□ 研 究 論 文 □

韓國國防經營分析學會誌

第 27 券, 第 1 號, 2001. 7. 2.

High Resolution Linear Graphs : Graphical Aids for Designing Off-Line Process Control

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Abstract

Designing high quality products and processes at a low cost is central technological and economic challenge to the engineer. The combination of engineering concepts and statistical implementations offered by Taguchi's off-line design technique has proven to be invaluable. By examining some deficiencies in designs from the Taguchi's highly fractional, orthogonal main effect plan based on orthogonal arrays, alternative method is proposed. The maximum resolution or the minimum aberration criterion is commonly used for selecting 2^{n-m} fractional factorial designs. We present new high resolution (low aberration) linear graphs to simplify the complexity of selecting designs with desirable statistical properties. The new linear graphs approach shows a substantial improvement over Taguchi's linear graphs and other related graphical methods for planning experiment. The new set of linear graphs will allow the experimenter to maintain the simple approach suggested by Taguchi while obtaining the best statistical properties of the resulting design such as minimum aberration as a by-product without dependency on complicated computational algorithm or additional statistical training.

1. Introduction

The predominant focus on designing experiments in robust design problems using Taguchi's [7,8,9] proposed sets of linear graphs result in those designs with less desirable statistical properties than alternative classical designs. In the context of robust parameter design, the presence of interactions among variables is usually avoided by Taguchi. He emphasizes additivity of effects by choosing the responses and experimental variables carefully to avoid inducing interactions. Thus the purpose of

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Taguchi's original linear graphs is focused on main effects only experiments. By this reason, he does not consider any particular statistical properties of the resulting designs derived from his proposed sets of linear graphs but only focus on the relationships among columns in the interaction table for each orthogonal arrays so that the proposed sets of linear graphs generally have simple regular shapes. Those are group of lines and triangular patterns for the small sized array, and triangular, rectangular and polygonal shapes for the moderately large sized orthogonal arrays.

However, most of the underlying physical mechanism of industry often necessitates the experimental plans that interactions among the variables are inevitable. In the presence of interactions, the two-level orthogonal arrays such as L_8, L_{16}, L_{32} and L_{64} are good candidate designs and simple to implement interactions as opposed to the three-level (L_9, L_{27} and L_{81}) and the mixed-level orthogonal arrays (L_{12}, L_{18}, etc.). The three-level orthogonal arrays are derived from the Latin square, Graeco-Latin square, and hyper-Graeco-Latin square designs. However, it should be noted that the interaction terms of designs bias all the estimated coefficients and the estimated variance. The mixed-level orthogonal arrays allow a very limited set of interactions. For example, an L_{18} orthogonal array allows only one interaction between the first two columns. However, the interaction effects are not biased by the presence of quadratic effects in 2^{n-m} designs since they bias only the constant term. Thus the two-level orthogonal arrays which are equivalent to the regular 2^{n-m} fractional factorials are useful to plan such experiments that include interactions.

By observing the absence of a goodness criterion of a design in Taguchi's linear graphs, it is a standard discipline to consider first the problem of maximizing the resolution of design. Box and Hunter[2] defined the concept of resolution of a design as one way to classify designs. A design which has no c-factor effect is confounded with any other effects containing less than $R-c$ factors results in resolution R. That is, a resolution III design does not confound main effects with one another but does confound main effects with two-factor interactions, and a resolution IV design does not confound main effects with two-factor interactions but does confound two-factor interactions with one another. Resolution V designs have main effects that are clear of three-factor interactions and all-lower-order effects, while two-factor interactions are estimated clear of other two-factor interactions. The higher the resolution of an experimental design, the less confounding between main factors and interactions so that an experimenter can get more accurate conclusions.

Table 1 summarizes Taguchi's proposed sets of linear graphs and their corresponding fractional factorial designs for two-level orthogonal arrays. The graph numbers are from Taguchi[8]. Note that most of the resulting designs are resolution Ⅲ even though they could be improved of higher resolution. Since designs with the same resolution are not equally good, we need a more refined criterion called minimum aberration[4]. The aberration of a design is the number of words of minimal length in the defining relation for the design. A minimum aberration design is one which minimizes aberration.

Another critical deficiency originated by the absence of the considerations for statistical pro-perties in Taguchi's linear graphs is that the enumeration of his selection of graph is not exhaustive in the realms of the fractional factorial designs. For example, the six linear graphs that proposed by Taguchi among more than eight hundreds types of graphs[8] for the 16-run orthogonal array missed some important designs such as 2^{6-2} and 2^{9-5} designs. As Taguchi[9] mentioned, there exist quite many possible types of linear graphs due to combinatorial multiplicity of the representations for arcs and nodes given certain number of factors and their specified interactions. As the bottom line however, the minimal graphs which can represent all possible fractional factorial designs under each specified number of factors should be enumerated for each orthogonal arrays.

