International Journal of Fuzzy Logic and Intelligent Systems, vol. 1, no. 1, June 2001 pp. 50-55

Fuzzy c-Continuous Mapppings

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Abstract

We generalize mainly the concept of c-continuity of a mapping due to Gentry and Hoyle III in fuzzy setting. And we investigate some properties of fuzzy c-continuous mappings.

Key words and phrases : q-coincident, FT_1, FT_2, fuzzy normal, fuzzy compact, fc-continuity.

0. Introduction

The concept of fuzzy sets was introduced by Zadeh in his classical paper[12]. Thereafter many investiga- tion have been carried out, in the general thearetical field and also in different application sides, based on this concept. Chang introduced the idea of fuzzy topo- logical spaces[2]. The idea is more or less a generali- zation of ordinary topological spaces. Different aspects of such spaces have been developed, by several investigators[2-6,8-11].

In this paper, we generalize mainly the concept of c-continuity of a mapping due to Gentry and Hoyle III[6] in fuzzy setting. And we investigate some properties of fuzzy c-continuous mappings.

I. Preliminaries

For a set X, each member of I^X is called a *fuzzy set* in X, where I denotes the unit interval. $\emptyset$ and X denote the fuzzy sets in X defined by $\emptyset(x)=0$ and $X(x)=1$ for each $x\in X$, respectively. Futhermore $(I^X,\cup,\cap,\emptyset,X)$ is a complete distribu- tive lattice satisfying the De Morgans' laws [12] : for $A,B\in I^X$

$(A\cap B)^c = A^c\cup B^c$ and

$(A\cup B)^c = A^c\cap B^c$,

where $(A^c)(x)=1-A(x)$ for each $x\in X$.

For $A\in I^X$, $\{x\in X: A(x)>0\}$ is called the *support* of A and denoted by A_0 or $S(A)$[12].

1991 Mathematics subject classification : 54A40

This paper was supported by Wonkwang University in 2001.

Definition 1.1[7]. A *fuzzy point* x_λ in a set X is a fuzzy set in X defined by

$x_\lambda(y)=\lambda$ if $y=x$,

$=0$ if $y\neq x$, for each $y\in X$.

In this case, we say that x_λ is a *fuzzy point with support* $x\in X$ and *value* $\lambda\in(0,1]$. We well denote the set of all fuzzy point in X as $F_p(X)$. Let $x_\lambda\in F_p(X)$ and let $A\in I^X$. Then x_λ is said to *belong to* A of $\lambda\le A(x)$. In this case, we shall use the notation $x_\lambda\in A$. A fuzzy set A in X is the union of all the fuzzy points belonging to A.

Two points x_λ and y_μ in X are said to be *distinct* if $x\neq y$(cf, [3]).

Definition 1.2[7]. Let $A,B\in I^X$ and let $x_\lambda\in F_p(X)$. Then x_λ is said to be *quasi-coincident*(in short, *q-coincident*) with A, denoted by $x_\lambda qA$ if $\lambda>A^c(x)$ or $\lambda+A(x)>1$. A is said to be *q-coincident with* B if there exists $x\in X$ such that $A(x)>B^c(x)$ or $A(x)+B(x)>1$. In this case, we also say that A and B are *q-coincident*.

Result 1.A[7]. Let $A,B\in I^X$ and let $x_\lambda\in F_p(X)$. Then $A\subset B$ if and only if A and B^c are *not q-coincident*(denoted by $A\bar{q}B^c$). Particularly $x_\lambda\in A$ if and only if $x_\lambda\bar{q}A^c$.

Definition 1.3[12]. Let f be a mapping from a set X into a Y, $A\in I^X$ and $B\in I^Y$. Then :

(i) The *image* of A under f, $f(A)$ is a fuzzy set in Y defined by for each $y\in Y$,

$[f(A)](y) = Sup_{x \in f^{-1}(y)} A(x)$ if $f^{-1}(y) \neq \emptyset$,
$= 0$ otherwise,
where $f^{-1}(y) = \{ x \in X : f(x) = y \}$.

(ii) The *inverse image* of B under f, $f^{-1}(B)$ is a fuzzy set in X defined by for each $x \in X$,
$f^{-1}(B)(x) = B(f(x))$.

