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Fuzzy Strongly r-Semineighborhoods

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Abstract

In this thesis, we introduce and investigate the notions of a fuzzy strongly r-semineighborhood and a fuzzy strongly r-quasi-semineighborhood in fuzzy topological spaces which are generalizations of a fuzzy strongly semineighborhood and a fuzzy strongly quasi-semineighborhood, respectively.

Key Words : fuzzy strongly r-semineighborhood, fuzzy strongly r-quasi-semineighborhood

1. Introduction and Preliminaries

E. P. Lee and S. J. Lee[5] introduced the concepts of fuzzy strongly r-semiopen sets and fuzzy strongly r-semicontinuous maps, which generalized the concepts of fuzzy strongly semiopen sets and fuzzy strongly semicontinuous maps of Shi-Zhong Bai[9].

In this thesis, we introduce and investigate the notions of a fuzzy strongly r-semineighborhood and a fuzzy strongly r-quasi-semineighborhood in fuzzy topological spaces which are generalizations of a fuzzy strongly semineighborhood and a fuzzy strongly quasi-semineighborhood, respectively.

We will denote the unit interval $[0,1]$ of the real line by I and $I_0=(0,1]$. A member μ of I^X is called a fuzzy set in X. For any $\mu\in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

A Chang's fuzzy topology on X is a family T of fuzzy sets in X which satisfies the following properties:

(1) $\tilde{0},\tilde{1}\in T$.
(2) If $\mu_1,\mu_2\in T$ then $\mu_1\wedge\mu_2\in T$.
(3) If $\mu_i\in T$ for each i, then $\bigvee\mu_i\in T$.

The pair (X,T) is called a Chang's fuzzy topological space.

A fuzzy topology on X is a map $T:I^X\to I$ which satisfies the following properties:

(1) $T(\tilde{0})=T(\tilde{1})=1$.
(2) $T(\mu_1\wedge\mu_2)\geq T(\mu_1)\wedge T(\mu_2)$.
(3) $T(\bigvee\mu_i)\geq\bigwedge T(\mu_i)$.

The pair (X,T) is called a fuzzy topological space.

Definition 1.1. ([6]) Let μ be a fuzzy set in a fuzzy topological space (X,T) and $r\in I_0$. Then μ is called

(1) a fuzzy r-open set in X if $T(\mu)\geq r$,
(2) a fuzzy r-closed set in X if $T(\mu^c)\geq r$.

Definition 1.2. ([3,6]) Let (X,T) be a fuzzy topological space. For each $r\in I_0$ and for each $\mu\in I^X$, the fuzzy r-closure is defined by

$$\mathrm{cl}(\mu,r)=\bigwedge\{\rho\in I^X:\mu\leq\rho,\ T(\rho^c)\geq r\},$$

and the fuzzy r-interior is defined by

$$\mathrm{int}(\mu,r)=\bigvee\{\rho\in I^X:\mu\geq\rho,\ T(\rho)\geq r\}.$$

Definition 1.3. ([6,7]) Let μ be a fuzzy set in a fuzzy topological space (X,T) and $r\in I_0$. Then μ is said to be

(1) fuzzy r-semiopen if there is a fuzzy r-open set ρ in X such that $\rho\leq\mu\leq\mathrm{cl}(\rho,r)$,
(2) fuzzy r-semiclosed if there is a fuzzy r-closed set ρ in X such that $\mathrm{int}(\rho,r)\leq\mu\leq\rho$,
(3) fuzzy r-preopen if $\mu\leq\mathrm{int}(\mathrm{cl}(\mu,r),r)$,
(4) fuzzy r-preclosed if $\mathrm{cl}(\mathrm{int}(\mu,r),r)\leq\mu$.

Definition 1.4. ([5]) Let μ be a fuzzy set in a fuzzy topological space (X,T) and $r\in I_0$. Then μ is said to be

(1) fuzzy strongly r-semiopen if there is a fuzzy r-open set ρ in X such that $\rho\leq\mu\leq\mathrm{int}(\mathrm{cl}(\rho,r),r)$,
(2) fuzzy strongly r-semiclosed if there is a fuzzy r-closed set ρ in X such that $\mathrm{cl}(\mathrm{int}(\rho,r),r)\leq\mu\leq\rho$.