<Table 1> Taguchi's Linear Graphs and Corresponding Fractional Factorials

Orthogonal Array	Taguchi(1987)'s Graph Number	Fractional Factorials	Resolution
L_8	(1)(2)	2^{4-1}	Ⅳ
L_{16}	(1)	2^{5-1}	V
	(2)	2^{7-3}	Ⅲ
	(3)(4)(6)	2^{8-4}	Ⅲ
	(5)	2^{10-6}	Ⅲ
L_{32}	(1)	2^{11-6}	Ⅲ
	(2)	2^{13-8}	Ⅲ
	(9)	2^{14-9}	Ⅲ
	(3)(4)(6)	2^{15-10}	Ⅲ
	(7)(11)(12)(13)	2^{16-11}	Ⅲ
	(5)	2^{17-12}	Ⅲ
	(8)	2^{18-13}	Ⅲ
	(10)	2^{20-15}	Ⅲ
L_{64}	(5)	2^{17-11}	Ⅲ
	(1)	2^{23-17}	Ⅲ
	(2)(3)	2^{26-20}	Ⅲ
	(4)(9)	2^{27-21}	Ⅲ
	(6)(7)	2^{29-23}	Ⅲ
	(10)	2^{31-25}	Ⅲ
	(8)	2^{32-26}	Ⅲ

This paper proposes high resolution and low aberration linear graphs which rectify the statistical deficiencies of the resulting designs that result from Taguchi's set of two-level linear graphs. Those are the best among the large set of linear graphs in terms of the maximum resolution and minimum aberration criteria, and can accommodate most of the two-level fractional factorial designs. By adapting the goodness criteria such as maximum resolution and minimum aberration, we can obtain the design with good statistical properties as well as maintain the simplicity of implementation. In section 2, we derive generators which guarantee a design of such good criteria for each orthogonal arrays. We present high resolution linear graphs for two-level orthogonal arrays in section 3. Those are high resolution linear graphs for L_8, L_{16}, L_{32} and L_{64} orthogonal arrays, respectively. In section 4, we discuss practical advantages of our high resolution linear graphs related to Taguchi's set of linear graphs as well as the other related graphical methods in fractional factorial design.

2. Derivation of Generators for Two-Level Orthogonal Arrays

An orthogonal array is an $r \times c$ array of s distinct elements which has a balanced property that every pair of columns, contains all possible s^2 ordered pairs of elements with the same frequency. Since most of the two-level orthogonal arrays ($s=2$; L_4, L_8, L_{16}, L_{32} and L_{64}) are regular fractional factorials, we can exploit the relationship between these designs and their orthogonal arrays to construct alternate linear graphs with higher resolution. Note that an arrangement of p-factors of a complete factorial can generate a two-level orthogonal array with 2^p rows, where $p=2,3,4,\cdots$.

We can obtain the generators of the orthogonal array corresponding to the 2^{n-m} fractional factorials as follow.

Step 1. Write the 2^p-1 treatment combinations in standard(Yates's) order. We call these column notation.

Step 2. Locate the p factors corresponding to the basic columns in the orthogonal array in reversed order. That is, for example, the first factor(say **a**) matches the column of 2^{p-1} symbols of **0** and 2^{p-1} symbols of **1** successively, which is associated with the pth factor in factorial plan.

Step 3. Match the labels of the column notation represented in Step 1 with the non-basic columns of the orthogonal array by modulo-2 arithmetic. That is

$$ab \cdots k \equiv a+b+\cdots+k \ (\text{mod}2).$$

Step 4. The generators of design for the orthogonal array corresponding to the 2^{n-m} fractional factorial can be obtained by equating the m extra factors to the appropriate column notation.

We illustrate these steps by obtaining the generator of the L_8 orthogonal array correspond-ing to the 2_{IV}^{4-1} factorial design. Here, we have $n=4$, $m=1$ and $p=n-m=3$.

Step 1. The column notation is : **a b ab c ac bc abc** .

Step 2. The three factors **a, b** and **c** correspond to the three basic columns in the L_8 orthogonal array and are located in column 1 **(a)** : $(00001111)^T$, column 2 **(b)** : $(00110011)^T$ and column 4 **(c)** : $(01010101)^T$, respectively.

Step 3. Match the non-basic column notations to the appropriate columns in orthogonal array by modulo-2 arithmetic. Those are column

3 **(ab)** : $(00111100)^T$, column **5** **(ac)** : $(01011010)^T$, column **6** **(bc)** : $(01100110)^T$ and column **7** **(abc)** : $(01101001)^T$, respectively.

***Step** 4*. The generator which can generate a design of resolution Ⅳ is **d=abc**. Thus we have to assign the extra factor **d** to the column **7** in order to obtain a design of resolution Ⅳ.

We call those columns that can be obtained a resolution Ⅳ design by assigning them as resolution Ⅳ columns. In the above example, the resolution Ⅳ columns are three basic columns 1, 2, 4 and an extra column 7. By knowing that assignment for main factors to the resolution Ⅳ columns only will result in designs of higher resolution if the number of factors does not exceed that of resolution Ⅳ columns, we can select such linear graphs that ensure the maximum resolution property. To ensure the minimum aberration property, we adapt the defining contrasts from Fries and Hunter[4]. We denote **a**,**b**, ⋯,**z** consecutively as the factor representations in the orthogonal array, and use **A**,**B**, ⋯,**Z** if the number of factors exceeds 26, and α, β, γ, ⋯ if they exceed 52.