Result 1.B[1,11]. Let $f: X \to Y$ be a mapping. Then :

(1) $f^{-1}(B^c) = [f^{-1}(B)]^c$ for each $B \in I^Y$.

(2) $[f(A)]^c \subset f(A^c)$ for each $A \in I^X$.
In particular, if f is bijective, then
$[f(A)]^c = f(A^c)$ for each $A \in I^X$.

(3) If $B_1 \subset B_2$, then $f^{-1}(B_1) \subset f^{-1}(B_2)$, where $B_1, B_2 \in I^Y$.

(4) If $A_1 \subset A_2$, then $f(A_1) \subset f(A_2)$, where $A_1, A_2 \in I^X$.

(5) $f(f^{-1}(B)) \subset B$ for each $B \in I^Y$.
In particular, if f is surjective, then
$f(f^{-1}(B)) = B$ for each $B \in I^Y$.

(6) $A \subset f^{-1}(f(A))$ for each $A \in I^X$.
In particular, if f is injective, then
$f^{-1}(f(A)) = A$ for each $A \in I^X$.

(7) If $\{B_\alpha\}_{\alpha \in \Lambda} \subset I^Y$, then
$f^{-1}(\bigcup_{\alpha \in \Lambda} B_\alpha) = \bigcup_{\alpha \in \Lambda} f^{-1}(B_\alpha)$ $\bigcup_{\alpha \in \Lambda} f^{-1}(B_\alpha)$ and
$f^{-1}(\bigcap_{\alpha \in \Lambda} B_\alpha) = \bigcap_{\alpha \in \Lambda} f^{-1}(B_\alpha)$.

(8) If $\{A_\alpha\}_{\alpha \in \Lambda} \subset I^X$, then
$f(\bigcup_{\alpha \in \Lambda} A_\alpha) = \bigcup_{\alpha \in \Lambda} f(A_\alpha)$.

(9) Let $g: Y \to Z$ be a mapping. Then $(g \circ f)^{-1}(C) = f^{-1}(g^{-1}(C))$ for each $C \in I^Z$.

Result 1.C[2]. Let $f: X \to Y$ be a mapping. Then :

(1) $f(x_\lambda) = [f(x)]_\lambda$ for each $x_\lambda \in F_p(X)$.

(2) If $A \in I^X$ and $x_\lambda \in A$, then $f(x_\lambda) \in f(A)$.

(3) If $A \in I^X$ and $y_\lambda \in f(A)$, then there exists $x \in X$ such that $f(x) = y$ and $x_\lambda \in A$.

(4) If $B \in I^Y$, $y \in f(X)$ and $y_\lambda \in B$, then for each $x \in f^{-1}(y)$, $x_\lambda \in f^{-1}(B)$.

(5) If $B \in I^Y$ and $x_\lambda \in f^{-1}(B)$, then $[f(x)]_\lambda \in B$.

Definition 1.5[7]. Let (X, T) be an fts and let Y a crisp subset of X. Then the family
$T_Y = \{A\mid_Y : A \in T\}$ is a fuzzy topology on Y. In this case, T_Y is called the *fuzzy relative topology* or *fuzzy subspace topology* of T to Y and the pair (Y, T_Y) is called a *fuzzy subspace* of (X, T).

It is clear that $A\mid_Y = A \cap Y$.

Definition 1.6. Two fuzzy sets A and B in a set X are said to be *disjoint* if $A \odot B = \emptyset$, where $(A \odot B)(x) = \max[0, A(x) + B(x) - 1]$ for each $x \in X$.
It is clear that $A \odot B = \emptyset$ if and only if $A \bar{q} B$.

Definition 1.7[3]. A fts X is said to be :

(1) *fuzzy* T_1(in short, FT_1) if for any two distinct fuzzy points x_λ and y_μ in X, there exists $U, V \in FO(X)$ such that $x_\lambda \in U \subset y_\mu^c$ and $y_\mu \in V \subset x_\lambda^c$.

(2) *fuzzy Hausdorff* or *fuzzy* T_2(in short, FT_2) if for any two distinct fuzzy points x_λ and y_μ in X, there exist $U, V \in FO(X)$ such that $x_\lambda \in U$, $y_\mu \in V$ and $U \odot V = \emptyset$.

