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Multi-Level Skip-Lot Sampling Plan

Gyo-Young Cho[1], Young Sun Ahn[2]

Abstact

This paper is a generalization of single and two-level skip-lot sampling plans to n -level. On every skipping inspection of the n-level skip-lot sampling plan, not only the number of consecutive lots to be accepted but also the fraction of lots to be inspected can be freely choosed. The general formulas of the operating characteristic function, average fraction inspected, average sample number and average outgoing quality in n-level skip-lot sampling plan are derived. The operating characteristic curves, average sample number and average outgoing quality of a referance plan, two-level and five-level skip-lot sampling plans are compared.

Keywords : Markov chain; Operating characteristic curves; Average fraction inspected; Average sample number; Average outgoing quality.

1. Introduction

The Skip-Lot Sampling Plan(SKSP) is a system of lot by lot inspection plans in which a provision is made for inspecting only some fraction of submitted lots. Dodge(1955) initially presented SKSP's as an extension of continuous sampling plan(CSP) type. Perry(1973a) developed it to single-level and two-level SKSP's of which the latter has no restriction on the fraction of lots inspected. Parker and Kessler(1981) presented a modified single-level SKSP. Hess and Kittleman(1989) applied Perry's(1973b) results to skip-period inspection plans to assure good performance of a plant.

In this paper, a general n-level SKSP (MLSKSP) is presented, which has no restriction on the level n and skipping parameters f_k and i_k, where f_k is the fraction of lots inspected on the kth skipping inspection, $k=1,2,\cdots,n$ and i_k is the number of lots consecutively inspected and accepted on the $(k-1)$th skipping inspection, $k=1,2,\cdots,n$. Note that

1) Professor, Department of Statistics, Kyungpook National University, Taegu, 702-701, Korea
E-mail : gycho@knu.ac.kr
2) Part-time lecturer, Department of Statistics, Kyungpook National University, Taegu, 702-701, Korea
E-mail : sunny65@chollian.net

$0<f_k\leq 1$ and i_k's are natural numbers for $k=1,2,\cdots,n$. Also note that Dodge(1943), Liberman and Solomon(1955) and Perry(1973b) has not taken into account the different numbers between i_k's

2. Procedure of the MLSKSP

MLSKSP uses a specified lot inspection plan called the reference-sampling plan, together with the following rules.

(1) Begin with normal inspection, using the reference plan. At this stage of operation, every lot is inspected.

(2) When i_1 consecutive lots are accepted on normal inspection, switch to the first skipping inspection at rate f_1.

(3) During the k-th skipping inspection, $k=1,2,\cdots,n-1$;

◉ When i_{k+1} consecutively inspected lots are accepted on normal inspection, switch to the first skipping inspection at rate f_{k+1}.

◉ When a lot is rejected, switch to normal inspection.

(4) During the n-th skipping inspection at rate f_k ;

When a lot is rejected, switch to normal inspection.

(5) Screen each rejected lot and correct or replace all defective units found.

3. Markov Chain Approach

The Markov chain approach can be used to obtain Operating Characteristic (OC) function, Average Fraction Inspected (AFI) and Average Outgoing Quality (AOQ) for a given value of process defective fraction. The states of the Markov chain for the MLSKSP are defined in terms of Perry(1973a, 1973b) as follows:

NR = state where lot is rejected under normal inspection.

N_j = states under normal inspection representing the number of consecutively accepted lots j, $j=1,2,\cdots,i_1$.

$SkAj$ = states that the number of consecutively inspected and accepted lots during k-th skipping inspection at rate f_k is j, $k=1,2,\cdots,n-1$, $j=1,2,\cdots,i_{k+1}$.

SkNj = states that lot is skipped during k-th skipping inspection at rate f_k, and previous number of inspected and accepted lots on k-th skipping inspection at rate f_k is j, $k=1,2,\cdots,n-1$, $j=0,1,\cdots,i_{k+1}-1$.

SkR = states that lot is rejected during k-th skipping inspection at rate f_k, $k=1,2,\cdots,n$.

SnA = state that lot is inspected and accepted during n-th skipping inspection at rate f_n.

