전달행렬법에 의한 변위를 허용하는 문형라멘의 영향선해석

Influence Lines of a Portal Frame with Joint Translations by Transfer Matrix Method

남 문 희* 하 대 환**
Nam, Moon-Hee Ha, Dae-Hwan

(논문접수일 : 2000년 2월 24일 : 심사종료일 : 2000년 1월 5일)

요 지

구조물설계에 있어서 영향선은 최대반력, 최대전단력, 최대휨모멘트 등을 계산하는데 아주 유용하게 사용된다. 모멘트분배법, 인도행렬법. 전달행렬법, 그리고 Müller-Breslau원리에 의한 단순보와 연속보의 영향선은 잘 알려져 있고 또 교량공학에서 널리 사용되고 있다. 그러나 변위를 허용하는 특별한 구조물의 영향선을 계산할 경우에는 약간의 어려움이 있다. 이 연구에서는 절점변위를 허용하는 문형라멘의 영향선을 전달행렬법에 의하여 구하고 유한요소법에 의하여 얻은 영향선과 비교하였고 그 결과는 좋은 일치를 보이고 있다.

핵심용어 : 전달행렬, 라멘의 영향선, 절점이동, 절점평형식

Abstract

The influence line is a helpful way to calculate the maximum reaction, shearing force and bending moment in the design of a structure. The influence line of a simple and continuous beam by the moment distribution method, leading matrix method, transfer matrix method and Müller-Breslau principle, etc, are well known and used widely in bridge engineering. But still there are some difficulties to determine the influence line of the particular structure which allows the side sway.

In this study the transfer matrix method is applied to get the influence lines of the portal frame with the joint displacement. The influence line of the portal frame calculated by the transfer matrix method is compared with that calculated by the finite element method. The results show a good agreement.

Keywords : *transfer matrix, influence line of rahmen, joint translation, joint equilibrium equation*

1. Introduction

The influence lines of the indeterminate structures are usually found by the Müller-Breslau principle, the consistent deformation method[1] and so on. These methods still have some problems to get the influence lines of the structures which have many degrees of indeterminacy or

* Regular Member, Dept. of Civil Engineering, Gyeongsang National University, Korea, Professor
** Dept. of Civil Engineering, Changwon College, Changwon, Korea, Associate Professor

• 이 논문에 대한 토론을 2001년 6월 30일까지 본 학회에 보내주시면 2001년 9월호에 그 결과를 게재하겠습니다.

allow the joint translations. Thus most of the well known influence lines are the influence lines of the determinate structures and indeterminate structures without the joint translations. Usually the influence lines of the indeterminate structures are obtained by using any structural analysis computer software involving the finite element method.

The transfer matrix method is applied to the analysis of the continuous beams, simple frames and plates. But the application of this method to the 3-D frames, plates and shells[2] is very limited until now and will be studied in the near future. The focus of this study is applying the transfer matrix method to the analysis of the indeterminate structures as a different way of the research in the structural engineering. Here a new application of the transfer matrix method produces the influence lines of the forces and displacements of the indeterminate structure by simple calculation without solving the simultaneous equations. Also the transfer matrix method is a simple method to solve rahmen structures with several intermediate supports in which joint translations are involved.

2. Theoretical Considerations

2.1 Transfer Matrix

Arbitrary vectors which act on the left-hand end's state vectors of a finite beam as shown in Fig.1 will reflect to vectors on the right-hand end's state vectors of the beam. This kind of relation of two side vectors is similar to a function that transfers one side vectors of the beam to the other side vectors. This can be written as a transfer matrix:

$$S_{2b} = f(S_{1b}) = G_b S_{1b} \tag{1}$$

where S_{1b} is the left-hand end's state vectors

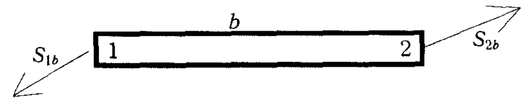

Fig. 1 State vectors of an element

of the member b, S_{2b} is the right-hand end's state vectors of the member b, f is a transfer function and G_b is the transfer matrix of the member b.

2.2 Equilibrium Equation of Joint

The equilibrium equation[3] at the joint X in Fig. 2(a) is

$$P_{X+} + P_X = P_{X-} \tag{2}$$

The relation between the state vectors is

$$\begin{bmatrix} d_{X+} \\ P_{X+} \end{bmatrix} = \begin{bmatrix} d_{X-} \\ P_{X-} \end{bmatrix} - \begin{bmatrix} 0 \\ P_X \end{bmatrix} \tag{3}$$

symbolically

$$S_{X+} = S_{X-} - S_X \tag{4}$$

In the case where the joint is rigid and is connected elastically to a foundation, the equilibrium equation is

