t) dt \right| < |Y_{pu}|, \quad (44)$$

or

$$t_{pc} = t_{e_2}, \quad (45)$$

$$|Y_{pu}| < |Y_{pc}| < \frac{1}{\alpha} \left| \int_0^{t_{z_2}} y(t) dt \right|, \quad (46)$$

where t_{e_2} is a second time point such that $y(t) = y(t - \alpha)$.

5. For $\alpha = t_i$,
it is the same as the case of II except there exists inflection point at time t_i .

6. For $t_i < \alpha$,

$$t_{rpc} = t_{z_1}, \quad (47)$$

$$Y_{rpc} = \frac{1}{\alpha} \int_0^{t_{z_1}} y(t) dt < Y_{rpc}, \quad (48)$$

$$t_{pc} = t_{z_2}, \quad (49)$$

$$|Y_{pc}| = \frac{1}{\alpha} \left| \int_0^{t_{z_2}} y(t) dt \right| < |Y_{pu}|. \quad (50)$$

In the case of overshoot:

This is the same as the case of overshoot in Theorem 1.

Proof: The proof of this theorem is similar to that of Theorem 1. But in the system with Type B undershoot, $y_c(t)$ can have two extrema except two undershoots and overshoot as the case of 4. Also, as the case of 5, $y_c(t)$ may have one inflection point according to set of $\alpha = t_i$. These phenomena arise from selected α for nearby value between t_{z_2} and t_i , and so there exist two or one time points such that $y(t) = y(t - \alpha)$ for $t_{z_2} < \alpha < t_m$. ■

This theorem can also be used to analysis of the compensated system with Type B undershoot as the Theorem 1. From Eq. (50), it seems that we can make Y_{pc} for positive value if $M_1 > M_2$, i.e., it seems that undershoot phenomena will not appear in this case, and so $y_c(t)$ is nonnegative. But for the system with Type B undershoot and at least one RHP real zero, M_2 is always larger than M_1 .

Theorem 4: Let us define $y(t)$ as the step response of the system with Type B undershoot and at least one RHP real zero. Then the areas of the undershoot have the following relation:

$$\int_0^{t_{z_1}} y(t) dt < \int_{t_{z_1}}^{t_{z_2}} [-y(t)] dt, \quad (51)$$

where t_{z_1} and t_{z_2} is a time point such that $y(t)$ firstly and secondly crosses over the zero level, respectively.

Proof: Let z_1 be one of the RHP real zero of the system. Similarly to Eq. (26), we can get the following relation:

$$\begin{aligned} 0 &= \int_0^{\infty} e^{-z_1 t} y(t) dt \\ &= \int_0^{t_{z_2}} e^{-z_1 t} y(t) dt + \int_{t_{z_2}}^{\infty} e^{-z_1 t} y(t) dt, \end{aligned} \quad (52)$$

and so it is satisfied the inequality as follows:

$$\int_0^{t_{z_2}} e^{-z_1 t} y(t) dt < 0. \quad (53)$$

Also we can obtain the lower bound for the areas of the undershoot as follows:

$$e^{-z_1 t_{z_1}} \int_0^{t_{z_1}} y(t) dt < \int_0^{t_{z_1}} e^{-z_1 t} y(t) dt \quad (54)$$

and

$$e^{-z_1 t_{z_1}} \int_{t_{z_1}}^{t_{z_2}} y(t) dt < \int_{t_{z_1}}^{t_{z_2}} e^{-z_1 t} y(t) dt, \quad (55)$$

since $e^{-z_1 t_{z_1}} \leq e^{-z_1 t}$, $y(t) \geq 0$ for $0 \leq t \leq t_{z_1}$, and $e^{-z_1 t_{z_1}} \geq e^{-z_1 t}$, $y(t) \leq 0$ for $t_{z_1} \leq t \leq t_{z_2}$, respectively. Hence the summation of Eqs. (54) and (55) is written by

$$e^{-z_1 t_{z_1}} \left[\int_0^{t_{z_1}} y(t) dt + \int_{t_{z_1}}^{t_{z_2}} y(t) dt \right] < \int_0^{t_{z_2}} e^{-z_1 t} y(t) dt, \quad (56)$$

and so, the result follows from Eq. (53) since $e^{-z_1 t_{z_1}} > 0$, which completes the proof. ■

From Theorem 1 and 3, it can seen that the peak undershoot Y_{pc} and the peak undershoot time t_{pc} of the compensated system are determined by M (or M_1 and M_2), t_z (or t_{z_1} and t_{z_2}) and α . To make it short, if the design parameter α is too large, then peak undershoot and maximum overshoot are very small at the price of rise time and *vice versa*. Thus, from the advanced information about the step response of the system without compensator, we can properly select the value of α .

IV. Simulation

In this chapter, the systems with Type A undershoot and Type B undershoot are taken for computer simulations to exemplify the results proposed.