3. High Resolution Linear Graphs for Two-Level Design

In this section we present a new set of linear graphs that rectifies the deficiencies of Taguchi's approach without losing its simplicity. The proposed construction and enumeration of the set of high resolution (low aberration) linear graphs for each two-level orthogonal arrays involve five steps.

***Step** 1*. Identify the generators by using the method in section 2.

***Step** 2*. Identify the resolution Ⅳ columns.

***Step** 3*. Enumerate all the possible corresponding fractional factorial designs. For each design, identify the maximum number of two-factor interactions that can be estimated.

***Step** 4*. For each design that the number of main factors is not greater than the number of resolution Ⅳ columns, identify the generators by the minimum aberration criterion.

***Step** 5*. For each design that can be achieved higher resolution, locate those nodes corresponding to the columns identified in step 4. Enumerate the possible patterns of arcs which can be represented all the estimable interactions. List only one graph from the set of graphs which have the same shape but different association if they exist.

3.1 $L_8(2^7)$ Orthogonal Array

By using the method in the previous section, the generators of the orthogonal array $L_8(2^7)$

can be obtained as follows; **d=ab**, **e=ac**, **f=bc** and **g=abc.** The confounding patterns are $l_a = a + bd + ce + fg$, $l_b = b + ad + cf + eg$, $l_c = c + ae + bf + dg$, $l_d = d + ab + ef + cg$, $l_e = e + ac + df + bg$, $l_f = f + bc + de + ag$, and $l_g = g + cd + be + af$. Since each main effect is confounded with three two-factor interactions, we can have at most four factors to obtain a design of resolution Ⅳ. The highest possible resolution that could be obtained using L_8 orthogonal array for various requirements set is summarized in Table 2. Note that the high resolution linear graphs for an L_8 orthogonal array in Figure 1-a, b are exactly same as Taguchi's. The generator **d=abc** also ensures a minimum aberration. Taguchi's two linear graphs for L_8 are comprehensive and accommodate both resolution Ⅲ and Ⅳ.

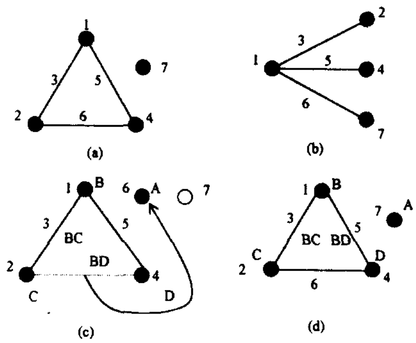
The experimenter under the Taguchi's procedure however, by chance, must select the appropriate linear graph without knowledge of de-sirable confounding patterns of interactions and then given the right choice must by chance again assign certain factors to certain columns when the requirements set has up to four main factors and up to three two-factor interactions in order to generate a resolution Ⅳ design.

<Table 2> Highest Possible Resolution by L_8

Case Number	Requirements Set		Highest Possible Resolution
	No. of Main Factors	Max no. of 2-factor Interactions	
1	3	3	Ⅴ
2	4	3	Ⅳ
3	5	2	Ⅲ
4	6	1	Ⅲ
5	7	0	Ⅲ

For example in Figure 1-c, Taguchi[8] considered an experimental plan which has four two-level factors (A, B, C, D) and 2 two-factor interaction effects (BC, BD). The main factors A, B, C and D were assigned to columns 6, 1, 2 and 4 respectively. Thus the interaction BC is associated with columns 3 and BD is associated with column 5. The generators of the resulting design is A=BD and the confounding patterns are $l_A = A + BD$, $l_B = B + AD$, $l_C = C$, $l_D = D + AB$, and $l_{BC} = BC$. This results in the three main factors being confounded with two-factor interactions so that the design is resolution Ⅲ.

However, the assignments of main factors to the resolution Ⅳ columns only in our approach will automatically ensure the association of interactions to the arcs of the linear graph so that the resulting design is of high resolution without trial and error(Figure 1-d). In order to obtain a design of resolution Ⅴ with three factors and up to three two-factor interactions, the experimenter must select the first linear graph(Figure 1-a) and do not use a separate node '7'.



[Figure 1] Linear Graphs for L_8

3.2 $L_{16}(2^{15})$ Orthogonal Array

The generators for the $L_{16}(2^{15})$ orthogonal array are $\boldsymbol{j = bd}$, $\boldsymbol{e = ab}$, $\boldsymbol{f = ac}$, $\boldsymbol{g = bc}$, $\boldsymbol{h = abc}$, $\boldsymbol{i = ad}$, $\boldsymbol{k = abd}$, $\boldsymbol{m = acd}$, $\boldsymbol{k = ad}$, $\boldsymbol{n = bcd}$, and $\boldsymbol{o = abcd}$. Since each main effect is confounded with seven two-factor interactions, the experimenter can use at most eight factors to obtain a design of resolution Ⅳ. The eight resolution Ⅳ columns in L_{16} orthogonal array are 1, 2, 4, 7, 8, 11, 13 and 14. The highest possible resolution designs that could be obtained for the L_{16} orthogonal array are listed in Table 3. As seen, the designs with more than eight factors are only resolution Ⅲ. A design in five factors, with up to ten two-factor interactions, of resolution Ⅴ can be obtained by selecting the first linear graph in the set of six linear graphs proposed by Taguchi for L_{16} orthogonal array.