SnN = state that lot is skipping during n-th skipping inspection at rate f_n.

SnR = state that lot is rejected during n-th skipping inspection at rate f_n.

Let P denote the probability of accepting a lot according to the reference plan and $Q=1-P$ throughout this paper. Then the one-step transition probability matrix M for the plan MLSKSP is given by

$$M = \begin{pmatrix} P_{00} & & & & & & \\ P_{10} & P_{11} & & & & & \\ P_{20} & & P_{22} & & & & \\ \cdot & & & \cdot & & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & & \cdot & \\ P_{(n-1)0} & & & & & P_{(n-1)(n-1)} & \\ P_{n0} & & & & & & P_{nn} \end{pmatrix},$$

where the submatrices of M are the followings. In these submatrices, all values of elements that have no entry equal zeros.

$$P_{00} = \begin{array}{c} \\ NR \\ N1 \\ N2 \\ \cdot \\ \cdot \\ \cdot \\ N_{i_1}-1 \end{array} \begin{array}{c} \begin{array}{cccccccc} NR & N1 & N2 & \cdot & \cdot & \cdot & N_{i_1} \end{array} \\ \begin{pmatrix} Q & P & & & & & \\ Q & & P & & & & \\ Q & & & P & & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & & \cdot & \\ \cdot & & & & & & \cdot \\ Q & & & & & & P \end{pmatrix} \end{array},$$

for $k=1,2,\cdots,n$

$$
P_{k0}=\begin{array}{c|c}
 & \begin{array}{cccccc} NR & N1 & N2 & \cdot & \cdot & \cdot N_{i_1} \end{array} \\
\begin{array}{c} S(k-1)A_{i_k} \\ SkA1 \\ SkA2 \\ \cdot \\ \cdot \\ \cdot \\ SkA(i_{k+1}-1) \\ SkR \\ SkN0 \\ SkN1 \\ SkN2 \\ \cdot \\ \cdot \\ \cdot \\ SkN(i_{k+1}-1) \end{array} & \left(\begin{array}{cc} & \\ & \\ & \\ & \\ & \\ & \\ & \\ Q & P \\ & \\ & \\ & \\ & \\ & \\ & \\ & \end{array}\right)
\end{array},
$$

where $S(k-1)A_{i_k}=N_{i_1}$ when $k=1$,

for $k=1,2,\cdots,n-1$,

$$
P_{kk}=\begin{array}{c|c}
 & \begin{array}{cccccccccc} A1 & A2 & \cdot & \cdot & A_{i_{k-1}} & R & N0 & \cdot & \cdot & N(i_{k+1}-1) \end{array} \\
\begin{array}{c} S(k-1)A_{i_k} \\ SkA1 \\ SkA2 \\ \cdot \\ \cdot \\ \cdot \\ SkA(i_{k+1}-1) \\ SkR \\ SkN0 \\ SkN1 \\ SkN2 \\ \cdot \\ \cdot \\ \cdot \\ SkN(i_{k+1}-1) \end{array} & \left(\begin{array}{cccccccccc}
f_kP & & & & & f_kQ & 1-f_k & & & \\
 & f_kP & & & & f_kQ & & \cdot & & \\
 & & \cdot & & & \cdot & & & \cdot & \\
 & & & \cdot & & \cdot & & & & \cdot \\
 & & & & \cdot & \cdot & & & & \\
 & & & & & \cdot & & & & \\
 & & & & f_kP & f_kQ & & & & 1-f_k \\
f_kP & & & & & f_kQ & 1-f_k & & & \\
 & f_kP & & & & f_kQ & & \cdot & & \\
 & & \cdot & & & \cdot & & & \cdot & \\
 & & & \cdot & & \cdot & & & & \cdot \\
 & & & & \cdot & \cdot & & & & \\
 & & & & & \cdot & & & & \\
 & & & & f_kP & f_kQ & & & & 1-f_k \\
 & & & & & & & & &
\end{array}\right)
\end{array},
$$

where $S(k-1)A_{i_k}=N_{i_1}$ when $k=1$, and

$$
P_{nn} = \begin{array}{c} \\ S(n-1)A_{i_n} \\ SnA \\ SnR \\ SnN \end{array}
\begin{array}{c} \begin{array}{ccc} SnA & SnR & SnN \end{array} \\ \begin{pmatrix} f_nP & f_nQ & 1-f_n \\ f_nP & f_nQ & 1-f_n \\ & & \\ f_nP & f_nQ & 1-f_n \end{pmatrix} \end{array} .
$$