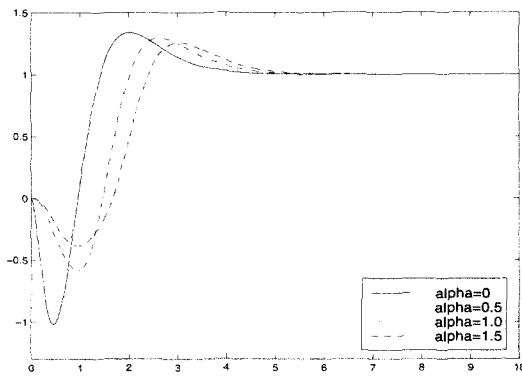
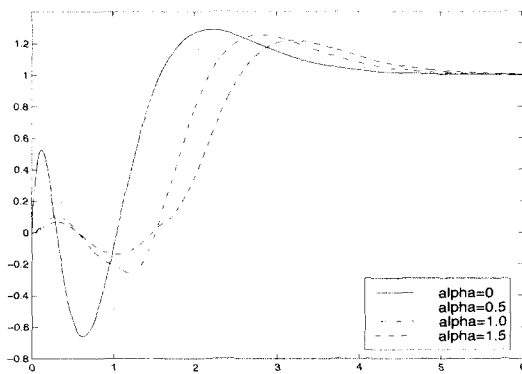
1. System with type A undershoot

Let us take the system $G_1(s)$ with Type A undershoot for numerical example as follows:

$$G_1(s) = \frac{K_1(s+0.8)(s-1)}{(s+2)(s+2.5+j)(s+2.5-j)(s+3)}, \quad (57)$$

where $K_1 = -54.375$ for the unity DC gain. It is noted that $G_1(s)$ has large Type A undershoot and overshoot because of one real RHP zero and LHP(left half plane) zero located between dominant pole and imaginary axis, respectively. The step response of the $G_1(s)$ is shown by the solid curve on Fig. 7. It has the following specifications: The peak undershoot is $Y_{pu} \simeq -1.01$ at the peak undershoot time $t_p \simeq 0.46$ sec, the maximum overshoot is $Y_{mo} \simeq 1.33$ at the maximum overshoot time $t_m \simeq 2.03$ sec, and zero crossing time $t_z \simeq 0.96$ sec.

Using the feedforward compensator (6), Theorem 1 can be applied to analyze effects for the changing α . The compensated

Fig. 7. Effects of α in the feedforward compensator for $G_1(s)$.Fig. 8. Effects of α in the feedforward compensator for $G_2(s)$.

step response according to the value of α is given by Fig. 7. It is shown that the amount of undershoot and overshoot is decreased as the value of α increases. It is noted that the peak undershoot time t_{pc} is not larger than t_z even if α is very large. Let us investigate when $\alpha = 1.5$. In this case, the step response of the $G_1(s)$ with compensator (6) is shown by the dashed curve on Fig. 7. It has the peak undershoot $Y_{pc} \approx -0.38$ at the peak undershoot time $t_{pc} = t_z \approx 0.96$ sec, and has the maximum overshoot $Y_{mc} \approx 1.125$ at the maximum overshoot time $t_{mc} \approx 3.04$ sec. The t_{mc} is the same as the time that $y(t) = y(t + 1.5)$ for $t > t_z \approx 0.96$ sec. The absolute magnitude of the peak undershoot and maximum overshoot is reduced about 62.4% and 15.4%, respectively, which exemplifies the effect of the compensator proposed.

2. System with type B undershoot

Let us consider the system $G_2(s)$ with Type B undershoot as follows:

$$G_2(s) = \frac{K_2(s + 2 + j)(s + 2 - j)(s - 1)}{(s + 2)(s + 2.5 + j)(s + 2.5 - j)(s + 3)}, \quad (58)$$

where $K_2 = 10.875$ for the unity DC gain. This system has Type B undershoot and overshoot because of two (complex) RHP zero and LHP zero located between dominant pole and imaginary axis, respectively. The step response of the $G_2(s)$ without compensator is shown by the solid curve on Fig. 8, which has the following specifications: The reverse peak undershoot $Y_{rpu} \approx 0.53$ at the reverse peak undershoot time $t_{rpu} \approx 0.12$ sec, the peak undershoot $Y_{pu} \approx -0.66$ at the peak undershoot time $t_p \approx 0.62$ sec, the maximum overshoot $Y_{mo} \approx 1.29$ at the maximum overshoot time $t_m \approx 2.17$ sec,

and zero crossing times $t_{z1} \approx 0.30$ sec and $t_{z2} \approx 1.04$ sec. In this system, the inflection point is appeared when $t_i = \alpha \approx 1.18$ sec, and so two extra extrema points is occurred for $t_{z2} \approx 1.04$ sec $< \alpha < t_i \approx 1.18$ sec.

