

Nonlinear Time Series Analysis Tool and its Application to EEG

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Abstract

Simply, Nonlinear dynamics theory means the complicated and noise-like phenomena originated from nonlinearity involved in deterministic dynamical system. An almost all the natural signals have nonlinear property. However, there exist few analysis software tool or package for a research and development of applications. We develop nonlinear time series analysis simulator is to provide a common and useful tool for this purpose and to promote research and development of nonlinear dynamics theory. This simulator is consists of the following four modules such as generation module, preprocessing module, analysis module and ICA module. In this paper, we applied to Electroencephalograph (EEG), as it turned out, our simulator is able to analyze nonlinear time series. Besides, we could get the useful results using the various parameters. These results are used to diagnostic the brain diseases.

Key Words : Nonlinear, Time series, Power spectrum, Bispectrum, Correlation dimension, Lyapunov exponents

1. Introduction

In the world, occur naturally to many significant time series has nonlinear property. The weather, stock, earthquake and bio signal is a good example to illustrate nonlinear time series. This signal analysis is very useful in order to know a natural and physical phenomenon.

The term nonlinear dynamics theory including chaos means, in the scientific, the complicated and noise-like phenomena originated from nonlinearity involved in deterministic dynamical system.[1-5] Nonlinear dynamics theory a fundamental concept is by now well established and described in a rich literature. The mere fact that simple deterministic systems generically exhibit complicated temporal behavior in the presence of nonlinearity has influenced thinking and in tuition in many fields. However, it has been questioned whether the relevance of chaos for the understanding of the time evolving world goes beyond that of a purely philosophical paradigm. The first question is if chaos theory can be used to gain a better understanding and interpretation of observed complex dynamical behavior. The second is if chaos theory can give an advantage in predicting or controlling such time evolution. Time evolution as a system property can be measured by recording time series. According to recent results, nonlinear dynamics theory will be the key to the answers

of the above questions. Nonlinear time series analysis based on this theoretical paradigm is described in many monographs and book, Abarane[6], Kantz and Schreiber[7], Grassberger et al.[8] and Kugiumtzis et al.[9,10]. So, Nonlinear dynamics is not only an important research field in science, but also has potential to be applied in many fields such as information processing[11-13], plant control[16], analysis and estimation of biological phenomenal[14,15], or even the prediction of economic market price[16,17]. Thus, there exist a number of research fields. However, there exist few analysis software tool or package for a research and development of applications incorporating nonlinear dynamics technology. The motivation for us to develop nonlinear time series analysis simulator is to provide a common and useful tool for this purpose and to promote research and development of nonlinear dynamics theory. This simulator is consists of the following four modules such as gene

In this paper, we describe the details of the simulator and its application to biomedical signal, EEG. As it turned out, our simulator is able to analyze generally nonlinear time series. And this simulator can choose the nonlinear parameter suitable for nonlinear time series property. It was known from the EEG analysis of result that EEG is clearly chaotic signal and its property.

II. Signal Generation and Processing Module

This module is preparations for the nonlinear time series analysis. First, signal generation module makes the chaotic time series such as the Lorenz and Rössler signal. So we estimated a value of the chaos simulator

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objectively. The following Eq.1, 2 show equation of the Lorenz and Roessler signal. This simulator can change each parameter and the initial value. We show the chaotic phenomenon according to the change of the values.

$$\begin{aligned} \frac{dX}{dt} &= Pr[Y - X] \\ \frac{dY}{dt} &= -XZ + rX - Y \\ \frac{dZ}{dt} &= XY - bZ \end{aligned} \quad (1) \text{ Lorenz Equation}$$

$$\begin{aligned} \frac{dX}{dt} &= -Y - X \\ \frac{dY}{dt} &= X + aY \\ \frac{dZ}{dt} &= b + Z(X - c) \end{aligned} \quad (2) \text{ Roessler Equation}$$