Result 1.D[4]. A fts X is FT_1 if and only if $x_\lambda \in FC(X)$ for each $x_\lambda \in F_p(X)$.

Definition 1.8[3]. A fts X is said to be *fuzzy normal* if for any two disjoint $F_1, F_2 \in FC(X)$, there exist $U, V \in FO(X)$ such that $F \subset U$, $F_2 \subset V$ and $U \odot V = \emptyset$. Hence $F_1 \subset U \subset V^c \subset F_2^c$.

Result 1.E[5]. Let X be a compact fts. If $A \in FC(X)$, then A is fuzzy compact in X.

Result 1.F[5]. Let X be a T_2-fts. If A is fuzzy compact in X, then $A \in FC(X)$.

Result 1.G[5]. Every F-continuous image of a compact fts is fuzzy compact.

Definition 1.9[8,10]. Let $\{(X_\alpha, T_\alpha) : \alpha \in \Lambda\}$ be a family of fts's, let $X = \prod_{\alpha \in \Lambda} X_\alpha$ the usual Cartesian product of $\{X_\alpha\}_{\alpha \in \Lambda}$ and let π_α the projection from X onto X_α for each $\alpha \in \Lambda$. Let $\mathscr{F}_f(\Lambda)$ be the family of all finite subsets of Λ, let $\mathscr{S} = \{\pi_\alpha^{-1}(B) : B \in T_\alpha, \alpha \in \Lambda\}$ and let $\mathscr{B} = \{ \bigcap_{\alpha \in F} \pi_\alpha^{-1}(U_\alpha) : U_\alpha \in T_\alpha, F \in \mathscr{F}_f(\Lambda)\}$. Then there exists a unique fuzzy topology T on X for which $\mathscr{B}$ is a base for T and $\mathscr{S}$ is a subbase for T. In fact, T is the family of all unions of members of $\mathscr{B}$.

In this case, T is called the *fuzzy product topology* on X and the pair (X, T) is called the fuzzy *product topological space*(in short, *product fts*.).

Result 1.I[10]. Let (X, T) be the product fts of the family $\{(X_\alpha, T_\alpha)\}_{\alpha\in\Lambda}$ of fts's. Then :

(1) π_α is F-continuous for each $\alpha\in\Lambda$.

(2) T is the smallest fuzzy topology on X for which (1) is true.

(3) Let Y be a fts and let $f: Y\to X$ a mapping. Then f is F-continuous if and only if $\pi_\alpha \circ f$ is F-continuous for each $\alpha\in\Lambda$.

Result 1.I[10]. let $\{(X_\alpha, T_\alpha)\}$, $\alpha=1,\cdots,n$, be a finite family of compact(countably compact) fts's. Then the product fts (X, T) is also compact (countably compact).

II. Basic properties of c-continuous mappings

Definition 2.1. Let X and Y be fts's, let $f: X\to Y$ be a mapping and let $x_\lambda \in F_p(X)$. Then f is said to be *fuzzy c-continuous*(in short, *fc-continuous*) at x_λ if for each $U\in FO(Y)$ such that $f(x_\lambda)\in U$ and U^c is fuzzy compact in Y, there exists a $V\in FO(X)$ such that $x_\lambda\in V$ and $f(V)\subset U$. The mapping f is said to be *fuzzy c-continuous*(on X) if f is fc-continuous at each fuzzy point in X.

It is clear that every F-continuous mapping is fc-continuous.

Theorem 2.2. Let X and Y be fts's and let $f: X\to Y$ be mapping. Then the following are equivalent :

(1) f is fc-continuous.

(2) If $U\in FO(Y)$ with fuzzy compact complement, then $f^{-1}(U)\in FO(X)$.

These statements are implied by :

(3) If C is a fuzzy compact set in Y, then $f^{-1}(C)\in FC(X)$.

Moreover, if Y is fuzzy Hausdorff, then all the statements are equivalent.

(Proof)(1)⟹(2): Suppose f is fc-continuous and let $U\in FO(Y)$ with compact complement. Let $x_\lambda\in f^{-1}(U)$. Then, by Result 1.C(5), $f(x_\lambda)\in U$. By the hypothesis, there exists a $V_{x_\lambda}\in FO(X)$ such that $x_\lambda\in V_{x_\lambda}$ and $f(V_{x_\lambda})\subset U$. Thus $V_{x_\lambda}\subset f^{-1}(U)$. So $f^{-1}(U)=\bigcup\{V_{x_\lambda}\in FO(X): x_\lambda\in f^{-1}(U)\}$. Hence $f^{-1}(U)\in FO(X)$.