Theorem 3 can be applied to analyze effects for changing α to use the feedforward compensator (6). The compensated step response according to α is given by Fig. 8. For example, if $\alpha = 1.5$, it has the reverse peak undershoot $Y_{rpu} \approx 0.07$ at the reverse peak undershoot time $t_{rpu} = t_{z1} \approx 0.30$ sec, the peak undershoot $Y_{pc} \approx -0.14$ at the peak undershoot time $t_{pc} = t_{z2} \approx 1.04$ sec, the maximum overshoot $Y_{mc} \approx 1.22$ at the maximum overshoot time $t_{mc} \approx 3.18$ sec. The absolute magnitude of the reverse peak undershoot, peak undershoot and maximum overshoot is reduced about 86.79%, 78.79% and 5.43%, respectively, compare to those of step response for the system without compensator.

V. Conclusions

In this paper, a simple feedforward compensator is proposed. Provided that the nonminimum phase system is properly stable, Theorem 1 and 3 have shown compensation effects according to changing of the design parameter α in the feedforward compensator (6). And it is shown that the step response of the compensated system is improved in the peak undershoot and maximum overshoot at the price of rise time. And, the usefulness of the feedforward compensator is shown via some numerical examples applied to two system which has Type A and Type B undershoot, respectively. It is noted that the feedforward compensator proposed has more effects on the undershoot than that on the overshoot. Although the amount of undershoots can be lessened by the proposed compensator, it is impossible to eliminate the undershoot phenomena, which still remains an open problem.

References

- [1] A. Bemporad, A. Casavola, and E. Mosca, "Nonlinear control of constrained linear systems via predictive reference management," *IEEE Trans. on Automat. Cont.*, vol. 42, no. 3, pp. 340–349, Mar., 1997.
- [2] K. S. Byun and J. B. Song, "Variable reference model for model reference control subject to bounded control signals," *J. of Contr., Automat. and Sys.*, vol. 6, no. 3, pp. 241–247, Mar., 2000.
- [3] O. Dahl, "Reference trajectory generation for single-loop controllers," *Contr. Engr. Practice*, vol. 5, no. 7, pp. 897–905, 1997.
- [4] W. Singhose, B. Mills, and W. Seering, "Vibration reduction with specified-swing input shapers," *IEEE Int. Conf., on Contr. Applications*, pp. 533–538, 1999.
- [5] W. Singhose and K. Grosser, "Limiting excitation of unmodeled high modes with negative input shapers," *IEEE Int. Conf., on Contr. Applications*, pp. 545–550, 1999.
- [6] S. Hara, N. Kobayashi, and T. Nakamizo, "Design of non-undershooting multivariable servo systems using dynamic compensators," *Int. J. of Control*, vol. 44, no. 2, pp. 331–342, 1986.
- [7] T. Mita and H. Yoshida, "Undershooting phenomenon and its control in linear multivariable servomechanisms," *IEEE Trans. on Automat. Contr.*, vol. 26, no. 2, pp. 402–407, 1981.

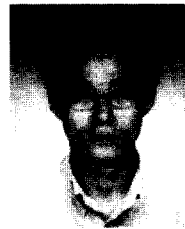
- [8] M. Vidyasagar, "On undershoot and nonminimum phase zeros," *IEEE Trans. on Automat. Contr.*, vol. 31, no. 5, p. 440, May, 1986.
- [9] B. A. León de la Barra, "On undershoot in SISO systems," *IEEE Trans. on Automat. Contr.*, vol. 39, no. 3, pp. 578-581, Mar., 1994.
- [10] B. M. Kwon, H. S. Ryu, and O. K. Kwon, "Transient response analysis and compensation of the second order system," *ICASE, Trans. on Contr., Automat. and Sys. Engr.*, vol. 2, no. 4, pp. 262-267, 2000.
- [11] R. H. Middleton, "Trade-offs in linear control system design," *Automatica*, vol. 27, no. 2, pp. 281-292, 1991.



Kwon, Byung-Moon

He was born in Pyongtaek, Korea, in 1976. He received B.S. degree in Electrical Engineering from Inha University in 1998. He is currently a Ph. D. candidate in the School of Electrical and Computer Engineering, Inha University. His main research

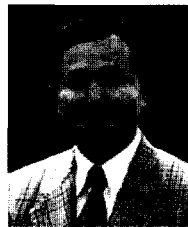
interests are in the areas of robust control, optimal control and nonminimum phase systems.



Ryu, Hee-Seob

He was born in Seoul, Korea, in 1971. He received B.S., M.S. and Ph.D. degree in Electrical Engineering from Inha University in 1994, 1996 and 2001, respectively. His main research interests are in the areas of robust estimation, control, and industrial

applications.



Kwon, Oh-Kyu

He was born in Seoul, Korea, in 1952. He received B.S. and M.S. degree in Electrical Engineering and his Ph.D. degree in Control and Instrumentation Engineering from Seoul National University in 1978, 1980 and 1985, respectively. He was at Dept. of

Electrical Engineering, Inha University, as Assistant Professor in 1984. He was also at Dept. of Electrical and Computer Engineering, the University of Newcastle, Australia as a Visiting Professor, during the academic year from Aug. 1988 to Aug. 1989. He is now a Professor at School of Electrical and Computer Engineering, Inha University. His main research interests are in the areas of robust estimation, fault detection and diagnosis, system identification and digital control applications.