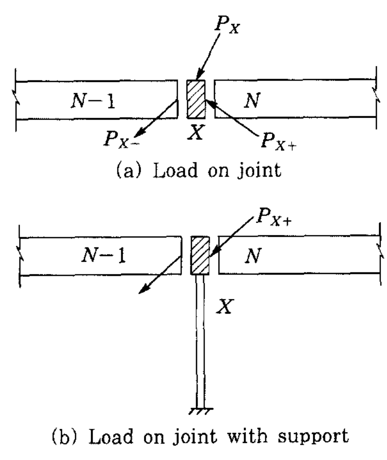

Fig. 2 State vectors on joint X

$$P_{X+}=P_{X-}+k_X d_X \quad (5)$$

where

k_X : the stiffness matrix of the support under the joint X

P_{X+} : the forces at the right hand side of the joint X

P_{X-} : the forces at the right hand side of the member $N-1$

d_X : the displacements at the joint X

The compatibility condition at the joint X is

$$d_{X+}=d_X=d_{X-} \quad (6)$$

The Eq.(5) and (6) may be written as

$$\begin{bmatrix} d_{X+} \\ P_{X+} \end{bmatrix} = \begin{bmatrix} I & 0 \\ k_X & I \end{bmatrix} \begin{bmatrix} d_{X-} \\ P_{X-} \end{bmatrix} \quad (7)$$

or as

$$S_{X+}=G_X S_{X-} \quad (8)$$

2.3 Application of Transfer Matrix[4)]

By applying Eq.(3) and (7) to the rahmen at Fig. 3 we obtain the state vectors S_{1b}' of the left-hand side of the member b as follows:

$$S_{1b}'=S_{A+}=\begin{bmatrix} I & 0 \\ k_A' & I \end{bmatrix} \begin{bmatrix} d_A \\ P_{A-} \end{bmatrix} - \begin{bmatrix} 0 \\ P_A \end{bmatrix} \quad (9)$$

where k_A' is the transformed stiffness matrix of the column under the joint A. The state vectors of the right-hand side of the member b can be obtained by

$$S_{2b}'=G_b' S_{1b}'=G_b' G_A S_{A-} - G_b' S_A \quad (10)$$

where G_b' : the transfer matrix of the member b.

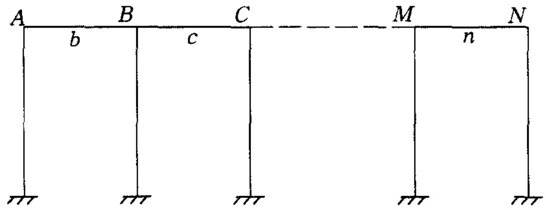

Fig. 3 A structure with intermediate supports

Progressing along the structures in the same way as above, we obtain the state vectors on the right side of joint N as

$$\begin{aligned} S_{N+} &= (G_N' G_n' G_M' G_m' \cdots G_B' G_b' G_A') S_{A-} \\ &\quad - (G_N' G_n' G_M' G_m' \cdots G_B' G_b' G_A') S_A \\ &\quad - \cdots - S_N \end{aligned} \quad (11)$$

or as

$$S_{N+}=G S_{A-}-C \quad (12)$$

C is the multiplication and summation of the known values, the transfer matrices and state vectors. G is the result of the first term. Considering the boundary condition, $P_{N+}=0$ and $P_{A-}=0$, the Eq.(12) can be written in the form

$$\begin{bmatrix} d_N \\ 0 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} d_A \\ 0 \end{bmatrix} - \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad (13)$$

that is

$$d_N=G_{11} d_A - C_1 \quad (14)$$

$$0=G_{21} d_A - C_2 \quad (15)$$

Once d_A has been found the state vectors at any member can be obtained easily by using Eq.(9) and (10). Due to this reason the values of influence lines, moment, shear force, axial force, horizontal displacement, vertical displacement and rotation, are obtained by simple multiplication of the transfer matrix when unit load is applied the given structure.

3. An Example and Results

3.1 Example

The rahmen shown in Fig. 4 is chosen as an example to prove the simplicity and validity of the proposed method. This method will derive the influence lines of the indeterminate structure involving the joint translation. The unit load is applied to the rahmen which is divided to 6 elements in the transfer matrix method. In the finite element method the structure is divided to 13 elements.