Second, signal processing module is based on the all signal processing. Consists of normalization, band pass filtering, difference, add random noise and surrogating. For the promote efficiency of analysis, this module makes suitable signal. For example, Fig. 1 shows the result of band pass filtering that the alpha wave abstract from the raw signal of EEG. And Fig. 2 is results of the normalization

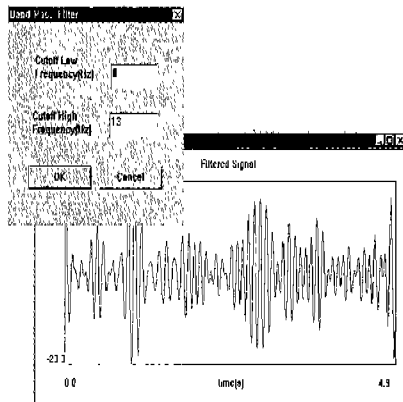


Fig. 1. Result of the band pass filtering

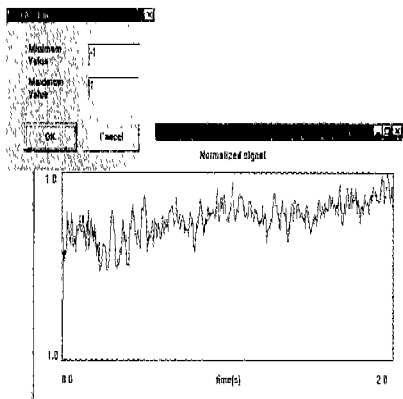


Fig. 2. Result of the normalization

III. Signal Analysis Module

Signal analysis module consists of four parts that is spectral analysis, phase space analysis, correlation analysis and mode analysis, respectively

3.1 Spectral analysis

3.1.1 Power spectrum and Bispectrum

Spectral analysis part can analyze into the spectrum property such as power spectrum and bispectrum. Particularly, bispectrum detect the presence of nonlinear properties such as quadratic phase coupling[18-20].

If $\{X(k)\} (k = 0, \pm 1, \pm 2, \dots)$ is a real stationary discrete-time signal and its moments up to order 2 and 3 exist, then each the second and third order cumulant can be written as

$$c_2^x(\tau_1) = m_2^x(\tau_1) - m_2^G(\tau_1) \quad (3)$$

$$c_3^x(\tau_1, \tau_2) = m_3^x(\tau_1, \tau_2) - m_3^G(\tau_1, \tau_2) \quad (4)$$

where $m_2^x(\tau_1), m_3^x(\tau_1, \tau_2)$ is each the second, third order moment and $m_2^G(\tau_1), m_3^G(\tau_1, \tau_2)$ is second, third order moment of an equivalent Gaussian signal that has the same mean value and autocorrelation sequence as $X(k)$. Clearly, if it is Gaussian $m_2^x(\tau_1, \tau_2) = m_2^G(\tau_1, \tau_2)$ and thus $c_3^x(\tau_1, \tau_2) = 0$. Power spectrum and bispectrum are defined in terms of cumulants as follows.

Power spectrum:

$$P(\omega) = \sum_{\tau_1=-\infty}^{\infty} c_2^x(\tau_1) \exp\{-j(\omega\tau_1)\} \quad (5)$$

Bispectrum:

$$B(\omega_1, \omega_2) = \sum_{\tau_1=-\infty}^{\infty} \sum_{\tau_2=-\infty}^{\infty} c_3^x(\tau_1, \tau_2) \exp\{-j(\omega_1\tau_1 + \omega_2\tau_2)\} \quad (6)$$

The bicoherence:

$$b_3^x(\omega_1, \omega_2) = \frac{B(\omega_1, \omega_2)}{\sqrt{P(\omega_1) + P(\omega_2) + P(\omega_1, \omega_2)}} \quad (7)$$

This bicoherence function is very useful in the detection and characterization of nonlinearities in time series and in discriminating linear process forms nonlinear ones.

