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Mathematical Knowledge Construction in Computer Based Learning

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Using computer technology in teaching school mathematics creates new instructional environments. The emphases on the use of computer technology in the classrooms and in particular the use of computer-based exploration as a context of mathematics instruction have been reflected in the recommendation of the NCTM (*Curriculum and Evaluation Standards for School Mathematics*, 1989). Although the power of using computer technology in the exploration of mathematical problems has been recognized and stressed by many educators, we do not have many research studies on mathematics in computer-based explorations. Especially research has failed to clarify how computer technology can contribute to the construction of procedural and conceptual knowledge of mathematics.

Up to now most researches on procedural and conceptual knowledge in computer environments have only focused on classifying programming languages which program language has more random access and rich interrelationship characteristic in relation to conceptual knowledge in humans, and which computer language has more characteristic flavor of procedural knowledge. How computer-based explorations affect the knowledge construction of mathematics, therefore, emerges as an issue of research on teacher education program for theoretical framework. This situation leads to do research on the effectiveness of using computer explorations in pre-service teacher education in terms of procedural and conceptual knowledge construction.

PURPOSE OF THE STUDY

One of the major issues of mathematics instruction is students' exploration in computer-based environments. Recently many popular calls for instructional purpose are not much based on research evidence concerning the potential effects of recommended changes. The purpose of this study was to examine several aspects of pre-service teacher's knowledge construction in computer-based environments by investigating;

1) procedural and conceptual knowledge on mathematics problems pre-service teachers hold,
2) how pre-service teachers obtain procedural and conceptual knowledge of mathematics through computer-based explorations, and
3) how pre-service teachers connect procedural and conceptual knowledge of mathematics during computer-based explorations.

This study focused on the investigation of pre-service teachers' knowledge on mathematics problem solving through computer explorations.

RESEARCH QUESTIONS

The research questions guiding the investigation were:

1. What procedural and conceptual knowledge of mathematics problems do pre-service teachers have?
2. What procedural and conceptual knowledge of mathematics problems are constructed through computer exploration?
3. What connections do pre-service teachers make between their procedural and conceptual knowledge of mathematics problems through computer explorations? What conditions influence pre-service teachers' connection of procedural and conceptual knowledge of mathematics problems during computer explorations?

METHODOLOGY

Three cases were studied to investigate a grounded theory of the knowledge construction of pre-service teachers through computer explorations.

An observation of classroom activity method with clinical interview (Ginsburg, 1981; Opper, 1977) was used to gain insight into the subject's knowledge of mathematics and knowledge construction through computer explorations. Researcher introspection was used to identify the sequence of mental processes as the subjects performed computer explorations. Typological strategy was used to analyze the data. The analysis of data consisted of category-based grouping for investigating how subjects constructed their mathematical knowledge. The category-based grouping was identified on the bases of Hiebert-Lefevre's definition and description on procedural and conceptual knowledge (Hiebert & Lefevre, 1986).

An EMT 486/668 classroom at the University of Georgia was selected for this study.

The class was designed for mathematical investigation using computers for pre-service teachers. The class emphasized computer exploration of various computer software applications. The class was scheduled for two hours from 11:00 a.m. to 1:00 p.m. The classroom has 16 computers, a scanner, a laser printer, and an overhead projector with a screen.

The lesson was mainly focused on how the students used software to introduce mathematical concepts, and to explore mathematical problems by using computer software such as Algebra Xpresser, Geometer's Sketchpad, Excel, Mathematics Teacher Workstation, Theorist, and xFunction.

SUBJECTS

Three pre-service teachers[1] participated in the study by random-pick-up method with volunteer basis. However, Charles, one of the participants, withdrew his willingness to be involved this study during the first exploration due to his personal reasons. The data of this study mainly based on the two subjects who completed all three explorations. All subjects took MAT 254 course that was a prerequisite for EMT 486/668 course at the University of Georgia. All subjects had more than average background in mathematics, and they showed positive attitude with enthusiasm toward this study.

PROCEDURE

This study observed three explorations among hand-in-write-ups and final projects for each subject. All participants' classroom activities and computer explorations were observed and video taped. If it was possible, an open-ended individual interview was performed before, during, and after each exploration. Open-ended clinical interviews along with thinking about procedures were used to observe and collect data on participants' knowledge process during the computer explorations. Finally an overall interview was held at the end of the explorations.