(2)⟹(1): Suppose the condition (2) holds. Let $x_\lambda\in F_p(X)$ and let $U\in FO(Y)$ such that $f(x_\lambda)\in U$ and U^c is fuzzy compact in Y. By the hypothesis, by Result 1.C (4) and Result 1.B (5), $f^{-1}(U)\in FO(X)$, $x_\lambda\in f^{-1}(U)$ and $f(f^{-1}(U))\subset U$. Thus f is fc-continuous at each fuzzy point x_λ in X. Hence f is fc-continuous.

(3)⟹(2): Suppose the condition (3) holds and let $U\in FO(Y)$ with fuzzy compact complement. Then $f^{-1}(U^c)\in FC(X)$ and $f^{-1}(U^c)=[f^{-1}(U)]^c$. Hence $f^{-1}(U)\in FO(X)$.

Now let Y be fuzzy Hausdorff.

(2)⟹(3): Suppose the condition (2) holds and let C be a fuzzy compact set in Y. Since Y is fuzzy Hausdorff, by Result 1.F, $C\in FC(Y)$. Thus $C^c\in FO(Y)$. By the hypothesis and Result 1. B (1), $f^{-1}(C^c)=[f^{-1}(C)]^c\in FO(X)$. Hence $f^{-1}(C)\in FC(X)$. ///

Theorem 2.3. If $f: X\to Y$ is fc-continuous and A a crisp subset of X, then $f|_A: A\to Y$ is fc-continuous.

(Proof) Let $U\in FO(Y)$ with compact complement. Then $f^{-1}(U)\in FO(X)$ and $(f|_A)^{-1}(U)=f^{-1}(U)\cap A=f^{-1}(U)|_A$. Thus $(f|_A)^{-1}(U)\in FO(A)$. Hence $f|_A$ is fc-continuous. ///

Theorem 2.4. If $f: X\to Y$ is F-continuous and $g: Y\to Z$ fc-continuous, then $g\circ f: X\to Z$ is fc-continuous.

(Proof) Let $U\in FO(Z)$ with compact complement. Then $g^{-1}(U)\in FO(Y)$. Since f is F-continuous, $f^{-1}(g^{-1}(U))\in FO(X)$ and $f^{-1}(g^{-1}(U))=(g\circ f)^{-1}(U)$ by Result 1. B (9). Thus $(g\circ f)^{-1}(U)\in FO(X)$. Hence $g\circ f$ is fc-continuous. ///

Theorem 2.5. Let X and Y be fts's and let $X=A\cup B$, where $A, B\in FO(X)$ (resp. $A, B\in FC(X)$) such that $A=A_0$ and $B=B_0$. If $f: X\to Y$ is a mapping such that $f|_A$ and $f|_B$ are fc-continuous, then f is fc-continuous.

(Proof) First suppose $A, B\in FO(X)$. Let $U\in FO(Y)$ with fuzzy compact complement. Then $f^{-1}(U)=(f|_A)^{-1}(U)\cup(f|_B)^{-1}(U)$. Since $f|_A$ and $f|_B$ are fc-continuous, $(f|_A)^{-1}(U), (f|_B)^{-1}(U)\in FO(X)$. Thus $f^{-1}(U)\in FO(X)$. Hence f is fc-continuous.

Now suppose $A, B\in FC(X)$. Let $x_\lambda\in F_p(X)$ and let $W\in FO(Y)$ with fuzzy compact complement containing $f(x_\lambda)$. Then either $x_\lambda\in A\cap B$, $x_\lambda\in A$ and $x_\lambda \bar{q} B$ or $x_\lambda \bar{q} A$ and $x_\lambda\in B$.