From Eq.(11) the state vector at the joint C is

$$S_{C+} = (G_C' G_c' G_b' G_B' G_a' G_A') S_{A-} - (G_C' G_c') S_G$$
$$= GS_{A-} - C \qquad (16)$$

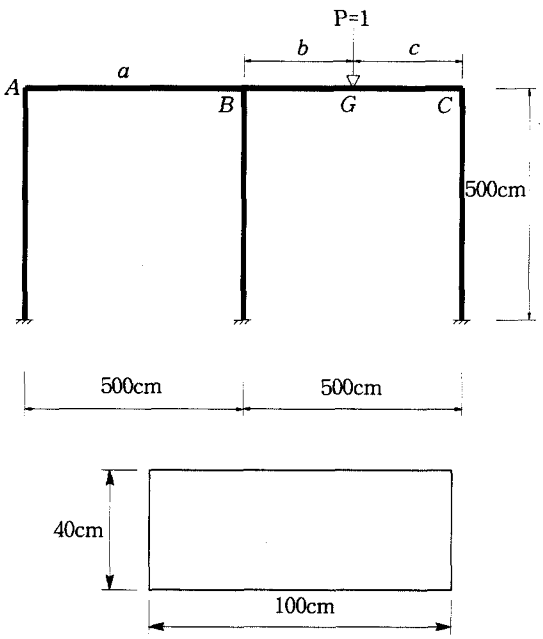

Fig. 4 Portal frame subject to unit load

where,

$$G_A = G_B' = G_C' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & 1 & 0 & 0 \\ 0 & \frac{EA}{L} & 0 & 0 & 1 & 0 \\ \frac{6EA}{L^2} & 0 & \frac{4EI}{L} & 0 & 0 & 1 \end{bmatrix}$$

$$G_a' = \begin{bmatrix} 1 & 0 & 0 & \frac{a}{EA} & 0 & 0 \\ 0 & 1 & a & 0 & -\frac{a^3}{6EI} & \frac{a^2}{2EI} \\ 0 & 0 & 1 & 0 & -\frac{a^2}{2EI} & \frac{a}{EI} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -a & 1 \end{bmatrix}$$

$$G_b' = \begin{bmatrix} 1 & 0 & 0 & \frac{b}{EA} & 0 & 0 \\ 0 & 1 & b & 0 & -\frac{b^3}{6EI} & \frac{b^2}{2EI} \\ 0 & 0 & 1 & 0 & -\frac{b^2}{2EI} & \frac{b}{EI} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -b & 1 \end{bmatrix}$$

$$G_c' = \begin{bmatrix} 1 & 0 & 0 & \frac{c}{EA} & 0 & 0 \\ 0 & 1 & c & 0 & -\frac{c^3}{6EI} & \frac{c^2}{2EI} \\ 0 & 0 & 1 & 0 & -\frac{c^2}{2EI} & \frac{c}{EI} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & c & 1 \end{bmatrix}$$

$$S_{A-} = \begin{Bmatrix} u_A \\ v_A \\ \theta_A \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \quad S_{C+} = \begin{Bmatrix} u_C \\ v_C \\ \theta_C \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \quad S_G = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{Bmatrix}$$

C is the known values by multiplication of the transfer matrices and state vectors from the dimension of the given structure and the external load, unit load. The unknown value S_{A-} is obtained by the equation above.

3.2 Results

The results are obtained by the transfer matrix method without solving the simultaneous equations. The results of the transfer matrix method and finite element method are shown in Tables.

The results in the tables are not the exact solutions. But the values of two methods show no significant difference. Even if there is a little difference, it is not important in the practical engineering. Thus one probably accepts this method as a valid method.

Table 1 The values of influence line of moment at the joint A

Load from joint A		0cm	100cm	200cm	300cm	400cm	500cm	600cm	700cm	800cm	900cm	1000cm
TM*	6els***	0.494	-29.716	-37.536	-30.378	-15.657	-0.789	6.289	5.520	1.321	-1.893	0.294
CS**	13els	0.494	-29.716	-37.536	-30.378	-15.658	-0.789	6.289	5.520	1.321	-1.893	0.294

*TM : Transfer Matrix Method, **CS : Commercial Software(SAP90), ***els : elements

Table 2 The values of influence line of shear force at the joint A

Load from joint A		0cm	100cm	200cm	300cm	400cm	500cm	600cm	700cm	800cm	900cm	1000cm
TM	6els	0.002	-0.808	-0.579	-0.345	-0.142	-0.004	0.055	0.061	0.037	0.010	0.002
FEM	13els	0.002	-0.808	-0.579	-0.345	-0.142	-0.004	0.055	0.061	0.037	0.010	0.002

Table 3 The values of influence line of axial force at the joint A

Load from joint A		0cm	100cm	200cm	300cm	400cm	500cm	600cm	700cm	800cm	900cm	1000cm
TM	6els	0.001	-0.080	-0.105	-0.088	-0.049	-0.002	0.020	0.014	-0.004	-0.014	0.001
FEM	13els	0.001	-0.080	-0.104	-0.088	-0.048	-0.002	0.020	0.014	-0.004	-0.014	0.001

Table 4 The values of influence line of rotation at the joint A(value : $\times 10^{-8}$)