4. Derivation of Operating Characteristic Function

Since it can be shown that this Markov chain is finite, recurrent, irreducible and aperiodic, the long run or stationary probabilities, Π_i's, of all the given states are the unique solutions to the following system of equations;

$$\Pi_i = \sum_{all\ states\ j} \Pi_j P_{ji} \qquad \text{for all states } i,$$

and

$$\sum_{all\ states\ i} \Pi_i = 1,$$

where P_{ji} is the one-step transition probability of going from state j to state i. The probability of acceptance for the plan MLSKSP can be obtained from

$Pa(f_1, f_2, \cdots, f_n; i_1, i_2, \cdots, i_n) = 1 - (\Pi_{NR} + \Pi_{S1R} + \cdots, \Pi_{SnR}),$

where Π_{NR}, $\Pi_{S1R}, \cdots$, Π_{SnR} are long-run probabilities of lot rejection in the normal, first, second, $\cdots$, and n-th skipping inspection, respectively.

After some tedious calculation we get the following solutions

$$
\begin{aligned}
&\Pi_{NR} = (1-P^{i_1})(\Pi_{NR} + \Pi_{S1R} + \cdots + \Pi_{SnR}),\\
&\Pi_{S1R} = P^{i_1}(1-P^{i_2})(\Pi_{NR} + \Pi_{S1R} + \cdots + \Pi_{SnR}),\\
&\Pi_{S2R} = P^{i_1}P^{i_2}(1-P^{i_3})(\Pi_{NR} + \Pi_{S1R} + \cdots + \Pi_{SnR}),\\
&\cdots,\\
&\Pi_{Sn-1R} = P^{i_1}P^{i_2}\cdots P^{i_n-1}(1-P^{i_n})(\Pi_{NR} + \Pi_{S1R} + \cdots + \Pi_{SnR}),\\
&\Pi_{SnR} = P^{i_1}P^{i_2}\cdots P^{i_n-1}(\Pi_{NR} + \Pi_{S1R} + \cdots + \Pi_{SnR}),
\end{aligned}
$$

and

$$\frac{1}{Q}\Pi_{NR} + \frac{1}{f_1Q}\Pi_{S1R} + \frac{1}{f_2Q}\Pi_{S2R} + \cdots + \frac{1}{f_nQ}\Pi_{SnR} = 1.$$

Thus

$$Pa(f_1, f_2, \cdots, f_n; i_1, i_2, \cdots, i_n) = 1 - (\Pi_{NR} + \Pi_{S1R} + \cdots, \Pi_{SnR}),$$
$$= 1 - \frac{Q}{A},$$

where

$$A = (1 - P^{i_1}) + \frac{P^{i_1}(1 - P^{i_2})}{f_1} + \frac{P^{i_1}P^{i_2}(1 - P^{i_3})}{f_2}$$
$$+ \cdots + \frac{P^{i_1}P^{i_2}\cdots P^{i_{n-1}}(1 - P^{i_n})}{f_{n-1}} + \frac{P^{i_1}P^{i_2}\cdots P^{i_n}}{f_n}.$$

From the above formula, if we let $f_1 = f_2 = \cdots = f_{n-1} = 1$, $f_n = f$, and $i_1 = i_2 = \cdots = i_{n-1} = 0$, $i_n = i$, $Pa(f_1, \cdots, f_n; i_1, \cdots, i_n)$ is reduced to

$$Pa(f, i) = \frac{fP + (1 - f)P^i}{f + (1 - f)P^i},$$

which is exactly Perry's(1973a) formula for the single-level skip-lot sampling plans SKSP-2.