(Case 1) Suppose $x_\lambda\in A\cap B$. Since $f|_A$ is fc-continuous at x_λ, there exists a $U\in FO(A)$ such that $x_\lambda\in U$ and $(f|_A)(U)\subset W$. Since $f|_B$ is fc-continuous

at x_λ, there exists a $V \in FO(B)$ such that $x_\lambda \in V$ and $(f|_B)(V) \subset W$. Since $U \in FO(A)$, there exists a $U' \in FO(X)$ such that $U = U'|_A = U' \cap A$. Since $V \in FO(B)$, there exists a $V' \in FO(X)$ such that $V = V'|_B = V' \cap B$. Let $Q = U' \cap V'$. Then $Q \in FO(X)$, $x_\lambda \in Q$ and $f(Q) \subset W$. Hence f is fc-continuous at x_λ.

(Case 2) Suppose $x_\lambda \in A$ and $x_\lambda \bar{q} B$. Since $f|_A$ is fc-continuous, there exists a $U \in FO(A)$ such that $x_\lambda \in U$ and $(f|_A)(U) \subset W$. Since $U \in FO(A)$, there exists a $U' \in FO(X)$ such that $U = U'|_A = U' \cap A$. Let $V = U' \cap B^c$. Then clearly $V \in FO(X)$. Since $x_\lambda \bar{q} B$, $x_\lambda \in B^c$ by Result 1.A. Thus $x_\lambda \in V$. Moreover $f(V) \subset W$. Hence f is fc-continuous.

(Case 3) Suppose $x_\lambda \bar{q} A$ and $x_\lambda \in B$. This case follows exactly like(Case2). ///

III. Further Results

Theorem 3.1. Let X be a fts, Y a Hausdorff fts, and $f: X \to Y$ fc-continuous. If $f(X)$ is a fuzzy subset of some fuzzy compact set in Y, then f is F-continuous.

(Proof) Let D be a fuzzy compact set in Y such that $f(X) \subset D$. Let $U \in FO(Y)$. Since Y is fuzzy Hausdorff, by Result 1.F, $D \in FC(Y)$. Thus $D^c \in FO(Y)$. So $U \cup D^c \in FO(Y)$. Furthermore $(U \cup D^c)^c = U^c \cap D \subset D$ and $U^c \cap D \in FC(Y)$. Since D is fuzzy compact in Y, by Result 1.E, $U^c \cap D$ is fuzzy compact in Y. So $(U^c \cup D)^c$ is a fuzzy compact subset of D. Now $f^{-1}(U) = f^{-1}(U \cup D^c)$ $f^{-1}(U) \in FO(X)$. Since f is fc-continuous, . Hence f is F-continuous. ///

Theorem 3.2. Let X be a fts and let Y a Hausdorff fts. If $f: X \to Y$ is F-continuous and bijective, then $f^{-1}: Y \to X$ is fc-continuous.

(Proof) Let C be a fuzzy compact subset of X. Since f is F-continuous, $f(C)$ is fuzzy compact in Y by Result 1.G. Since Y is fuzzy Hausdorff, by Result 1.F, $f(C) \in FC(Y)$. Hence, by Theorem 2.2. f^{-1} is fc-continuous. ///

Corollary 3.2. Let X be a compact fts and let Y a Hausdorff fts. If $f: X \to Y$ is bijective and F-continuous, then f is an F-homeomorphism.

(Proof) By Theorem 3.2, f^{-1} is fc-continuous. Thus, by Theorem 3.1, f^{-1} is F-continuous. Hence f is an F-homeomorphism. ///

Theorem 3.3. A mapping $f: X \to Y$ is fc-continuous if and only if the inverse image of each fuzzy closed compact subset of Y is F-closed in X.

(Proof)($\Rightarrow$): Suppose $f: X \to Y$ is fc-continuous and let U be a fuzzy closed compact subset of Y. Then $U^c \in FO(Y)$. Since f is fc-continuous, by Theorem 2.2. $f^{-1}(U^c) \in FO(X)$. By Result 1.B (1) $f^{-1}(U^c) = [f^{-1}(U)]^c$. Thus $[f^{-1}(U)]^c \in FO(X)$. Hence $f^{-1}(U^c) \in FC(X)$.
($\Leftarrow$): Suppose the necessary condition holds. Let $U \in FO(Y)$ with fuzzy compact complement. Then U^c is a fuzzy closed compact subset of Y. By the hypothesis, $f^{-1}(U^c) \in FC(X)$. Thus $f^{-1}(U) \in FO(X)$. Hence f is fc-continuous. ///

Theorem 3.4. Let $f: X \to Y$ be fc-continuous and injective. If Y is FT_1, then X is FT_1.