Load from joint A		0cm	100cm	200cm	300cm	400cm	500cm	600cm	700cm	800cm	900cm	1000cm
TM	6els	0.086	-3.926	-4.656	-3.396	-1.440	-0.081	0.481	0.851	0.948	0.690	-0.005
FEM	13els	0.086	-3.926	-4.656	-3.396	-1.440	-0.081	0.481	0.851	0.948	0.690	-0.005

Table 5 The values of influence line of vertical displacement at the joint A(value : $\times 10^{-7}$)

Load from joint A		0cm	100cm	200cm	300cm	400cm	500cm	600cm	700cm	800cm	900cm	1000cm
TM	6els	-5.422	-4.390	-3.145	-1.875	-0.771	-0.022	0.299	0.330	0.203	0.052	0.009
FEM	12els	-5.422	-4.390	-3.145	-1.875	-0.771	-0.021	0.299	0.330	0.203	0.052	0.009

Table 6 The values of influence line of horizontal displacement at the joint A(value : $\times 10^{-6}$)

Load from joint A		0cm	100cm	200cm	300cm	400cm	500cm	600cm	700cm	800cm	900cm	1000cm
TM	6els	-0.119	2.993	2.769	1.002	-0.518	0.001	0.533	-0.962	-2.710	-2.941	0.118
FEM	12els	-0.119	2.993	2.769	1.002	-0.518	0.001	0.533	-0.962	-2.710	-2.941	0.118

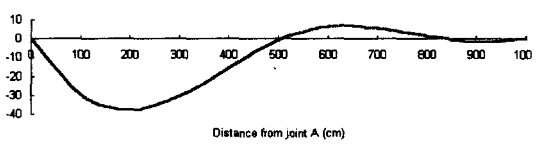

Fig. 5 Influence line of moment at joint A

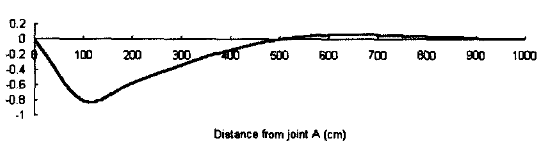

Fig. 6 Influence line of shear force at joint A

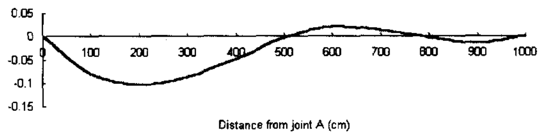

Fig. 7 Influence line of axial force at joint A

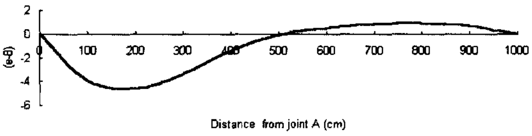

Fig. 8 Influence line of rotation at joint A

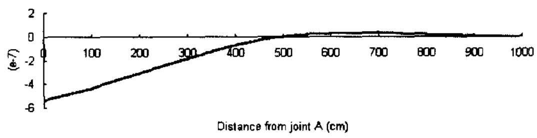

Fig. 9 Influence line of vertical displacement at joint A

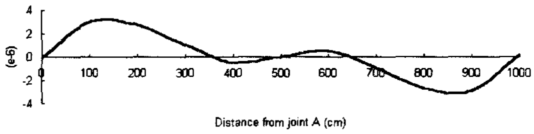

Fig. 10 Influence line of horizontal displacement at joint A

4. Conclusions

The present transfer matrix method is applied to the portal frame structure. The results of the two methods demonstrates that there is no significant difference between them. The simplicity and validity of the proposed method is proved.

The conclusions can be stated as follows :

1. The results of two methods, transfer matrix and finite element method show no significant difference.
2. The influence lines of the indeterminate structures can be simply obtained by the transfer matrix method without solving simultaneous equations.
3. The influence lines of the indeterminate structures with joint translations can be easily derived by using this method.
4. The influence lines of the rahmen composed of varying section will be produced by changing the elements of the transfer matrix of a gradually varying member.

Terminology

d_X : the displacements at the joint X

G_b' : the transfer matrix of the member b

k_X : the stiffness matrix of the support under the joint A

P_{X+} : the forces at the right hand side of the joint X

P_{X-} : the forces at the right hand side of the member $N-1$

References

1. Hsieh, Y.Y., *Elementary Theory of Structures*, Prentice-Hall, U.S.A., 1982, p.416
2. Ha, D.H, "An Analysis of Axisymmetric Tank on Elastic foundation by Transfer Matrix Method," Ph. Degree, Gyeongsang National University, 1966, p.93
3. Livesley, R.K., *Matrix Methods of Structures Analysis*, Pergamon Press, England, 1975, p.277
4. Nam, M.H., Ha, D.H. and Lee, K.H., "The Influence Line of a Portal Frame by Transfer Matrix Method," *The First International Conference on Advances in Structural Engineering and Mechanics*, Vol.1, 1999 August, pp.549~554