3.2 Phase space analysis

This part can analyze spatio-temporal property of the nonlinear time series. It consists of four classes parameter as shown below.

3.2.1 Delayed Axis phase space

This parameter called to put it another way the strange attractor. The strange attractor is simply the pattern of the

pathway, in visual form, produced by graphing the behavior of a system. Several researchers have defined and studied the strange attractor. The first was Lorenz in Deterministic nonperiodic flow [21], and Later Ruelle in Sensitive dependence on initial condition and turbulent behavior of dynamical systems [22]. Also strange attractor has property as following. Nonlinear time series exhibit a fractal structure in computer simulations of the strange attractor. Cycled around randomly without any particular set number of times, never crossing itself staying in the same phase space, and displaying self-similarity at any scale[23,24]. The attractor acts on the system as a whole and collects the trajectories of perturbation in the time series. Though these systems are unstable, they have patterned order and boundary. And if time series has not property of the chaos, attractor has not patterned order and boundary. Hence, we can judge chaos signal or not. Also, we can understand spatio-temporal property of the nonlinear time series utilized in the attractor topological property. Fig.3 shows the attractor of Lorenz signal. This attractor shows from graphing the change in weather systems modeled by Lorenz.

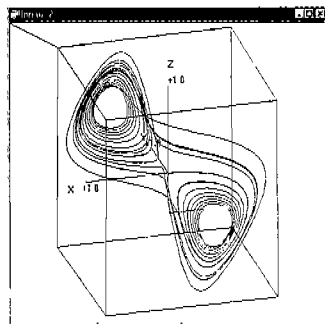


Fig. 3. Attractor of Lorenz signal

3.2.2 Correlation dimension

Correlation dimension, D2, is a measure of complexity of the process being investigated, which characterizes the distribution of pints in the phase space. Simply, In the case of steady-state behavior, the correlation dimension of the attractor is zero. In the case of periodic behavior, the correlation dimension of the attractor is one. And in chaotic states, the dimension usually takes on non-integer values[25].

The existence of an attractor and the evaluation of its correlation dimension may be achieved in the following manner[26]. By introducing vector notation, \vec{x}_i stands for a point of phase space whose coordinates are (x_i, y_i, z_i) . A reference point \vec{x}_j from these data is chosen and all its distances $|\vec{x}_i - \vec{x}_j|$ from the $N-1$ r from the point emaining points are computed. This allows us to count the data points that are within a prescribed distance r , one arrives at the quantityn the

phase space. Repeating the process for all values of

$$C(r) = \frac{1}{N^2} \sum_{\substack{i,j=1 \\ i \neq j}}^N \theta(r - |\vec{x}_i - \vec{x}_j|) \tag{8}$$

where θ is the Heaviside function, $\theta(x) = 0$ if $x < 0$ and $\theta(x) = 1$ if $x > 0$.

$C(r)$ measures the extent to which the presence of a data point \vec{x}_i affects the position of the other points. $C(r)$ may thus be referred to as the integral correlation function of the attractor. And $C(r)$ measures the spatial correlation of points on the attractor obtained form time series data. For small r , it is known that $C(r)$ behaves according to a power law. In a system with f degrees of freedom, $C(r)$ scales as r^f for a signal arising from noise, but as r^v (with $v < f$) when the signal arises from deterministic chaos[27].

The slope of $\log C(r)$ versus $\log r$ therefore gives the correlation dimension of the attractor.

$$D_{GP} = \frac{d \log C(r)}{d \log r} \tag{9}$$

This method for calculating correlation dimension is Grassberger Procaccia Algorithm[27]. With the help of relation Eq. 9, a correlation dimension D_{GP} is computed by considering successively higher values of embedding dimension d of the phase space. If the D_{GP} versus d dependence is saturated beyond some relatively small d the system represented by the time series data should possess an attractor. The saturation value D_{GP} is regarded as the correlation dimension of the attractor represented by the time series data[28].