However, none of Taguchi's set of L_{16} linear graphs result in a resolution Ⅳ design for experimental situations described by cases 2, 3 and 4 in Table 3. Additional graphs from large set are needed to replace the six suggested by Taguchi to capture designs with better statistical properties.

For the case 2 which Taguchi does not consider, a design in six factors with up to seven two-factor interactions, the two linear graphs in Figure 3 result in a design of resolution Ⅳ. The generators for the high resolution linear graphs are **e=abc** and **f=bcd**, which are same as those for the minimum aberration 2^{6-2}_{IV} design. The resulting confounding patterns are $l_a = a$, $l_b = b$, $l_c = c$, $l_d = d$, $l_e = e$, $l_f = f$, $l_{ab} = ab + ce$, $l_{ac} = ac + be$, $l_{bc} = bc + ae$, $l_{ad} = ad + ef$, $l_{bd} = bd + cf$, $l_{cd} = cd + bf$ and $l_{de} = de + af$. Hence, all main effects are clear from two-factor interactions and two-factor interaction effects are aliased with each other.

For the case 3 which Taguchi does not consider as in the case 2, an experiment in seven factors with up to seven two-factor interactions, the linear graph in Figure 4 generates a resolution Ⅳ design. The generators are $\boldsymbol{e = abc}$, $\boldsymbol{f = acd}$ and $\boldsymbol{g = bcd}$, which are used to construct the 2^{7-3}_{IV} minimum aberration design.

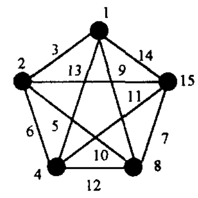
<Table 3> Highest Possible Resolution by L_{16}

Case Number	Requirements Set		Highest Possible Resolution
	No. of Main Factors	Max no. of 2-factor Interactions	
1	5	10	V
2	6	7	IV
3	7	7	IV
4	8	7	IV
5	9	6	III
6	10	5	III
7	11	4	III
8	12	3	III
9	13	2	III
10	14	1	III
11	15	0	III

For the case 4, experiments in eight main factors with up to seven two-factor interactions which is very frequently used in practical applications, Figure 5 shows a set of seven high resolution linear graphs. The generators are

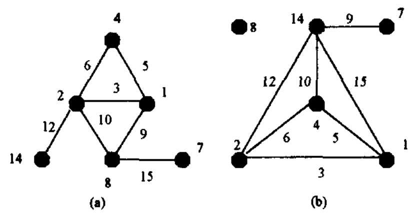
$e = abc$, $g = abd$, $h = acd$, and $i = bcd$,

which are essentially same as in Box, Hunter and Hunter[3] and lead to the design of minimum aberration also. Note that the linear graphs corresponding to the high resolution (Figure 5 a, b, c) and low resolution[8] have the same shape, but differ in the assignment of factors to column of the orthogonal array. When there are more than eight factors, these graphs can be used but they result in resolution III design. It should be noted that our collection of linear graphs from Figure 2 to Figure 5 are exhaustive for the 16-run design. Furthermore,