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Theorem 1.5. ([5])

(1) Any union of fuzzy strongly r-semiopen sets is fuzzy strongly r-semiopen.
(2) Any intersection of fuzzy strongly r-semiclosed sets is fuzzy strongly r-semiclosed.

Definition 1.6. ([5]) Let $(X, \mathcal{T})$ be a fuzzy topological space. For each $r \in I_0$ and for each $\mu \in I^X$, the fuzzy strongly r-semiclosure is defined by

$$\mathrm{sscl}(\mu, r) = \bigwedge\{\rho \in I^X : \mu \leq \rho, \rho \text{ is fuzzy strongly } r\text{-semiclosed}\},$$

and the fuzzy strongly r-semiinterior is defined by

$$\mathrm{ssint}(\mu, r) = \bigvee\{\rho \in I^X : \mu \geq \rho, \rho \text{ is fuzzy strongly } r\text{-semiopen}\}.$$

For $x \in X$ and for each $\alpha \in (0,1]$, a fuzzy point x_α in X is a fuzzy set in X defined by

$$x_\alpha(y) = \begin{cases} \alpha & \text{if } y = x, \\ 0 & \text{if } y \neq x. \end{cases}$$

In this case, x and α are called the support and the value of x_α, respectively. A fuzzy point x_α is said to belong to a fuzzy set μ in X, denoted by $x_\alpha \in \mu$, if $\alpha \leq \mu(x)$. A fuzzy set μ in X is the union of all fuzzy points which belong to μ.

A fuzzy point x_α in X is said to be quasi-coincident with μ, denoted by $x_\alpha q \mu$, if $\alpha + \mu(x) > 1$. A fuzzy set ρ in X is said to be quasi-coincident with a fuzzy set μ in X, denoted by $\rho q \mu$, if there is an $x \in X$ such that $\rho(x) + \mu(x) > 1$.

Definition 1.7. ([4,7]) Let x_α be a fuzzy point of a fuzzy topological space $(X, \mathcal{T})$ and $r \in I_0$. Then a fuzzy set μ in X is called

(1) a fuzzy r-neighborhood (fuzzy r-semineighborhood, fuzzy r-preneighborhood, respectively) of x_α if there is a fuzzy r-open (fuzzy r-semiopen, fuzzy r-preopen, respectively) set ρ in X such that $x_\alpha \in \rho \leq \mu$,
(2) a fuzzy r-quasi-neighborhood (fuzzy r-quasi-semineighborhood, fuzzy r-quasi-preneighborhood, respectively) of x_α if there is a fuzzy r-open (fuzzy r-semiopen, fuzzy r-preopen, respectively) set ρ in X such that $x_\alpha q\, \rho \leq \mu$,

2. Fuzzy strongly r-semineighborhoods

We are going to define the concepts of a fuzzy strongly r-semineighborhood and a fuzzy strongly r-quasi-semineighborhood in a fuzzy topological space.

Definition 2.1. Let x_α be a fuzzy point of a fuzzy topological space $(X, \mathcal{T})$ and $r \in I_0$. Then a fuzzy set μ in X is called

(1) a fuzzy strongly r-semineighborhood of x_α if there is a fuzzy strongly r-semiopen set ρ in X such that $x_\alpha \in \rho \leq \mu$,
(2) a fuzzy strongly r-quasi-semineighborhood of x_α if there is a fuzzy strongly r-semiopen set ρ in X such that $x_\alpha q\, \rho \leq \mu$.

Clearly, if μ is a fuzzy strongly r-semineighborhood (strongly r-quasi-semineighborhood) of x_α and $r \geq t$, then μ is also a fuzzy strongly t-semineighborhood (strongly t-quasi-semineighborhood) of x_α.