3.2.3 Lyapunov exponent

Lyapunov exponent measure the mean exponential contraction or expansion of a spheres axis describing a set of different initial conditions. In other words, they estimate the mean exponential divergence or convergence of nearby trajectories of the attractor. Mathematically, they are defined by logarithm of the eigenvalues of the matrix which determine the development of trajectories starting in an infinitesimal neighborhood of a fiducially trajectory[29,30]. In general, an n-dimensional system has n-Lyapunov exponents. The set of all Lyapunov exponents gives the average rate of volume growth under the flow in the systems phase-space. Given a continuous dynamical system in an n-dimensional phase space, we monitor the long-term evolution of an infinitesimal n-sphere of initial conditions; the sphere will become an n-ellipsoid due to the locally deforming nature of the flow. The ith one-dimensional Lyapunov exponent is then defined in term of the length of the ellipsoidal principal

axis $P_i(t)$:

$$\lambda_i = \lim_{t \rightarrow \infty} \frac{1}{t} \log_2 \frac{P_i(t)}{P_i(0)} \quad (10)$$

where the λ_i are ordered from largest to smallest.

We are particularly interested in the largest Lyapunov exponent λ_1 : this is a direct measure of the predictability of the time course of a system. It stands for the average rate of divergence in phase-space of two adjacent trajectories. For example, periodic systems have a λ_1 of zero. The case $\lambda_1 > 0$ can be used for the identification of chaotic systems. Since stochastic signals may reveal positive Lyapunov exponents as well, it is necessary to further distinguish both cases by checking for nonlinear predictability.

The most widespread method, used to compute the Lyapunov exponent was provided by Wolf and colleagues[31]. From the delayed time series two adjacent trajectories are observed over time. From the average divergence rate $\lambda_1 (= \lambda_{\max})$ is extracted. The Lyapunov exponents were estimated by Wolf's algorithm with the fixed evolution time program for λ_1 . Determination of the largest Lyapunov exponents indicates the sensitive dependence on initial conditions. The positive values of the Lyapunov exponents mean that the underlying control mechanisms are nonlinearly coupled and reveal chaotic behavior.

3.2.4 Recurrence plot

Recurrence plot are a useful processing parameter since they provide a comprehensive view of the dynamic courses within a time series signal. A recurrence plot is a 2-dimensional $N \times N$ pattern of points where N is the number of embedding vectors \vec{x}_i obtained from the delay coordinates of the time series signal. A point i, j in this plot is set if

$$\vec{x}_j \in U_i \cap \{\vec{x}_k \mid r > \|\vec{x}_i - \vec{x}_k\|\} \quad (11)$$

where for some partition on the real numbers P_i is the probability to find a time series value in the i th interval, and $P_{ij}(\tau)$ is the joint probability that an observation falls into the i th interval and the observation time γ later falls into the j th time. In theory this expression has no systematic dependence on the size of the partition elements and can be quite easily computed. There exist good arguments that if the time delayed mutual information exhibits a marked minimum at a certain value of γ , then this is a good candidate for a reasonable time delay[38,39].

3.4 Mode analysis

This part is principal component analysis (PCA)

parameter. PCA is a statistical technique falling under the general title of factor analysis[40]. The purpose of PCA is to identify the dependence structure behind a multivariate stochastic observation in order to obtain a compact description of it[41, 42].

Singular Value Decomposition (SVD) is used to derive the PCA. Multi-channel signal can be expressed by a P (time points) \times N (channels) matrix, E , and decomposed as a product of three matrixes, $E = USV^T$, where U is an $P \times N$ matrix such that $U^T U = 1$, S is an $N \times N$ diagonal matrix, and V is $N \times N$ matrix such that $V^T V = V V^T = 1$. If E is an input signal of N channels and P time points, U contains its N normalized principal component waveforms which are linearly decorrelated and can be remixed to reconstruct the original signal. S contains the N amplitudes of the N principal component waveforms. We can define the non-normalized principal component waveforms as the columns of $P = US$. PCA finds orthogonal directions of greatest variance in the data[43].