DATA COLLECTION AND ANALYSIS

The sources of data collection were observation, interviewing, field notes, students' written assignments, computer works, and audio and video tapes of students' computer

[1] Nicolson, Charles, and Raychel

explorations. The main sources of data were observation, interview and subjects' written assignments. If it was not enough to collect information of subjects' mental process by observation and interview, this study used subjects' written assignments and written projects for data collection.

Throughout the period of the research, data collection and analysis were processed together. Typological strategy was used for data analysis. Hiebert-Lefevre's definition and description of procedural and conceptual knowledge (cf. Hiebert & Lefevre, 1986) were used to develop guidelines to analyze the data. For the supplement of the guidelines, analytic induction and constant comparison developed enumerative coding categories.

Guidelines for procedural knowledge

P1) Used/showed written symbols.
P2) Used rules, formula, algorithms.
P3) Carried out linear sequences.
P4) Had knowledge of skills.
P5) Used syntactic configurations.
P6) Aware of exterior features.
P7) Used step-by-step method.
P8) Knowledge by rote learning.

Guidelines for conceptual knowledge

C1) Used facts, properties, relations.
C2) Had proposition.
C3) Connected web of knowledge.
C4) Made relationships between pieces of information.
C5) Learned meaningfully.
C6) Rote learning did not generate knowledge.

Guidelines for linkage between procedural and conceptual knowledge

L1) Used standard process, but varied or extended.
L2) Generalized processes.
L3) Used non-routine process.
L4) Could explain their explorations with correct answers.
L5) Reconstruct a new process.
L6) Created, extended, and generalized their solutions.

Nicolson's exploration problems

1) Explore $x^n + y^n = 1$ (for different n).
2) Record temperature of a cup of hot water for 30 minutes. Enter the data on a spreadsheet and construct a function that will model the data.
3) Explore $XY = aX + bY + c$ (for different a, b and c).

Charle's exploration problems

1) Find two linear functions $f(x)$ and $g(x)$ such that their product $h(x) = f(x)g(x)$ is tangent to each of $f(x)$ and $g(x)$ at two distinct points.

Raychel's exploration problems

1) Examine graphs of $Y = a \sin (bX + c)$ (for different a, b, and c)
2) The same as Nicolson's third exploration
3) Given a rectangular cardboard of dimension15 by 25, if a small square of the same size is cut from each corner and each side folded up along the cuts to form a lidless box, what is the maximum volume of the box? (Use Algebra Xpresser, GSP, and Excel.)

SUMMARY OF FINDINGS

Case of Nicolson[2]

At the beginning of the first exploration, Nicolson already had a previous procedural and conceptual knowledge of the graph $X^2+Y^2= 1$. But he did not have procedural and conceptual knowledge for the graph of $X^n + Y^n = 1$ for every n. Through a step-by-step method by carrying out linear sequences as n increases. And visual information from computer screens. He becomes aware of exterior features of the graphs $X^n + Y^n = 1$. That representing some concepts and found a pattern to predict a general case for n. But he still did not show enough evidence of exhibition of conceptual knowledge.

After exploring odd and even numbers for n separately, he could generalize his process and extended his generalization. Eventually during the computer exploration, he showed procedural and conceptual knowledge and a connection between two types of knowledge. When procedural and conceptual knowledge were constructed, the two types of knowledge were linked together. Because of his lack of previous knowledge of Algebra, he could not fully connect his exploration results to new mathematical knowledge.

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[2] See Footnote 1.

Restricted time in the computer explorations was a critical factor to constructing procedure and conceptual knowledge and their connection in computer exploration for mathematics. Computer exploration for mathematics could give him a wide view of mathematical knowledge and could help him discover open-ended mathematical fields. All three factors positively influenced on the linkage of procedural and conceptual knowledge. At the beginning of the second exploration Nicolson did not have any experience on making a model by using data, and he did not show any procedural knowledge for the question. Group discussion obviously helped his procedural knowledge construction.

He constructed a model function $y = A*(\exp(-b*C)) + D$. During exploring parameter A and B he tried to connect his previous knowledge on parameter A with parameter B. but he still could not grasp the whole concept of the problem. Even though he somewhat had procedural and conceptual knowledge, he could not strongly connect the two types of knowledge. After finishing the exploration of parameter D, he connected knowledge of each parameter and applied the knowledge to make a model function. He achieved the construction of relationships between pieces of information and he could exploration. His knowledge construction developed from procedural knowledge to conceptual knowledge and the connected two types of knowledge. His computer exploration provided plenty of sub-information about the role of parameters in graphing functions. The sub-information affected positively on the link of procedural and conceptual knowledge. He constructed a kind of procedural and conceptual knowledge and linked the two types of knowledge. But he could not extend or create new solutions. His restricted solution can be explained by his lack of previous knowledge on functions and researcher' quick-hints at the beginning of the computer exploration.