Theorem 2.2. Let $(X, \mathcal{T})$ be a fuzzy topological space and $r \in I_0$. Then a fuzzy set μ in X is fuzzy strongly r-semiopen if and only if μ is a fuzzy strongly r-semineighborhood of x_α for every fuzzy point $x_\alpha \in \mu$.

Proof. Let μ be any fuzzy strongly r-semiopen set of X and $x_\alpha \in \mu$. Put $\rho = \mu$. Then ρ is a fuzzy strongly r-semiopen set in X and $x_\alpha \in \rho \leq \mu$. Thus μ is a fuzzy strongly r-semineighborhood of x_α.

Conversely, let $x_\alpha \in \mu$. Since μ is a fuzzy strongly r-semineighborhood of x_α, there is a fuzzy strongly r-semiopen set ρ_{x_α} in X such that $x_\alpha \in \rho_{x_\alpha} \leq \mu$. So we have

$$\mu = \bigvee\{x_\alpha : x_\alpha \in \mu\} \leq \bigvee\{\rho_{x_\alpha} : x_\alpha \in \mu\} \leq \mu.$$

Thus $\mu = \bigvee\{\rho_{x_\alpha} : x_\alpha \in \mu\}$. Since arbitrary join of fuzzy strongly r-semiopen sets is fuzzy strongly r-semiopen, μ is fuzzy strongly r-semiopen.

Theorem 2.3. Let $(X, \mathcal{T})$ be a fuzzy topological space and $r \in I_0$. Then a fuzzy set μ in X is fuzzy strongly r-semiopen if and only if μ is a fuzzy strongly r-quasi-semineighborhood of x_α for every fuzzy point $x_\alpha q\, \mu$.

Proof. Let μ be any fuzzy strongly r-semiopen set of X and $x_\alpha q\, \mu$. Put $\rho = \mu$. Then ρ is a fuzzy strongly r-semiopen set in X and $x_\alpha q\, \rho \leq \mu$. Thus μ is a fuzzy strongly r-quasi-semineighborhood of x_α.

Conversely, let x_α be any fuzzy point in μ such that $\alpha < \mu(x)$. Then $x_{1-\alpha} q \mu$. By the hypothesis, μ is a fuzzy strongly r-quasi-semineighborhood of $x_{1-\alpha}$. Thus there is a fuzzy strongly r-semiopen set ρ_{x_α} in X such that $x_{1-\alpha} q \rho_{x_\alpha} \leq \mu$. Hence $\alpha < \rho_{x_\alpha}(x)$ and $\rho_{x_\alpha} \leq \mu$. So we have

$$\begin{aligned} \mu &= \bigvee\{x_\alpha : x_\alpha \text{ is a fuzzy point in } \mu \text{ such that } \alpha < \mu(x)\} \\ &\leq \bigvee\{\rho_{x_\alpha} : x_\alpha \text{ is a fuzzy point in } \mu \text{ such that } \alpha < \mu(x)\} \\ &\leq \mu. \end{aligned}$$

Thus $\mu=\bigvee\{\rho_{x_\alpha}: x_\alpha$ is a fuzzy point in μ such that $\alpha<\mu(x)\}$. Since each ρ_{x_α} is fuzzy strongly r-semiopen, μ is fuzzy strongly r-semiopen.

Theorem 2.4. Let x_α be a fuzzy point in a fuzzy topological space (X, T) and $r\in I_0$. Then $x_\alpha\in \mathrm{sscl}(\mu, r)$ if and only if $\rho q \mu$ for all fuzzy strongly r-quasi-semineighborhood ρ of x_α.

Proof. Suppose that there is a fuzzy strongly r-quasi-semineighborhood ρ of x_α such that $\rho \not{q} \mu$. Then there is a fuzzy strongly r-semiopen set λ such that $x_\alpha q\lambda\leq\rho$. So $\lambda \not{q} \mu$ and hence $\mu\leq\lambda^c$. Since λ^c is fuzzy strongly r-semiclosed, $\mathrm{sscl}(\mu, r)\leq \mathrm{sscl}(\lambda^c, r)=\lambda^c$. On the other hand, since $x_\alpha q\lambda$, we have $x_\alpha\notin\lambda^c$. Hence $x_\alpha\notin \mathrm{sscl}(\mu, r)$. This is a contradiction.