IV. ICA Module

Independent component analysis (ICA) has received attention because of its potential signal processing applications such as speech enhancement systems, telecommunications and medical signal processing. The goal of ICA is to recover independent sources given only sensor observations that are unknown linear mixtures of the unobserved independent sources signals. In contrast to correlation-based transformations such as PCA, ICA reduces the statistical dependencies of the signals, attempting to make the signal as independent as possible. Many researchers in neural networks and statistical signal processing have studied the blind source separation problem. One of them, we applied extended infomax algorithm in Bell and Sejnowski[44].

4.1 Extended ICA Algorithm

Assume that there is an M -dimensional zero-mean vector $s(t) = [s_1(t), \Lambda, s_M(t)]^T$, such that the components $s_i(t)$ are mutually independent. The vector $s(t)$ corresponds to M independent scalar-valued source signals $s_i(t)$. We can write the multivariate p.d.f. of the vector as the product of marginal independent distributions.

$$p(s) = \prod_{i=1}^M p_i(s_i) \quad (14)$$

A data vector $x(t) = [x_1(t), \Lambda, x_N(t)]^T$ is observed at such time point t ,

$$x(t) = As(t) \quad (15)$$

where A is a full rank $N \times M$ scalar matrix. As the components of the observed vectors are no longer independent, the multivariate p.d.f. will not satisfy the p.d.f. product equality. We shall consider the case where, the number of sources is equal to the number of sensors $N = M$. If the components of $s(t)$ are such that at most one source is normally distributed then it is possible to extract the sources $s(t)$ from the received mixture $x(t)$ [45]. The mutual information of the observed vector is given by the Kullback-Leibler(KL) divergence of the multivariate density from the product of the marginal densities:

$$I(x) = \int p(x) \log \frac{p(x)}{\prod_{i=1}^N p_i(x_i)} dx \quad (16)$$

The mutual information will always be positive and will only equal zero when the components are independent[46].

The goal of ICA is to find a linear mapping W such that the unmixed signal u

$$u(t) = W(t) = WAs(t) \quad (17)$$

as statistically independent. The sources are recovered up to scaling and permutation. There are many ways for learning W . Discussed earlier, Bell and Sejnowski[44] independently derived stochastic gradient learning rules for this maximization and applied them, respectively to time series analysis and blind separation of sources.

Consider the joint entropy of two nonlinearly transformed components of y :

$$H(y_1, y_2) = H(y_1) + H(y_2) - I(y_1, y_2) \quad (18)$$

where $y_i = g(u_i)$ and $g()$ is an invertible, bounded nonlinearity. The nonlinearity function provides, through its Taylor series expansion, higher-order statistics that are necessary to establish independence. Maximizing this joint entropy involves maximizing the individual entropies, $H(y_1)$ and $H(y_2)$, while minimizing the mutual information, $I(y_1, y_2)$, shared between the two. Thus, maximizing $H(y)$, in general, minimizing $I(y)$.

When this latter quantity is zero, the two variable are statistically independent.

The algorithm attempts to maximize the entropy $H(y)$ by iteratively adjusting the elements of the square matrix, W using small batches of data vectors drawn randomly from $\{x\}$ without substitution, according to

$$\Delta W \propto \frac{\partial H(y)}{\partial W} W^T W = [I + \phi u^T] W \quad (19)$$

where $\phi_i = (\partial / \partial u_i) \ln(\partial y_{ii} / \partial u_i)$. The $(W^T W)$ natural gradient term[47] avoid matrix inversions and speeds convergence. The form of the nonlinearity $g(u)$ plays an

essential role in the success of the algorithm. The ideal form for $g(u)$ is the cumulative density function of the distributions of independent sources.