(Proof) Let x_λ and y_μ be distinct fuzzy points in X. Since Y is FT_1, by Result 1.D, $f(x_\lambda), f(y_\mu) \in FC(Y)$. Moreover $f(x_\lambda)$ and $f(y_\mu)$ are fuzzy compact in Y. Since f is fc-continuous, by Theorem 3.3, $f^{-1}(f(x_\lambda)), f^{-1}(f(y_\mu)) \in FC(X)$. Since f is injective, $f^{-1}(f(x_\lambda)) = x_\lambda$ and $f^{-1}(f(y_\mu)) = y_\mu$ by Result 1 B (6). Thus $x_\lambda, y_\mu \in FC(X)$. Hence X is FT_1. ///

Lemma 3.5. (1) Let $f: X \to Y$ be F-closed. Then for any $S \in I^Y$ and any $U \in FO(Y)$ containing $f^{-1}(S)$, there exists a $V \in FO(X)$ such that $S \subset V$ and $f^{-1}(V) \subset U$.
(2) Let $f: X \to Y$ be F-open. Then for any $S \in I^Y$ and any $A \in FC(Y)$ containing $f^{-1}(S)$, there exists a $B \in FC(Y)$ such that $S \subset B$ and $f^{-1}(B) \subset A$.

(Proof) We prove (1) only, since the proof of (2) is similar. Let $S \in I^Y$ and let $U \in FO(Y)$ such that $f^{-1}(S) \subset U$. Let $V = [f(U^c)]^c$. Since $f^{-1}(S) \subset U$, $S \subset V$. Since f is F-closed, $V \in FO(Y)$. Observing that $f^{-1}(V) = f^{-1}([f(U^c)]^c) = [f^{-1}(f(U^c))]^c \subset (U^c)^c$ $= U$. This completes the proof. ///

Theorem 3.6. Let $f: X \to Y$ be fc-continuous, F-closed and surjective. If X is fuzzy normal and Y is FT_1, then Y is FT_1.

(Proof) Let $y_{1,\lambda}$ and $y_{2,\mu}$ be distinct fuzzy point in Y.

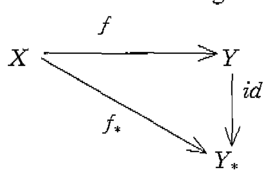
Since Y is FT_1, by Result 1.D, $y_{1,\lambda}$ and $y_{2,\mu}$ are fuzzy closed compact sets in Y. Since f is fc-continuous, by Theorem 3.3, $f^{-1}(y_{1,\lambda}), f^{-1}(y_{2,\mu}), \in FC(X)$. Moreover $f^{-1}(y_{1,\lambda}) \odot f^{-1}(y_{2,\mu}) = \emptyset$. Since X is fuzzy normal, there exists $U_1, U_2 \in FO(X)$ such that $f^{-1}(y_{1,\lambda}) \subset U_1, f^{-1}(y_{2,\mu}) \subset U_2$ and $U_1 \odot U_2 = \emptyset$. Since f is F-closed, by Lemma 3.5, there exist $V_1, V_2 \in FO(Y)$ such that $y_{1,\lambda} \in V_1$, $f^{-1}(y_{1,\lambda}) \subset f^{-1}(V_1) \subset U_1, y_{2,\mu} \in V_2$ and $f^{-1}(y_{2,\mu}) \subset f^{-1}(V_2) \subset U_2$. Moreover $V_1 \odot V_2 = \emptyset$. Hence Y is FT_2. ///

Lemma 3.7. Let (X, T) be a fts and let $\mathscr{B} = \{U \in T: U^c$ is fuzzy compact in $X\}$. Then there exists a uniquely fuzzy topology T_* on X for which $\mathscr{B}$ is a fuzzy base for T.

Furthermore $T_* \subset T$ and (X, T_*) is always a compact fuzzy topological spaces. In this case (X, T_*) is called the *induced compact fuzzy topological spaces* by (X, T) and will be denoted by X_*.