Conversely, suppose $x_\alpha\notin \mathrm{sscl}(\mu, r)$. Then there is a fuzzy strongly r-semiclosed set η such that $\mu\leq\eta$ and $x_\alpha\notin\eta$. Then since η^c is fuzzy strongly r-semiopen and $x_\alpha q\, \eta^c$, η^c is a fuzzy strongly r-quasi- semineighborhood of x_α. By the hypothesis, $\eta^c q\, \mu$. Hence $\mu\not\leq(\eta^c)^c=\eta$. This is a contradiction.

Theorem 2.5. Let x_α be a fuzzy point in a fuzzy topological space (X, T) and $r\in I_0$. Then $x_\alpha\in \mathrm{ssint}(\mu, r)$ if and only if there is a fuzzy strongly r-semineighborhood ρ of x_α such that $\rho\leq\mu$.

Proof. Let $x_\alpha\in \mathrm{ssint}(\mu, r)$. Then there is a fuzzy strongly r-semiopen set ρ such that $x_\alpha\in\rho$ and $\rho\leq\mu$.

Conversely, suppose that there is a fuzzy strongly r-semineighborhood ρ of x_α such that $\rho\leq\mu$. Then there is a fuzzy strongly r-semiopen set λ such that $x_\alpha\in\lambda\leq\rho\leq\mu$. Thus $x_\alpha\in \mathrm{ssint}(\mu, r)$.

Remark 2.6. (1) Every fuzzy r-neighborhood (r-quasi-neighborhood) of x_α is also a fuzzy strongly r-semineighborhood (strongly r-quasi-semineighborhood) of x_α.

(2) Every fuzzy strongly r-semineighborhood (strongly r-quasi-semineighborhood) of x_α is also a fuzzy r-semineighborhood (r-quasi-semineighborhood) of x_α.

(3) Every fuzzy strongly r-semineighborhood (strongly r-quasi-semineighborhood) of x_α is also a fuzzy r-preneighborhood (r-quasi-preneighborhood) of x_α.

Following examples show that their converses need not be true in general.

Example 2.7. Let $X=\{a, b\}$ and μ_1 and μ_2 be fuzzy sets in X defined by $\mu_1(a)=\frac{3}{5}$, $\mu_1(b)=\frac{1}{10}$ and $\mu_2(a)=\frac{7}{10}$, $\mu_2(b)=\frac{9}{10}$. Define $T: I^X\rightarrow I$ by

$$T(\mu)=\begin{cases} 1 & \text{if } \mu=\tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu=\mu_1, \\ 0 & \text{otherwise}. \end{cases}$$

Then clearly T is a fuzzy topology on X. Let $x=b$ and $\alpha=\frac{1}{5}$. Then $x_\alpha\in\mu_2$ and μ_2 is fuzzy strongly $\frac{1}{2}$-semiopen. Thus μ_2 is a fuzzy strongly $\frac{1}{2}$-semineighborhood but not fuzzy $\frac{1}{2}$-neighborhood. Also μ_2 is a fuzzy strongly $\frac{1}{2}$-quasi-semineighborhood of x_α which is not a fuzzy $\frac{1}{2}$-quasi-neighborhood of x_α.

Example 2.8. Let $X=\{a, b\}$ and μ_1 and μ_2 be fuzzy sets in X defined by $\mu_1(a)=\frac{1}{2}$, $\mu_1(b)=\frac{2}{5}$ and $\mu_2(a)=\frac{1}{2}$, $\mu_2(b)=\frac{3}{5}$. Define $T: I^X\rightarrow I$ by

$$T(\mu)=\begin{cases} 1 & \text{if } \mu=\tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu=\mu_1, \\ 0 & \text{otherwise}. \end{cases}$$

Then clearly T is a fuzzy topology on X. Let $x=b$ and $\alpha=\frac{1}{2}$. Then $x_\alpha\in\mu_2$ and μ_2 is fuzzy $\frac{1}{2}$-semiopen. Thus μ_2 is a fuzzy $\frac{1}{2}$-semineighborhood but not fuzzy strongly $\frac{1}{2}$-semineighborhood. Also μ_2 is a fuzzy $\frac{1}{2}$-quasi-semineighborhood of x_α which is not a fuzzy strongly $\frac{1}{2}$-quasi-semineighborhood of x_α.