Note that although a nonlinear function is used in determining W , once the algorithm converges and W is found, the decomposition is a linear transformation, $u = Wx$. This extended infomax algorithm was used to analyze the nonlinear time series in this study[48].

V. Application to EEG

EEG is a complex and aperiodic time series, which is a sum over a very large number of neuronal dendritic potentials. It is an important problem to decide whether the EEG is filtered noise or a deterministic signal. If the EEG is deterministic, then we can extract a lot of information on brain dynamics from the EEG, and then study brain functions with dynamical models from the EEG. Thus, application of nonlinear dynamical measures of EEG has gained importance in understanding brain functions in various behavioral states. A detailed perspective of this new approach has been reviewed earlier[49]. We used our nonlinear time series analysis tool to analysis of normal state EEG. This EEG was evoked by five kinds of stimuli, such as non-stimulus, auditory, finger and toe movement and visual stimulus. According to the International EEG nomenclature 10/20 systems, the subjects were twenty normal people. We analyzed the characteristics of EEG using many parameters as shown below. And Fig. 4 shows the raw signal of EEG that we measured on 20 channels.

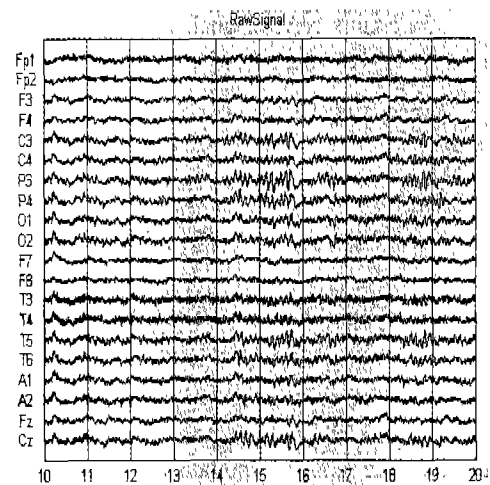


Fig. 4. EEG raw signal on 20 channels

5.1 Power spectrum and Bispectrum

The spectral analysis parameter used in study is performed by means of the linear analysis in which Fourier transformation algorithm produces the power and frequency information of brain waves that consist of a sinusoidal unit. But bispectrum includes the concept of phase coupling which is not contained in the power

spectrum. Thus, bispectrum is a method that analyzes a signal as the result of phase coupling interaction produced the nonlinear time series, such as EEG.

Fig.5 shows the analysis result of the power spectrum and bispectrum of the non-stimulus. As a result, power spectrum shows rate of the frequency band to the non-stimulus. And bispectrum appeared in the region of the phase coupling.

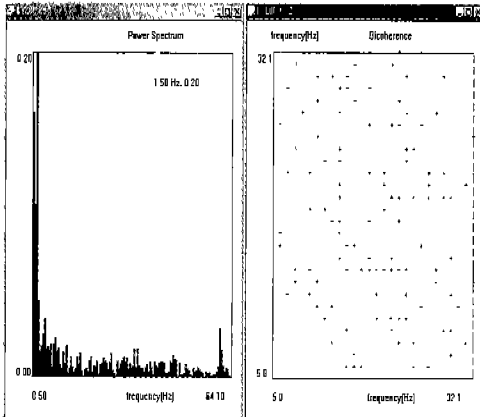


Fig. 5. Power spectrum and Bispectrum of the non-stimulus

5.2 Attractor

We have discussed earlier that attractor shows activity and collection the trajectories of perturbation on the system. Fig. 6 shows three kinds of the attractor. First attractor appeared in the EEG to the non-stimulus, second is auditory stimulus and last is visual stimulus. This attractor is shape of difference in each stimulus. In the case of visual stimulus, appeared in large trajectory than the different of stimulus.