(Proof) Clearly $X \in \mathscr{B}$. So $\bigcup \mathscr{B} = X$. Let $B_1, B_2 \in \mathscr{B}$ and let $x_\lambda \in B_1 \cap B_2$. Then $B_1, B_2 \in T$. B_1^c and B_2^c are fuzzy compact in X. Thus $B_1 \cap B_2 \in T$ and $(B_1 \cap B_2)^c$ is fuzzy compact in X. So $B_1 \cap B_2 \in \mathscr{B}$. Hence this completes the proof. ///

Proposition 3.8. Let X and Y be fts's, let Y_* the induced compact fts by Y and let $f: X \to Y$ a mapping. Define the mapping $f_*: X \to Y_*$ by $f_*(x) = f(x)$ for all $x \in X$. Consider the following diagram :



Then f is fc-continuous if and only if f_* is F-continuous. Also id is F-continuous and id^{-1} is fc-continuous.

Theorem 3.9. Let $f: X \to Y$ be fc-continuous. If $f_*: X \to Y_*$ is F-closed (resp. F-open), then f is F-closed(resp. F-open).

(Proof) Let C be any F-open(resp. closed) set in X. Then $f_*(C)$ is F-open(resp. F-closed) in Y_*. Since f is fc-continuous, by Proposition 3.8, $id^{-1}(f_*(C)) = f(C)$ is F-open(resp. F-closed) in Y. Hence this completes the proof. ///

We might note that under the hypotheses of Theorem 3.9, if f is bijective, then f^{-1} is F-continuous.

For a mapping $f: X \to Y$, the mapping $g: X \to X \times Y$, given by $g(x) = (x, f(x))$, is called the *graph mapping with respect to* f.

Theorem 3.9. Let $f: X \to Y$ be a mapping and let X a compact fts. If the graph mapping $g: X \to X \times Y$ is fc-continuous, then f is fc-continuous.

(Proof) Let $x_\lambda \in F_p(X)$ and let $V \in FO(Y)$ such that $f(x_\lambda) \in V$ and V^c is fuzzy compact in Y. Then $\pi_Y^{-1}(V)$ is F-open in $X \times Y$ by Result 1.H(1). On the other hand, $[\pi_Y^{-1}(V)]^c = X \times V^c$. Since X and V^c are fuzzy compact, $X \times V^c$ is fuzzy compact in $X \times Y$ by Result 1.I. Since X is a compact fts, $[\pi_Y^{-1}(V)]^c$ is fuzzy compact in $X \times Y$ by Result 1.E. Since g is fc-continuous, there exists a $U \in FO(X)$ such that $x_\lambda \in U$ and $g(U) \subset \pi_Y^{-1}(V)$. Thus $\pi_Y(g(U)) = f(U) \subset V$. Hence f is fc-continuous. ///

Reference

[1] C.L.Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24(1968)182-189.

[2] Z.Deng, Fuzzy pseudo-metric spaces, J. Math. Anal. Appl. 86(19820 74-95.

[3] A.A.Fora, Fuzzy separation axioms and fuzzy continuity, Arab Gulf J. Scient. Res., math. phys. Sci., A5(3)(1987) 307-318.

[4] S.Ganguly and S.Saha, On separation axioms and T_i-fuzzy continuity, Fuzzy Sets and Systems 16(1985) 265-275.

[5] ＿＿＿＿＿＿, A note on compactness in a fuzzy setting, Fuzzy Sets and Systems 34(1990) 117-124.

[6] K. R. Gentry and H. B. Hoyle. III, c-continuous functions, Fundamenta Mathematicae 76(1972) 9-17.

[7] Pu Pao-Ming and Liu Ying-Ming, Fuzzy topology, J. Math. Anal. Appl.76(1980)571-599.

[8] ＿＿＿＿＿＿, Fuzzy topology II, J. Math. Anal. Appl. 77(1980)20-37.

[9] S. Saha, Fuzzy δ-continuous mappings, J. Math. Anal. Appl. 126(1987)130-142.

[10] C.K.Wong, Fuzzy topology : product and quotient theorem, J. Math. Anal. Appl. 45(1974)512-521.

[11] T.H.Yalvac, Fuzzy sets and functions or fuzzy spaces, J. Math. Anal. Appl.126(1987)409-423.

[12] L.A.Zadah, Fuzzy sets, Inform and Control 8(1965) 338-353.

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