Example 2.9. Let $X=\{a, b\}$ and μ_1 and μ_2 be fuzzy sets in X defined by $\mu_1(a)=\frac{1}{2}$, $\mu_1(b)=\frac{1}{3}$ and $\mu_2(a)=\frac{1}{2}$, $\mu_2(b)=\frac{11}{15}$. Define $T: I^X\rightarrow I$ by

$$T(\mu)=\begin{cases} 1 & \text{if } \mu=\tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu=\mu_1, \\ 0 & \text{otherwise}. \end{cases}$$

Then clearly T is a fuzzy topology on X. Let $x=b$ and $\alpha=\frac{8}{15}$. Then $x_\alpha\in\mu_2$ and μ_2 is a fuzzy $\frac{1}{2}$-preopen. Thus μ_2 is a fuzzy $\frac{1}{2}$-preneighborhood but not fuzzy strongly $\frac{1}{2}$-semineighborhood. Also μ_2 is a fuzzy $\frac{1}{2}$-quasi-preneighborhood of x_α which is not a fuzzy strongly $\frac{1}{2}$-quasi-semineighborhood of x_α.

Let (X, T) be a fuzzy topological space. For each

$r \in I_0$, an r-cut

$$T_r = \{\mu \in I^X : T(\mu) \geq r\}$$

is a Chang's fuzzy topology on X.

Let (X, T) be a Chang's fuzzy topological space and $r \in I_0$. A fuzzy topology $T^r : I^X \to I$ is defined by

$$T^r(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ r & \text{if } \mu \in T - \{\tilde{0}, \tilde{1}\}, \\ 0 & \text{otherwise.} \end{cases}$$

The next two theorems show that a fuzzy strongly semineighborhood [9] is a special case of a fuzzy strongly r-semineighborhood.

Theorem 2.10. Let x_α be a fuzzy point of a fuzzy topological space (X, T) and $r \in I_0$. Then a fuzzy set μ is a fuzzy strongly r-semineighborhood (strongly r-quasi-semineighborhood) of x_α in (X, T) if and only if μ is a fuzzy strongly semineighborhood (strongly quasi-semineighborhood) of x_α in (X, T_r).

Proof. Straightforward.

Theorem 2.11. Let x_α be a fuzzy point of a Chang's fuzzy topological space (X, T) and $r \in I_0$. Then a fuzzy set μ is a fuzzy strongly semineighborhood (strongly quasi-semineighborhood) of x_α in (X, T) if and only if μ is a fuzzy strongly r-semineighborhood (strongly r-quasi-semineighborhood) of x_α in (X, T^r).

Proof. Straightforward.

The product fuzzy set $\mu \times \rho$ of a fuzzy set μ of X and a fuzzy set ρ of Y is defined by

$$(\mu \times \rho)(x, y) = \mu(x) \wedge \rho(y)$$

for all $(x, y) \in X \times Y$.

Let (X, T) and (Y, U) be fuzzy topological spaces and $r \in I_0$. Then X is r-product related to Y if any fuzzy set μ of X and any fuzzy set ρ of Y,

$$\mathrm{cl}(\mu \times \rho, r) = \mathrm{cl}(\mu, r) \times \mathrm{cl}(\rho, r).$$

Let $\{(X_i, T_i)\}_{i \in J}$ be a family of fuzzy topological spaces. Let $X = \prod X_i$ and $p_i : X \to X_i$, $i \in J$, denote the projection map. Let $(T_i)_r$ denote the Chang's fuzzy topology on X_i for $i \in J$, $r \in I_0$. Let

$$\prod (T_i)_r = \sup_{i \in J} p_i^{-1}((T_i)_r)$$

be the Chang's fuzzy topology generated by $\{p_i^{-1}((T_i)_r)\}_{i \in J}$ as a subbase.