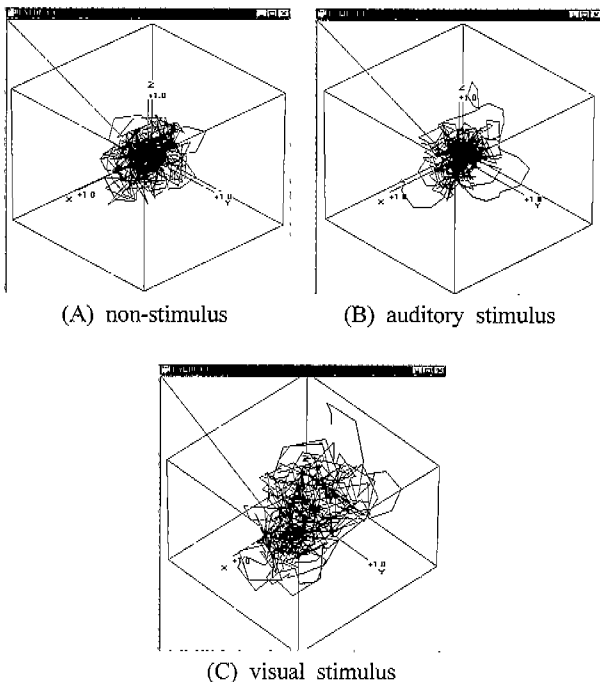


Fig. 6. Attractor according to each stimulus

5.3 Correlation Dimension

Correlation dimension is a measure of complexity. It estimates the degree of freedom of the EEG signal in our study, and further, it determines the number of independent variables that are necessary to describe the dynamics of the central nervous system. Table. 1 shows the correlation dimension of the five states stimuli, each non-stimulus, auditory, finger and toe movement and visual stimulus. As a result, correlation dimension has high value at part of the frontal lobe than the occipital lobe. And the auditory stimulus as noise has high value than the different stimuli. Also, Fig. 7 shows the analysis result of the correlation dimension according to the embedding dimension from 1 to 10.

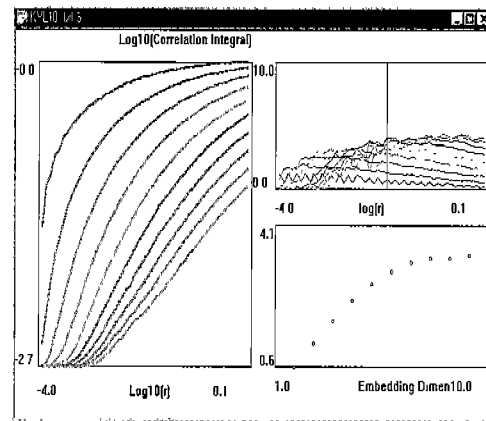


Fig. 7. Correlation dimension of the visual stimulus

Table 1. Correlation dimensions for the each stimulus

Channels	Non-stimulus	Auditory	Finger move	Toe move	Visual
Fp1	5.1	5.16	4.68	5.21	4.99
Fp2	4.87	5.15	5.02	5.16	5.09
F3	4.64	4.89	4.69	4.98	4.58
F4	4.58	4.89	4.42	4.84	4.49
C3	4.57	4.76	4.5	4.5	4.46
C4	4.61	4.69	4.47	4.8	4.48
P3	4.53	4.69	4.41	4.62	4.42
P4	4.51	4.63	4.43	4.57	4.46
O1	4.44	4.58	4.32	4.54	4.35
O2	4.22	4.7	4.54	4.39	4.45
F7	4.32	4.73	4.39	4.57	4.51
F8	4.12	4.46	4.25	4.75	4.48
T3	4.49	4.86	4.63	4.8	4.61
T4	4.49	4.81	4.55	4.82	4.54
T5	4.39	4.65	4.44	4.52	4.47
T6	4.4	4.67	4.45	4.46	4.44
A1	4.36	4.72	4.32	4.56	4.31
A2	4.18	4.71	4.37	4.49	4.35
Fz	4.55	4.77	4.52	4.82	4.54
Cz	4.63	4.6	4.5	4.87	4.27
mean	4.5	4.76	4.49	4.7	4.51

5.4 Lyapunov exponents

The Lyapunov exponents of EEG give the valuable information about the brain system in addition to correlation dimension. If a system is known to be deterministic, a positive Lyapunov exponents value can be taken as proof of a chaotic system. Fig. 8 shows the analysis result of the Lyapunov exponents according to the embedding dimension from 1 to 10. We can know that this EEG of the finger movement stimulus is deterministic chaotic signal from this result.