5. Derivation of Average Fraction Inspected (AFI) and Average Sample Number (ASN)

A very important property of skipping lot sampling plan is that of reduced inspection, and it will be investigated here using the average sample number of the plan as a basis. The average sample size or average sample number is defined as the average sample number of sample units inspected per lot. For the skipping lot plan, lots that are not sampled will have an average sample size of zero. It will be assumed that there is no curtailment in the inspection of a sample ; i.e. even when the acceptance number is exceeded and rejection is certain, the remainder of the sample is nevertheless inspected. For the plan MLSKSP, lots that are not sampled will have an average sample number of zero. For the lots that are sampled, the average sample number will be that of the reference sampling plan. Let ASN(MLSKSP) be the average sample number of the plan MLSKSP and ASN(R) be that of the reference plan. Perry(1973a) has shown that ASN(SKSP) = ASN(R) × AFI, $0 < \text{AFI} < 1$, where AFI is the average fraction of lots inspected which are sampled. Thus

$$\text{ASN(MLSKSP)} = \text{ASN(R)} \times \text{AFI}, \qquad 0 < \text{AFI} < 1,$$

and it is obvious that ASN(MLSKSP) < ASN(R) since $0 < \text{AFI} < 1$. This shows that MLSKSP yields a reduction in inspection as based upon the reference plan's ASN. Now we can derive AFI of MLSKSP as follows. Let Π_k be the long-run stationary probabilities of level k, $k = 1, 2, \cdots, n$. And

$$\Pi_k' = \Pi_k - \sum_{l=0}^{i_{k+1}-1} \Pi_{SkNl}, \quad k=1,2,\cdots,n.$$

Then

$$\begin{aligned}
\Pi_0' &= \Pi_0, \\
\Pi_1' &= \Pi_1 - (\Pi_{S1N0} + \Pi_{S1N1} + \cdots + \Pi_{S1Ni_1-1}) \\
&= \Pi_{S1A1} + \Pi_{S1A2} + \cdots + \Pi_{S1Ai_1} + \Pi_{S1R}, \\
\Pi_2' &= \Pi_2 - (\Pi_{S2N0} + \Pi_{S2N1} + \cdots + \Pi_{S2Ni_2-1}) \\
&= \Pi_{S2A1} + \Pi_{S2A2} + \cdots + \Pi_{S2Ai_2} + \Pi_{S2R}, \\
&\cdots, \\
\Pi_{n-1}' &= \Pi_{n-1} - (\Pi_{Sn-1N0} + \Pi_{Sn-1N1} + \cdots + \Pi_{Sn-1Ni_{n-1}-1}) \\
&= \Pi_{Sn-1A1} + \Pi_{Sn-1A2} + \cdots + \Pi_{Sn-1Ai_{n-1}} + \Pi_{Sn-1R}, \\
\Pi_n' &= \Pi_{SnA} + \Pi_{SnR}.
\end{aligned}$$

It can be shown that

$$\begin{aligned}
\text{AFI} &= \Pi_0' + \Pi_1' + \Pi_2' + \cdots + \Pi_n' \\
&= \frac{1}{A},
\end{aligned}$$

where

$$\begin{aligned}
A = (1-P^{i_1}) &+ \frac{P^{i_1}(1-P^{i_2})}{f_1} + \frac{P^{i_1}P^{i_2}(1-P^{I_3})}{f_2} \\
&+ \cdots + \frac{P^{i_1}P^{i_2}\cdots P^{i_{n-1}}(1-P^{i_n})}{f_{n-1}} + \frac{P^{i_1}P^{i_2}\cdots P^{i_n}}{f_n}.
\end{aligned}$$

From the above formula, if we let $f_1 = f_2 = \cdots = f_{n-1} = 1$, $f_n = f$, and $i_1 = i_2 = \cdots = i_{n-1} = 0$, $i_n = i$ then

$$\text{AFI} = \frac{f}{(1-f)P^i + f},$$

which has been shown by Perry(1973a).