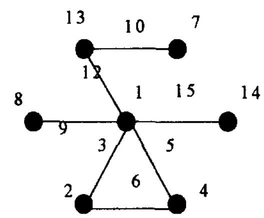


[Figure 2] Resolution V Linear Graph with Five Factors for L_{16}

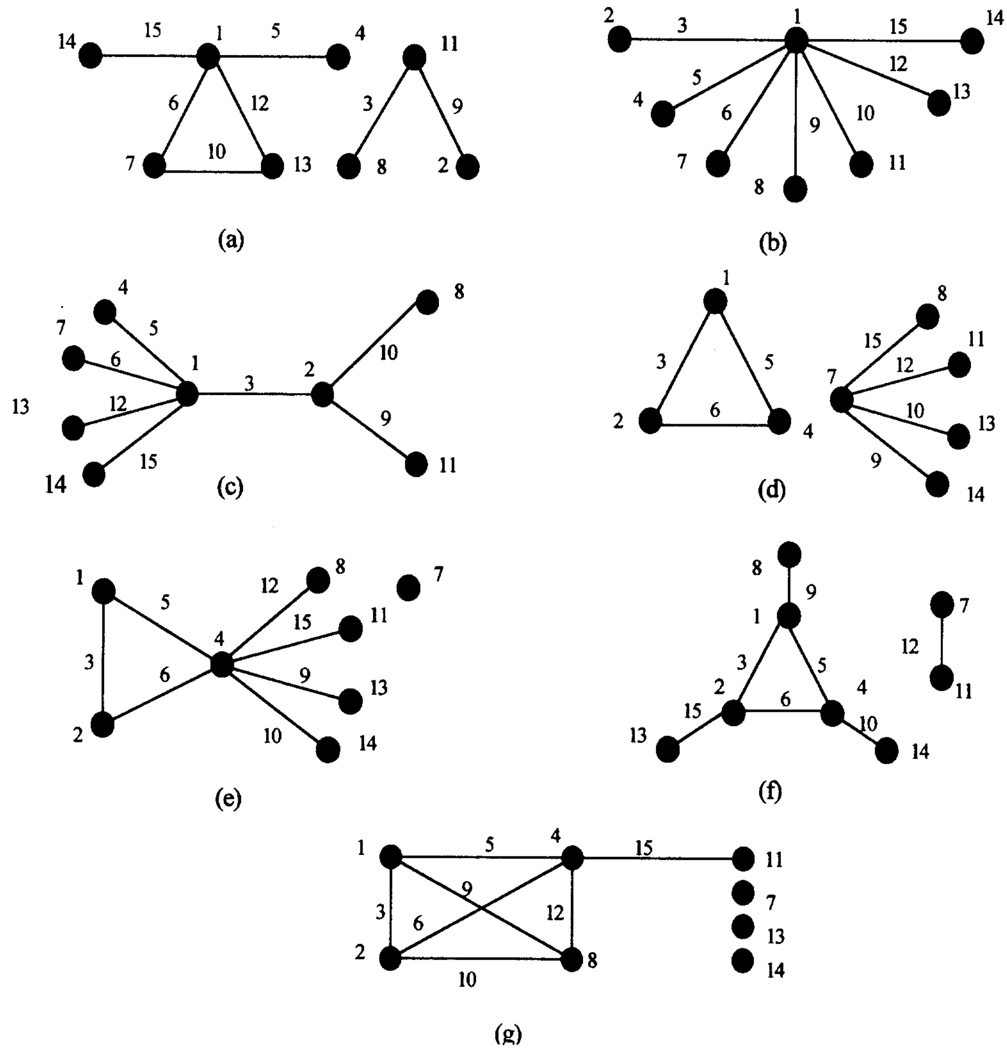
in addition to maintaining high resolution and low aberration properties, the proposed set of graphs is quite appealing to accommodate the various patterns of two-factor interactions in practical applications.



[Figure 3] Resolution IV Linear Graphs with Six Factors for L_{16}



[Figure 4] Resolution IV Linear Graph with Seven Factors for L_{16}



[Figure 5] Resolution Ⅳ Linear Graphs with Eight Factors for L_{16}

3.3 $L_{32}(2^{31})$ Orthogonal Array

By adapting the method in section 2, the generators for the $L_{32}(2^{31})$ orthogonal array are **f=ab, g=ac, h=bc, i=abc, j=ad, k=bd, l=abd, m=cd, n=acd, o=bcd, p=abcd, q=ae, r=be, s=abe, t=ce, u=ace, v=bce, w=abce, x=de, y=ade, z=bde, A=abde, B=cde, C=acde, D=bcde** and **E=abcde**. Since main effect is confounded with fifteen two-factor interactions, the experimenter can have at most sixteen factors to obtain a design of resolution Ⅳ. The 16 resolution Ⅳ columns are 1, 2, 4, 7, 8, 11, 13, 14, 16, 19, 21, 22, 25, 26, 28 and 31.

The hierarchical relationship between increasing sized orthogonal arrays can be used to generalize undesirable aspects of designs result from Taguchi's sets of linear graphs in an optimal sense. That is, most of the designs by his set of linear graphs are of resolution Ⅲ, which could be better designs in terms of resolution or aberration if hierarchical relationship of higher resolution were maintained between increasing sized orthogonal arrays. Linear graphs for a large sized orthogonal array, like an L_{32}, highlight this phenomenon. The highest possible resolution designs that could be obtained for an L_{32} orthogonal array are listed in Table 4.

From the set of linear graphs for an L_{32} orthogonal array given by Taguchi[8], none of his thirteen linear graphs can generate a resolution Ⅳ design since they do not include high resolution L_{16} linear graphs as a part. However, ten of his thirteen linear graphs could be improved to a resolution Ⅳ by choosing different experiment with more than sixteen factors will be a resolution Ⅲ in nature.

First, an experimental study in six factors with up to fifteen two-factor interactions, a design of resolution Ⅳ can be obtained by a linear graph in Figure 6. The generator of the design is **f=abcde**. Similarly, linear graphs which do not included in this paper due to the space limit can be used to obtain a design of resolution Ⅳ for the cases 2, 3, 4 and 5 in Table 4. For the case 2, a design in seven factors with up to eighteen two-factor interactions, the corresponding linear graph will generate a resolution Ⅳ design by the generators **f=acde** and **g=bcde**. The generators are **f=abc**, **g=abd**, **h=bcde** for the case 3 and **f=bcde**, **g=abce**, **h=abde**, **i=acde** for the case 4, and **f=abcd**, **g=abce**, **h=abde**, **i=acde** and **j=bcde** for the case 5. Note that all high resolution linear graphs for the cases 2, 3, 4 and 5 include a pentagonal shape which is a resolution V linear graph in the L_{16} orthogonal array(Figure 2) as a part. From the case 6 to 11, all the generators can generate a resolution Ⅳ design are consisted of three-letter. For example, for a case 6 which has eleven factors with up to fifteen two-factor interactions, the generators **f=abc**, **g=bcd**, **h=cde**, **i=acd**, **j=ade** and **k=bde** guarantees a resolution Ⅳ design.