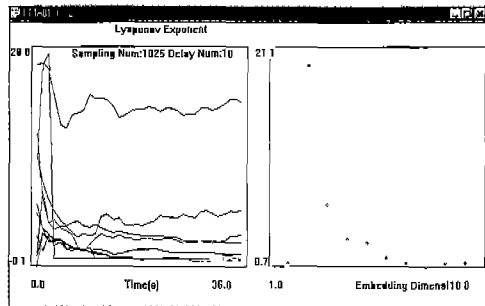


Fig. 8. Lyapunov exponents of the finger movement stimulus

5.5 Independent component analysis

We demonstrate the ability of the ICA algorithm to decompose multi-channel EEG data into temporally independent components by applying ICA to EEG data. Fig. 9(A) shows a 10-sec portion of the recorded EEG of the toe movement stimulus time series. Fig. 9(B),(C) shows the derived ICA component activations and scalp topographies for five selected ICA components. As a result, we can estimate the region of brain activity from each ICA components using topological map. This information can use a standard of judgment for removing the artifact.

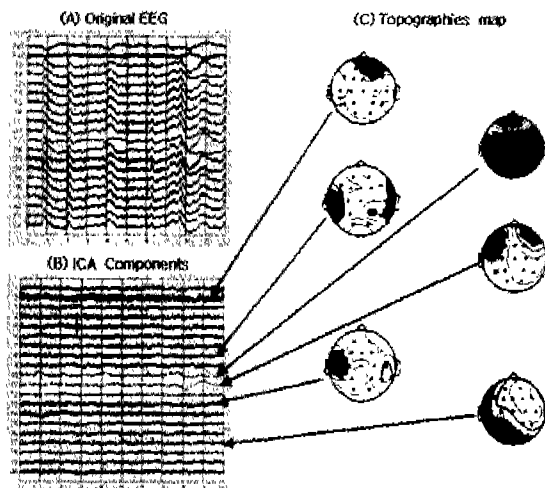


Fig. 9. (A) The EEG raw signal of toe movement stimulus (B) Corresponding ICA component (C) the scalp topographies for five selected ICA components

VI. Conclusion

In the clinical point, EEG is very important to diagnostic the brain disease as like dementia, epilepsy, Alzheimer, etc. However it was difficult to find some critical points in the EEG signals. It should be need a well-trained clinical instruction to analysis EEG signal in brain diseases. Most of the diagnosis of EEG in the clinic depends on the chart recoding and experience only. We developed the nonlinear time series analysis tool to improve the method of the diagnosis of EEG signals.

First, we show that this tool has the four modules, such as signal generation, processing, analysis and ICA module, respectively. Signal generation module consist of the Lorenz and Rosseler signal generation parts. Signal processing module consist of normalization, band pass filtering, surrogating, etc. Signal analysis & ICA module consist of the spectral analysis, phase space analysis, correlation analysis, and extended ICA module was implemented.

Second, we show the results of the analysis of the EEG that was evoked by various stimuli. Each stimulus is non-stimulus, auditory, visual, finger and toe movement stimulus, respectively. We could get the useful results for the various parameters and revealed the characteristics that are not shown at the recording chart.

Third, we show the ICA results and could get each independent component of EEG. And it was possible to estimate the position of where the independent components are occurs. Also, we could remove the various artifacts such as eye blinking, muscle, eye movement artifact, etc. After removed artifacts, we evaluate the nonlinear dynamics for the reconstructed EEG signals.

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