6. Deviation of Average Outgoing Quality (AOQ)

The AOQ is widely used for the evaluation of a rectifying sampling plan. The AOQ is the quality in the lot that results from the application of rectifying inspection. It is the average value of lot quality that would be obtained over a long sequence of lots from a process with defective fraction p_{dl}. Since the AOQ of reference plan is

$$\mathrm{AOQ(R)} = \frac{P \cdot p_{dl} \cdot (N-n)}{N},$$

The AOQ of MLSKSP is

$$\mathrm{AOQ(MLSKSP)} = \mathrm{AFI} \cdot \mathrm{AOQ(R)} + (1-\mathrm{AFI}) \cdot p_{dl},$$

where N is lot size, n is sample size. Note that as the lot size N becomes large relative to the sample size n, we may write AOQ(R) and AOQ(MLSKSP) as

$$\mathrm{AOQ(R)} \simeq P \cdot p_{dl},$$

and

$$\mathrm{AOQ(MLSKSP)} \simeq \mathrm{AFI} \cdot P \cdot p_{dl} + (1-\mathrm{AFI}) \cdot p_{dl}.$$

7. Numerical Examples

In order to obtain the OC function, ASN, AOQ of more higher-level MLSKSP than two-level, it is sufficient to adjust the skipping parameters f_k and i_k similarly to the cases of the single- and two-level plans.

Figure 1 shows the operating characteristic curves of reference plans, $Pa(\frac{1}{2},\frac{1}{4};4,4)$, $Pa(\frac{1}{2},\frac{1}{4};4,6)$, $Pa(\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10};4,4,4,4,4)$, and $Pa(\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10};2,4,6,8,10)$, for the reference plans of $n=20$, $c=1$ and $n=50$, $c=2$, respectively.

Figure 2 shows the ASN curves of $\mathrm{ASN}(\frac{1}{2},\frac{1}{4};4,4)$, $\mathrm{ASN}(\frac{1}{2},\frac{1}{4};4,6)$, $\mathrm{ASN}(\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10};4,4,4,4,4)$, $\mathrm{ASN}(\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10};2,4,6,8,10)$, for the reference plans of $n=20$, $c=1$, and $n=50$, $c=2$, respectively.

Figure 3 shows the AOQ curves of reference plans $\mathrm{AOQ}(\frac{1}{2},\frac{1}{4};4,4)$, $\mathrm{AOQ}(\frac{1}{2},\frac{1}{4};4,6)$, $\mathrm{AOQ}(\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10};4,4,4,4,4)$, $\mathrm{AOQ}(\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10};2,4,6,8,10)$, for the reference plans of $n=20$, $c=1$, and $n=50$, $c=2$, respectively.

From the above figures, we can see that (1) the acceptance probabilities of the five-level skip-lot sampling plan with same i_k are largest among all the plans considered when the defective rate is less than about 0.02 and are almost same with the skip-lot plans except the five-level skip-lot sampling plan with different i_k when the defective rate is greater than about 0.02, (2) the ASNs and AOQs of the five-level skip-lot sampling plan with same i_k are almost same with the skip-lot sampling plans considered except the five-level skip-lot sampling plan with different i_k for all the defective rate.

8. Concluding Remarks

A general multi-level skip-lot sampling plan applicable to more than two-level skip-lot sampling plans is developed. The developed multi-level skip-lot sampling plan has merits that we can freely choose not only the number of consecutive lots to be accepted but also the fraction of lots to be inspected.

It has seen that the five-level skip-lot sampling plan with same i_k seemes more resonable than the lower-level skip-lot sampling plans considered in the aspect of the acceptance probability, ASN and AOQ, and the five-level skip-lot sampling plans considered in the aspect of ASN, although the AOQ is largest. Also, it seems that the higher-level skip-lot sampling plan seems more reasonable than the lower-level skip-lot sampling plans considereded in the aspect of the acceptance probability, ASN and AOQ since the skip-lot sampling plans are made when the quality of product is good.