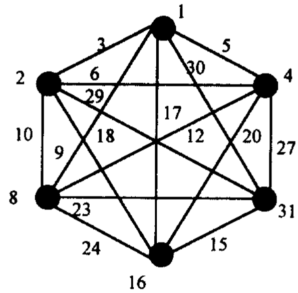
We present high resolution linear graphs only for the case 11 which has sixteen factors with up to fifteen two-factor interactions. The generators are **f=abc**, **g=abd**, **h=acd**, **i=bcd**, **j=abe**, **k=ace**, **l=bce**, **m=ade**, **n=bde**, **o=cde** and **p=abcde**. The fourteen high resolution linear graphs in the Figure 7 represent the best linear graphs which can accommodate a variety of desired interaction patterns as well as generate a design of resolution Ⅳ.

As an example, an experimenter wants to plan a design has a total of 15 factors, all two-factor interactions among the four specific factors(call group A) and among the different three specific factors (call group B) and between one of the three factors in the latter and the remaining six factors (call group C).

<Table 4> Highest Possible Resolution by L_{32}

Case Number	Requirements Set		Highest Possible Resolution
	No. of Main Factors	Max no. of 2-factor Interactions	
1	6	15	Ⅳ
2	7	18	Ⅳ
3	8	20	Ⅳ
4	9	21	Ⅳ
5	10	21	Ⅳ
6	11	15	Ⅳ
7	12	15	Ⅳ
8	13	15	Ⅳ
9	14	15	Ⅳ
10	15	15	Ⅳ
11	16	15	Ⅳ
12	17	14	Ⅲ
13	18	13	Ⅲ
14	19	12	Ⅲ
15	20	11	Ⅲ
16	21	10	Ⅲ
17	22	9	Ⅲ
18	23	8	Ⅲ
19	24	7	Ⅲ
20	25	6	Ⅲ
21	26	5	Ⅲ
22	27	4	Ⅲ
23	28	3	Ⅲ
24	29	2	Ⅲ
25	30	1	Ⅲ
26	31	0	Ⅲ

As we do in the requirement graph, the linear graph in Figure 7-f will satisfy this experimental requirements. We assign the four factors in group A to the rectangular shaped part in a linear graph at Figure 7-f, three factors in group B to the triangular shaped part and six factors in group C to the upper part of the triangular shaped part, and the remaining two fact-



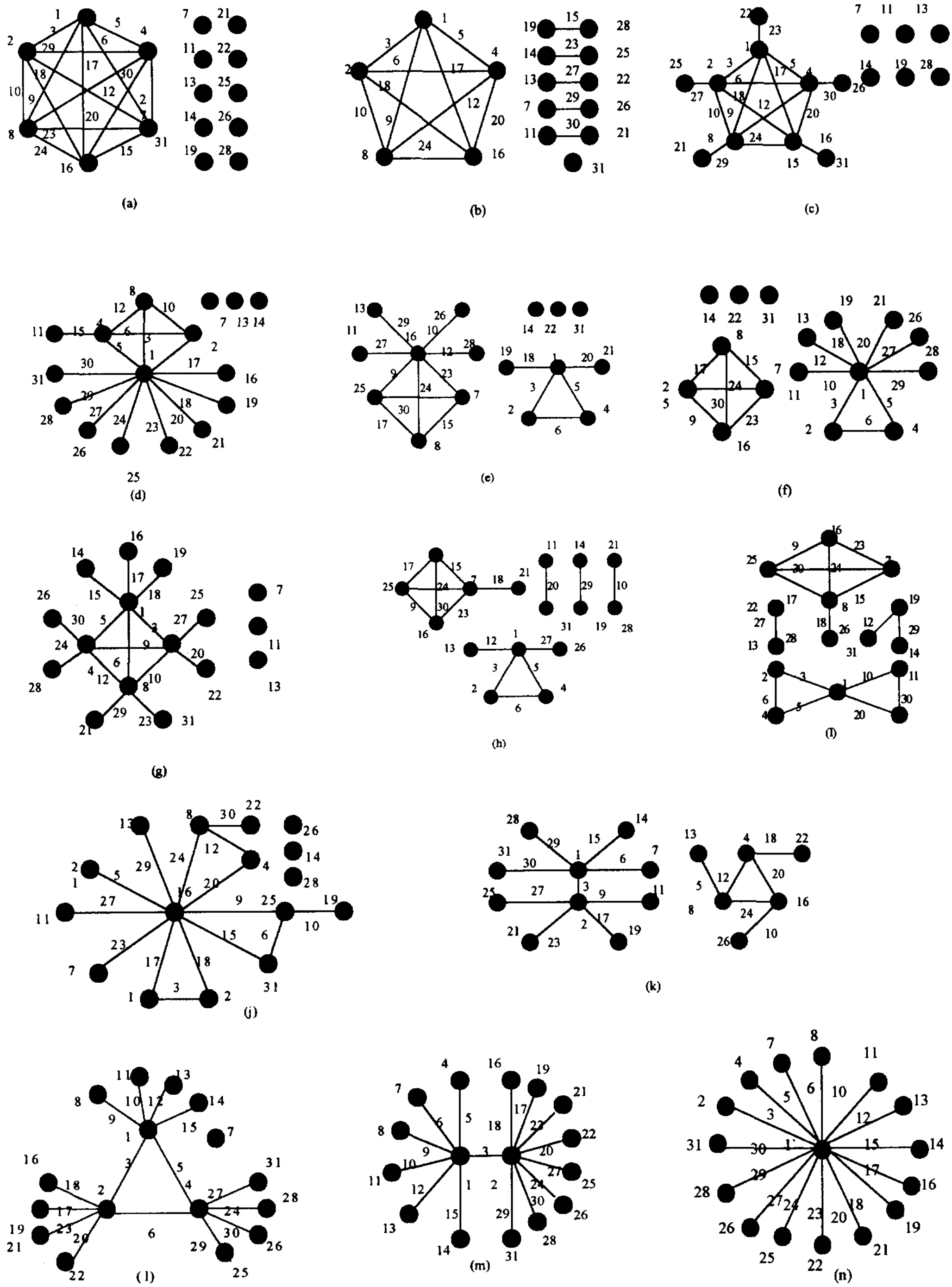
[Figure 6] Resolution Ⅳ Linear Graph with Six Factors for L_{32}