At the beginning of third exploration, he did not show any knowledge on the graph of the equation $xy = ax + by + c$. He just got basic procedural knowledge without conceptual knowledge on the problem. He explored the equation $xy = ax + by + c$ with various substitution of coefficients a, b, and c with real numbers. His conception on this mathematical syntax such as, intercept, led to a correct answer resulting from a strong conceptual knowledge construction. Even though he did not try to verify the asymptotes by algebraic proof, he could find the properties of asymptotes only by the visual verification from the computer exploration. His previous knowledge on basic mathematical concepts verified by visual information induced strong procedural and conceptual knowledge construction.

In the middle of the exploration of the graph of $xy = 3x + 3y + c$ for $c = 0, 1, 2$, he could not explain the intercepts 0, – 1/3, and – 2.3 for $c = 0$, 1, and 2 respectively. During the exploration he had procedural and conceptual knowledge. He could not fully link them together. After exploring $xy = 3x + 2y + c$ for $c = 0, 1, 2, 3$, he could confirm his discovery by using algebraic calculation. His analysis and conclusion on the asymptotes came from algebraic proof and visual information through computer exploration. Multiple approaches helped his mathematical understanding on the asymptotes by linking procedural and conceptual knowledge through computer exploration. For the exploration of $x^2y^2 = ax^2 + by^2 + c$ for $c = 0, 1, 2,$ he remarkably generalized the properties of the asymptotes such that if $a = 5$ and $b = 7$, the asymptotes would occur at $x = \sqrt{7}$ and $y = \sqrt{5}$. He linked procedural and conceptual knowledge on the asymptotes. But he could not explore various characteristics of the graph such as the differences between negative and positive value of the parameter, the role of constant, etc. He could not find any specific relationship between the intercepts and the values of *a, b,* and *c.* He reported on the occurrence of the intercepts for the case of $c < 0$.

He used propositions for shifting curves that were stored in his memory. He had strong conceptual knowledge on shifting functions. He finished his exploration with a satisfactory result. He exhibited procedural and conceptual knowledge, and he could link them appropriately.

Case of Charles[3]

In the beginning of the first exploration, he showed that he had the meaning of tangent. And he had weak procedural and conceptual knowledge on the problem. During computer exploration, his trial and error method and group work helped his procedural and conceptual knowledge construction. After the first exploration Charles withdrew his willingness to participate in this study for personal reasons.

Case of Raychel[4]

She already had basic procedural and conceptual knowledge about a simple sine function. But she did not have extended knowledge about a general sine function $y = a \sin(bx + c)$. After she explored the function with changing parameters *a, b*, and *c*, she realized the value of *a* affects amplitude. She also achieved the construction of relationship between different values of *a*. She constructed procedural and conceptual knowledge by systematic way in which was achieved a linkage between two types of knowledge on parameter *a*.

She successfully found out period of the sine function changing the *b* parameter. She had knowledge of skills needed to carry out the exploration and she connected it to *a* web of knowledge about the period. She could not, however, yet fully extend her solution. Somehow she linked procedural and conceptual knowledge on parameter and period of

[3] See Footnote 1.
[4] See Footnote 1.

sine functions. At each exploration she predicted, explained, and represented the mathematical concepts for sine functions in terms of the value *c*. But she could not generalize shifting of sine functions for all cases.

She was mainly dependent on visual information for measuring a numerical position of a graph. Her lack of prerequisite knowledge on graphing was a factor for hindering a perfect knowledge construction. She already had procedural and conceptual knowledge about shifting graphs. She just tested and verified her conjecture for shifting by using computer exploration. The process of verification by using computer exploration helped her build connection of her previous knowledge on shifting a general function to new knowledge of shifting sine functions.

She did not notice that parameter *b* influences shifting magnitude of the graph. So she could not construct strong linkage of procedural and conceptual knowledge on shifting. Various attempts with different variables were important to view the whole picture of the sine function. She strongly connected procedural and conceptual knowledge through the computer exploration.