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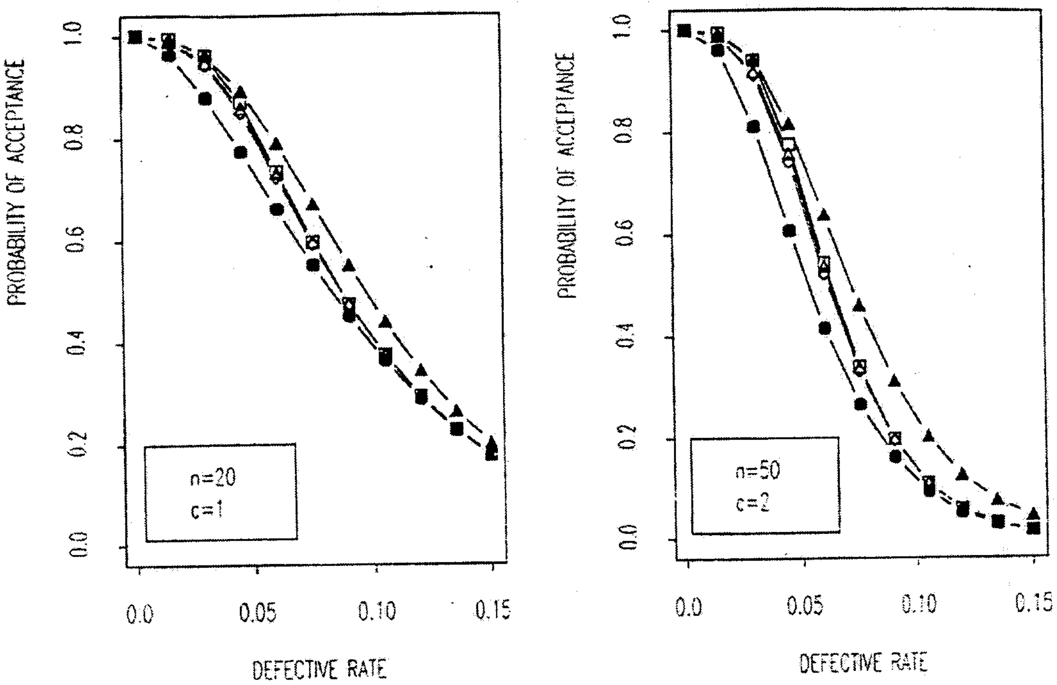

Figure 1. OC Curves with Reference Plan.
● : Reference plan, △ : $Pa(\frac{1}{2}, \frac{1}{4}; 4, 4)$, ◇ : $Pa(\frac{1}{2}, \frac{1}{4}; 4, 6)$,
□ : $Pa(\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}; 4, 4; 4, 4, 4)$, ▲ : $Pa(\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}; 2, 4; 6, 8, 10)$