ors to any two nodes of the separate three nodes.

It should be noted that a resolution Ⅳ design for the cases 6 to 10 which have eleven to fifteen factors with up to fifteen two-factor interactions can be easily obtained from the set of fourteen high resolution linear graphs in Figure 7. The experimenter can choose a linear graph which satisfies a desired interaction pattern and then ignores the remaining nodes or arcs. Furthermore, from the cases 12 which are designs of resolution Ⅲ, we can easily assign all main factors and interactions from those linear graphs by the appropriate modification rules. Thus, for the two-level designs with 32-runs, our collection of graphs is exhaustive as well as the better than Taguchi's in terms of the maximum resolution or minimum aberration.

3.4 $L_{64}(2^{63})$ Orthogonal Array

The generators for the $L_{64}(2^{63})$ orthogonal array are $g=ab, h=ac, i=bc, j=abc,\ k=$



[Figure 7] High Resolution Linear Graphs for L_{32}

$ad, l = bd$, $m=abd$, $n=cd$, $o=acd$, $p=bcd$, $q=abcd$, $y=de$, $z=ade$, $A=bde$, $B=abde$, $C=cde$, $D=acde$, $E=bcde$, $F=abcde$, $G=af$, $H=bf$, $I=abf$, $J=cf$, $K=acf$, $L=bcf$, $M=abcf$, $N=df$, $O=adf$, $P=bdf$, $Q=abdf$, $R=cdf$, $S=acdf$, $T=bcdf$, $U=abcdf$, $V=ef$, $W=aef$, $X=bef$, $Y=abef$, $Z=cef$, $\alpha = acef$, $\beta = bcef$, $\gamma = abce$, $\delta = def$, $\varepsilon = bdef$, $\zeta = abdef$, $\eta = cdef$, $\theta = acdef$, $\iota = bcde$, $x = abcdef$.

Since each main effect is confounded with 31 two-factor interactions, the experimenter can have at most thirty two factors to obtain a design of resolution Ⅳ. Including the 16 columns in lower sized array L_{32}, the sixteen additional columns 32, 35, 37, 38, 41, 42, 44, 47, 49, 50, 52, 55, 56, 59, 61 and 62 are the resolution Ⅳ columns in L_{64} orthogonal array. The highest possible resolution designs can be listed same as the lower-sized orthogonal arrays.

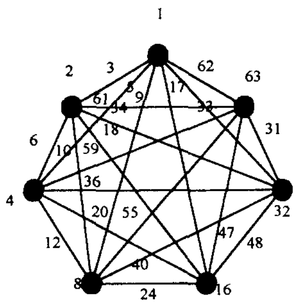
First, for seven factors with up to 21 two-factor interactions, a design of resolution Ⅶ can be obtained by choosing a linear graph in Figure 8. The generator of the design is **g=abcdef**. Similarly, up to nine factors, we can easily obtain the resolution Ⅳ or V design by the single linear graph[6].

From the design which has more than nine factors, no single polygonal shaped linear graph exists such that all lines can represent their interactions by connecting every pair of nodes. Hence, by using the lower-sized orthogonal array's (e.g. L_{32}) high resolution linear graphs, we can easily obtain a type of graph having the high resolution property. For example, a design which has 10 factors and with up to 39 two-factor interactions, the generators **g=abce, *h=abde, i=acdf*** and **j=bcdf** can generate a design of resolution Ⅳ.

The high resolution linear graph for this case can be obtained by adding these four nodes (23, 27, 45, 46) to the high resolution linear graph with six factors for L_{32} orthogonal array in Figure 6. The fifteen resulting high resolution linear graphs[6] are not listed in this paper due to the space limit. However, the set of these linear graph has a complex system of internal lines so that it seems to be impractical.