For the second exploration she also, very similar to Nicolson, understood what the problem meant, but did not have strong procedural and conceptual knowledge on the problem. She used step-by-step and structured method to collect a set of graphs and compared the shape of the graphs to fine the pattern of *d*. Her knowledge construction obviously came from visual information through computer exploration.

She showed knowledge of properties and relations in graphing $xy = ax + by + c$ in terms of *b* related to other existing knowledge that she had in terms of the parameter a. She achieved a construction of relationships between parameters *a* and *bs*. She could compare a family of curves with appropriately changing parameters. These procedures could help her understanding of what features of the graph were controlled by parameters. She could extend and generalize how parameters *a* and *b* control the function $xy = ax + by + c$. She showed strong linkage between procedural and conceptual knowledge on how the graph was controlled by parameters *a* and *b* through visual information from the computer exploration.

She was aware of exterior features of a set of curves with two different values of *a* and *b* respectively, and got knowledge on meaning of the dependence of parameters *a* and *b*. With carrying out linear sequences of the exploration for different parameters *a*, *b*, and *c*, she acquired meaningful relationships between each parameter and connected previously discovered knowledge to the new knowledge. She showed strong linkage of procedural and conceptual knowledge acquired from the computer exploration.

For the third exploration, she already had background knowledge about the problems. Through computer exploration with Excel and with a joint exploration on the TI-82 calculator, she reinforced a construction of procedural and conceptual knowledge. She used TI-82 calculator to help her exploration. The use of TI-82 graphing calculator obviously helped her construct knowledge that could not be generated by rote learning. It was a kind of triangulation approach to a problem. Even though she used non-standard processes, the processes had been varied and extended to fit this problem.

With the cubic equation $y = 4x^3 - 80x^2 + 375x$ already found, she easily carried out the exploration with Algebra Xpresser. Through the Algebra Xpresser she observed the maximum value of somewhere around 500 $inch^3$ occurs when the length of the cut-out is 3.036 *inch*. In this exploration she mainly depended on visual information using trace function. She showed the maximum value theorem that was stored in her memory, and connected the proposition to the knowledge that was discovered during the computer exploration. Her procedural knowledge was quite dependent on the knowledge of how to use the computer software. But her mathematical conceptual knowledge was independent on the knowledge of computer software. Her conceptual knowledge on this problem relied on the visual information that was one of the special features computer technology could provide.

To find a maximum volume of a box using GSP, she used basic geometry construction that she already had and followed step-by-step method. She did not use a set of formula or algorithm but create an animation drawing to generalize her solution. This was non-routine process but was as efficient as standard process. Through three different approaches she created, extended, and generalized her solution. She showed positive attitude to the problem, and felt comfortable in using computer technology for mathematical exploration. All of these factors were very helpful for her construct strong procedural and conceptual knowledge and their linkage in computer exploration. If students had some previous procedural and conceptual knowledge in non-computer environments, it was very helpful for students construct strong procedural and conceptual knowledge and their linkage in computer exploration.

Other Findings

Positive factors for doing computer explorations

- Setting up strategies
- Viable visual information
- Strong background knowledge
- Group discussion
- Multiple approaches
- Positive attitude for using computer

Negative factors for doing computer explorations

· Restricted time for computer explorations
· Weak basic mathematical knowledge
· Quick hint
· Misconception

DISCUSSION

For a pre-service teacher who has not enough graphing and trigonometry experience, computer technology can show how certain parameters control different aspects of the graphing, stretching, shrinking, shifting, and amplitudes. Much like the inspection Raychel did, pre-service teachers can explore the tendencies of the parameters. Through exploration, pre-service teachers could be guided to the desired goal. By the computer exploration, pre-service teachers will have a concrete foundation of the concept of parameters since they explored the aspects of them for themselves.

For pre-service teachers who have been exposed to trigonometry, technology can serve several purposes including a review of existing concepts, and a facilitator of expansions of the previously learned concepts. Computer technology speeds up the routine of plotting points and allows students to explore more complex situations using basic concept that connected conceptual knowledge.

Using computer to explore functions is a good opportunity for pre-service teachers because it allows them to study the parts of behavior of a function and it's relationship with parameters. Also, they can easily explore how the composition of two linear functions or many is predicted. There are many different possible approaches in computer exploration that students can work in groups to see how many solutions they can come up with. Group cooperation has plenty of advantage in computer-based exploration. At the end of class each group could explain what they have found and discuss what they have produced as results. In explaining and listening to fellow classmates, students will learn more. Working with computer environments, students will get a better feel for what is happening when they factor a quadratic expression.