Let T be the fuzzy topology generated by $\{\prod (T_i)_r\}_{0 < r \leq 1}$. That is

$$T(\mu) = \vee\{r \in I_0 : \mu \in \prod (T_i)_r\}.$$

Then T is called the product fuzzy topology on X and denoted by $\prod T_i$.

Lemma 2.12. Let $r \in I_0$ and a fuzzy topological space (X, T) be r-product related to a fuzzy topological space (Y, U). Then for any fuzzy set μ of X and any fuzzy set ρ of Y, $\mathrm{int}(\mu \times \rho, r) = \mathrm{int}(\mu, r) \times \mathrm{int}(\rho, r)$.

Proof. Let μ be any fuzzy set in X and ρ any fuzzy set in Y. Then

$$\begin{aligned}
&\mathrm{int}(\mu \times \rho, r) \\
&= \mathrm{int}(((\mu \times \rho)^c)^c, r) \\
&= \mathrm{cl}((\mu \times \rho)^c, r)^c \\
&= \mathrm{cl}((\mu^c \times \tilde{1}) \vee (\tilde{1} \times \rho^c), r)^c \\
&= [\mathrm{cl}(\mu^c \times \tilde{1}, r) \vee \mathrm{cl}(\tilde{1} \times \rho^c, r)]^c \\
&= [(\mathrm{cl}(\mu^c, r) \times \mathrm{cl}(\tilde{1}, r)) \vee (\mathrm{cl}(\tilde{1}, r) \times \mathrm{cl}(\rho^c, r))]^c \\
&= [(\mathrm{cl}(\mu^c, r) \times \tilde{1}) \vee (\tilde{1} \times \mathrm{cl}(\rho^c, r))]^c \\
&= [(\mathrm{int}(\mu, r)^c \times \tilde{1}) \vee (\tilde{1} \times \mathrm{int}(\rho, r)^c)]^c \\
&= [(\mathrm{int}(\mu, r) \times \mathrm{int}(\rho, r))^c]^c \\
&= \mathrm{int}(\mu, r) \times \mathrm{int}(\rho, r).
\end{aligned}$$

Hence the theorem follows.

Theorem 2.13. Let (X, T) and (Y, U) be fuzzy topological spaces and $r \in I_0$. If X is r-product related to Y, then the product $\mu \times \rho$ of a fuzzy strongly r-semiopen(strongly r-semiclosed) set μ in X and a fuzzy strongly r-semiopen(strongly r-semiclosed) set ρ in Y is fuzzy strongly r-semiopen(strongly r-semiclosed) in the product fuzzy topological space $X \times Y$.

Proof. Let μ and ρ be fuzzy strongly r-semiopen sets in X and Y respectively. Then there are fuzzy r-open sets μ_1 in X and ρ_1 in Y, such that $\mu_1 \leq \mu \leq \mathrm{int}(\mathrm{cl}(\mu_1, r), r)$ and $\rho_1 \leq \rho \leq \mathrm{int}(\mathrm{cl}(\rho_1, r), r)$. Then $\mu_1 \times \rho_1$ is fuzzy r-open in $X \times Y$. Since X is r-product related to Y, from the Lemma 2.12, we have

$$\begin{aligned}
\mu_1 \times \rho_1 \leq \mu \times \rho &\leq \mathrm{int}(\mathrm{cl}(\mu_1, r), r) \times \mathrm{int}(\mathrm{cl}(\rho_1, r), r) \\
&= \mathrm{int}(\mathrm{cl}(\mu_1, r) \times \mathrm{cl}(\rho_1, r), r) \\
&= \mathrm{int}(\mathrm{cl}(\mu_1 \times \rho_1, r), r).
\end{aligned}$$

Hence $\mu \times \rho$ is fuzzy strongly r-semiopen in $X \times Y$. Similarly, if μ and ρ are fuzzy strongly r-semiclosed sets then $\mu \times \rho$ is also fuzzy strongly r-semiclosed.

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