Similarly, for the 64-runs design cases until 21 factors with up to 41 two-factor interactions whose generators are consisted of extra variables with all interaction columns containing a four-letter word, the linear graphs that can be implemented designs of high resolution have no practical attraction. In order to accommodate all the maximum number of two-factor interactions, all graphs are represented by a single polygonal shape which has a complex system of lines that cross each other. We believe that this is not a serious limitation of high resolution linear graphs because which the number of two-factor interactions to be studied exceed twice that of main factors are not common. Furthermore, two-level fractional factorial experiments with 8, 16 and 32 runs are most widely used. From the 64-runs design with 22 factors to 32 factors and with up to all 31 two-factor interactions, whose generators include all three-letters, we can use the set of fifteen resolution Ⅳ linear graphs in [6].

Similarly in the L_{32} orthogonal array, an experimenter can easily obtain a design of resolution Ⅳ by ignoring the extra nodes after identifying linear graph which satisfies the required pattern of two-factor interactions for the design cases that have less than 32 factors. For the 64-runs design with more than 32 factors, the resolution Ⅲ design can be easily constructed by using the appropriate modification rules. Hence, for the two-level designs with 64 runs, our collection of high resolution linear graphs are almost exhaustive except for a few cases which require a large amount of estimation



[Figure 8] Resolution Ⅶ Linear Graph with Seven Factors for L_{64}

for two-factor interactions with relatively small number of factors.

4. Comparison with Taguchi's and Other Related Graphical Methods

The proposed set of new high resolution linear graphs eliminates most of deficiencies of designs from the Taguchi's set of linear graphs. First, good statistical properties of the designs which his graphs do not consider are guaranteed. Most of the designs for his linear graphs of resolution Ⅲ are improved to those of the maximum resolution or minimum aberration criteria.

Secondly, the proposed sets of high resolution linear graphs are quite exhaustive to cover almost of the important designs which are missed by Taguchi's collection of linear graphs, such as 2^{6-2}, 2^{9-5}, 2^{12-7}, 2^{7-1}, 2^{8-2} and so on. The only impractical designs in our collection of graphs are those of the 64-runs from 10 factors to 21 factors with considerably large amount of interactions as discussed in the last section.

By comparing the other graphical approaches related to this subject, we can see that it is difficult to make a practical approach for the above design cases that involve an implementation of assignments of all interactions whose number exceeds twice that of main factors. The interaction graph, which is a graph version of the interaction table, is suggested by Kacker and Tsui[5]. As noted by the authors, the interaction graph method is difficult to implement for more than 16-runs. The nonisomorphic graph is suggested by Wu and Chen[10]. The design is represented by searching for the nonisomorphic graph from the feasible graphs set that is required for a preliminary stage under the minimum aberration criterion, which is isomorphic to the required linear graph that representing all main effects and two-factor interactions in the requirement set by nodes and arcs same as in Taguchi's procedure.

For the 16-run design, all three methods provide a complete list of fractional factorial designs. For the 32-run design, the interaction graph method cannot be implemented, the nonisomorphic graph method can handle up to 10 factors by manual version, but our high resolution linear graph method provides a complete solution for the all cases. For the 64-run design, Wu and Chen's method can handle up to only 11 factors. Our proposed method can handle up to 10 factors and also more than 21 factors, respectively.

Thirdly, the proposed sets of high resolution linear graphs for each orthogonal arrays are very efficient tools to estimate the various selected patterns of two-factor interactions which are known to be important in the context of quality engineering. By considering the total number of graphs which can match all the subsets of specified interactions is too large to be enumerated even in a single small-sized array, our collection of linear graphs fairly covers practical needs of the experimenter.

Furthermore, the method to implement for constructing design is not changed from the framework of Taguchi's approach, which is very straightforward to assign factors to the columns so that no additional computational effort is required to obtain the designs of good overall statistical properties.

References

[1] Box, G.E.P., "Signal-to-Noise Ratios, Performance Criteria, and Transformations", Technometrics, Vol.30, pp.1-17, 1988.

[2] Box, G.E.P. and Hunter,T.S., "The 2^{k-p} Fractional Factorial Designs", Technometrics, Vol.3, pp.311-352, 1961.

[3] Box,G.E.P., Hunter,W.G. and Hunter,J.S., Statistics for Experimenter, John Wiley and Sons ,Inc. New York, NY, 1978.

[4] Fries, A. and Hunter,W.G., "Minimum Aberration 2^{k-p} Designs", Technometrics, Vol

22, pp.601-608, 1980.

[5] Kacker, R.N. and Tsui, K.L., "Interaction Graphs Graphical Aids for Planning Experiments", Journal of Quality Technology, Vol.22, pp.1-14, 1990.

[6] Lee, S.H., "Recent Developments in Off-Line Quality Control", Journal of Statistical Planning and Inference, Vol.25, pp.26-37, 1991.

[7] Taguchi, G., Introduction to Quality Engineering, Asian Productivity Organization, Tokyo, Japan, 1986.

[8] Taguchi, G., Systems of Experimental design, 2 Vols., Translated and Published in English by UNIPUB, New York, NY, 1987.

[9] Taguchi, G. and Wu, Y., Introduction to Off-Line Quality Control, Central Japan Quality Control Association, Nagoya, Japan, 1980.

[10] Wu,C.F.J. and Chen,Y., "Graph-Aided Assignment of Interactions in Two-Level Fractional Factorial Designs",The Institute for Improvement in Quality and Productivity Research Report RR-91-10, University of Waterloo, Waterloo, Canada, 1992.