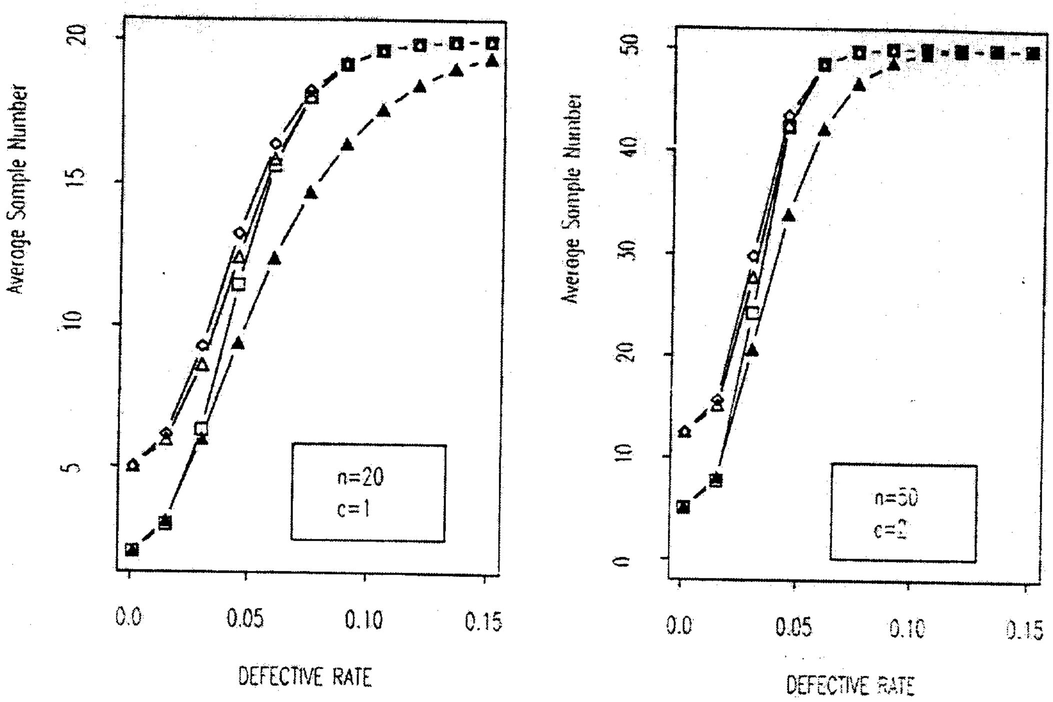

Figure 2. Average Sample Number (ASN) Curves with Reference Plan.
△ : ASN($\frac{1}{2}$, $\frac{1}{4}$; 4, 4), ◇ : ASN($\frac{1}{2}$, $\frac{1}{4}$; 4, 6), □ : ASN($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$; 4, 4, 4, 4, 4), ▲ : ASN($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$; 2, 4, 6, 8, 10)

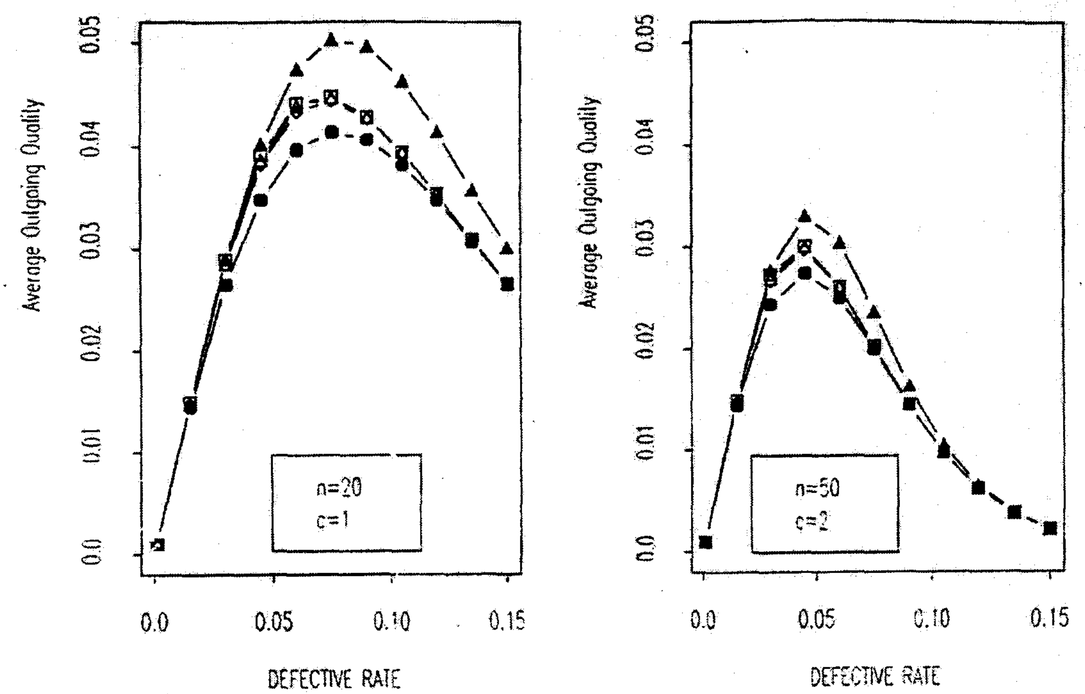

Figure 3. Average Outgoing Quality (AOQ) Curves with Reference Plan.

● : **Reference plan**, : AOQ($\frac{1}{2}, \frac{1}{4}; 4, 4$), ◇ : AOQ($\frac{1}{2}, \frac{1}{4}; 4, 6$),

□ : AOQ($\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}; 4, 4, 4, 4, 4$), ▲ : AOQ($\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}; 2, 4, 6, 8, 10$)