Like Charles' first exploration, a discussion to get students' thinking of how to go about solving the problem could start with what types of combination of linear functions is necessary to get a parabola tangent to both at the same time. This would involve a discussion of slope since one function needs to have a positive slope and the other needs to have a negative slope. With this being the case then the discussion could lead to what direction the parabola would have. By using this type of discussion students would be able to make some connections between linear and quadratic functions as well as what controls the slope and direction of the functions. Using Algebra Xpresser makes this type of exploration possible because students would be able to experiment without being slowed down by having to graph all of the different functions. Also, once they found one that looked like a solution then they would need to prove it.

In computer exploration students learn mathematics by using concepts that aid them in understanding and directing the visual information. The fundamental basis for this claim is that appropriate computer environments can help students elaborate on, and become cognizant of, mathematics implicit in certain kinds of intuitive thinking. When "intuition is translated into a program it becomes more obstructive and more accessible to reflection" (Papert, 1980, p.145). Verbal information supplemented by visual helps adults learn new concepts better than verbal information alone (Cantu & Herron, 1978; Dwayer, 1982; Holliday, 1975; Rigney & Lutz, 1976). This research result also supports their arguments. Obviously computer exploration tremendously helped students connect procedural and conceptual knowledge that are indication of mathematical understanding.

Krutetskii (1976) mentioned that a solution to a mathematical problem could be obtained in several ways, adding that one cannot discover much about mathematical thinking by analyzing test results. Too much emphasis in the classroom on the results instead of on the mathematical thinking that generated the results through computer explorations will give students a false conception of mathematics. This argument illustrates the importance of the mathematical thinking that can be generated by computer exploration. Computer exploration can help students learn to think mathematically. The exploration focused on mathematical thinking can "develop a mathematical point of view valuing the processes of mathematization, abstraction which as the predilection to apply them. They can develop competence with the tools of the trade in which using those tools in the service of good of understanding structure-mathematical sense making (Schoenfeld, 1992)."

In solving mathematical problems in which the basic facts, propositions, or relations of the different attribute functions, sequences, finding roots of a function is a part of conceptual knowledge only if these basic facts, propositions, and relations are related to some other existing knowledge. This knowledge construction can be promoted by computer explorations. In conclusion, computer exploration for mathematics is a powerful tool to assist pre-service teachers' mathematical understanding by constructing and linking procedural and conceptual knowledge. Thus, we can say that computer exploration for mathematics can enhance mathematical competence.

REFERENCES

Cantu, L. & Herron, J. (1978): Concrete and formal Piagetian stages and science concept attainment. *Journal of Research in Science Teaching* **15**, 135–143.

Dwayer, F. M. (1982): The program of systematic evaluation a brief review. *International Journal of Instructional Media* **10**, 23–38.

Ginsburg, H. E. (1981): The clinical interview in psychological research on mathematical thinking: Aims, rationales, techniques. *Learn. Math.* **1(3)**, 4–11. MATHDI **1981l.** 01565

Hiebert, J. & Lefevre, P. (1986): Conceptual and procedural knowledge in mathematics: An introductory analysis. In: J. Hiebert (Ed.), *Conceptual and procedural knowledge: the case of mathematics* (pp. 1–27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Holliday, W. G. (1975): The effects of verbal and adjunet pictorial verbal information in science instruction. *Journal of Research in Science Teaching* **12**, 77–83.

Krutetskii, V. A. (1976): *The psychology of mathematical abilities in school children.* Chicago: University of Chicago Press.

National Council of Teachers of Mathematics (1989): *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Opper, S. (1977): Piaget's clinical method. *Journal of Children's Mathematical Behavior* **5**, 90–107.

Papert, S. (1980): *Mind storms: Children, Computers, and powerful ideas.* Harvester Studies in Cognitive Science, 14. Brighton: Harvester Press. MATHDI **1982j.** 09086

Rigney, J. W. & Lutz, K. (1976): Effect of graphic analogies of concepts in chemistry on learning and attitude. *Journal of Educational Psychology* **68**, 305–311.

Schoenfeld, A. H. (1992): Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In: D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370), A project of the National Council of Teachers of Mathematics. New York, NY: Macmillan. MATHDI **1992f.** 01809