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Mated Fuzzy Topological Spaces

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ABSTRACT

We introduce the concept of mated fuzzy topological spaces and then investigate some of their properties.

Key Words : mated fuzzy topology, fuzzy (r, s)-open set, fuzzy (r, s)-interior

1. Introduction

As a generalization of fuzzy sets, the notion of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker and his colleagues [5, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets.

In this paper, we introduce the concept of mated fuzzy topological spaces as a generalization of intuitionistic fuzzy topological spaces and smooth topological spaces, and then investigate some of their properties. Also we introduce the notions of fuzzy (r, s)-interiors and fuzzy (r, s)-closures in mated fuzzy topological spaces.

2. Preliminaries

In this paper, I denotes the unit interval $[0, 1]$ of the real line. For a set X, I^X denotes the collection of all maps from X to I. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1} - \mu$. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair $A = (\mu_A, \gamma_A)$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership respectively, and $\mu_A + \gamma_A \le \tilde{1}$.

Obviously every fuzzy set μ of X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1 [1] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on X. Then

(1) $A \subseteq B$ iff $\mu_A \le \mu_B$ and $\gamma_A \ge \gamma_B$.
(2) $A = B$ iff $A \subseteq B$ and $A \supseteq B$.
(3) $A^c = (\gamma_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
(6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Definition 2.2 [6] An intuitionistic fuzzy topology on X is a family $\mathfrak{T}$ of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in \mathfrak{T}$.
(2) If $A_1, A_2 \in \mathfrak{T}$ then $A_1 \cap A_2 \in \mathfrak{T}$.
(3) If $A_i \in \mathfrak{T}$ for all i, then $\bigcup A_i \in \mathfrak{T}$.

The pair $(X, \mathfrak{T})$ is called an *intuitionistic fuzzy topological space.*

Let $(X, \mathfrak{T})$ be an intuitionistic fuzzy topological space. Then members of $\mathfrak{T}$ are called *intuitionistic fuzzy open sets* of X and their complements *intuitionistic fuzzy closed sets.*

Definition 2.3 [6] Let $(X, \mathfrak{T})$ be an intuitionistic fuzzy topological space and A an intuitionistic fuzzy set of X. Then the *fuzzy closure* is defined by

$$\mathrm{Cl}(A) = \bigcap \{F \mid A \subseteq F,\ F^c \in \mathfrak{T}\}$$

and the *fuzzy interior* is defined by

$$\mathrm{Int}(A) = \bigcup \{G \mid A \supseteq G,\ G \in \mathfrak{T}\}$$

Proposition 2.4 [6] For an intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X, \mathfrak{T})$, we have

(1) $\mathrm{Int}(A)^c = \mathrm{Cl}(A^c)$.
(2) $\mathrm{Cl}(A)^c = \mathrm{Int}(A^c)$.

3. Mated fuzzy topological spaces

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X.

Definition 3.1 [7] Let X be a nonempty set. A *mated fuzzy topology* $\mathfrak{T} = (\tau, \tau_c)$ on X is two maps $\tau : I(X) \to I$ and $\tau_c : I(X) \to I$ which satisfy the following properties:

(1) $\tau(A) + \tau_c(A) \le 1$ for any $A \in I(X)$.
(2) $\tau(0_\sim) = \tau(1_\sim) = 1$ and

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$\tau_c(0_{\sim})=\tau_c(1_{\sim})=0$.

(3) $\tau(A\cap B)\geq\tau(A)\wedge\tau(B)$ and $\tau_c(A\cap B)\leq\tau_c(A)\vee\tau_c(B)$.

(4) $\tau(\bigcup A_i)\geq\bigwedge\tau(A_i)$ and $\tau_c(\bigcup A_i)\leq\bigvee\tau_c(A_i)$.

The $(X,\Im)=(X,\tau,\tau_c)$ is called a *mated fuzzy topological space*. And, we call τ a *gradation of openness* and τ_c a *gradation of nonopenness*.

Definition 3.2 [7] Let X be a nonempty set. A *mated fuzzy family of closed sets* $\Omega=(\omega,\omega_c)$ on X is two maps $\omega: I(X)\to I$ and $\omega_c: I(X)\to I$ which satisfy the following properties :

(1) $\omega(A)+\omega_c(A)\leq 1$ for any $A\in I(X)$.

(2) $\omega(0_{\sim})=\omega(1_{\sim})=1$ and $\omega_c(0_{\sim})=\omega_c(1_{\sim})=0$.

(3) $\omega(A\cup B)\geq\omega(A)\wedge\omega(B)$ and $\omega_c(A\cup B)\leq\omega_c(A)\vee\omega_c(B)$.

(4) $\omega(\bigcap A_i)\geq\bigwedge\omega(A_i)$ and $\omega_c(\bigcap A_i)\leq\bigvee\omega_c(A_i)$.

In this case, we call ω a *gradation of closedness* and ω_c a *gradation of nonclosedness*.

Proposition 3.3 Let $\Im=(\tau,\tau_c)$ be a mated fuzzy topology on X and $\Omega_{\Im}=(\omega_\tau,\omega_{\tau_c})$ defined by

$$\omega_\tau(A)=\tau(A^c) \text{ and } \omega_{\tau_c}(A)=\tau_c(A^c).$$

Then $\Omega_{\Im}$ is a mated fuzzy family of closed sets on X.

Proof. (1) For any $A\in I(X)$,

$$\omega_\tau(A)+\omega_{\tau_c}(A)=\tau(A^c)+\tau_c(A^c)\leq 1.$$

(2) $\omega_\tau(0_{\sim})=\tau(1_{\sim})=1$, $\omega_\tau(1_{\sim})=\tau(0_{\sim})=1$ and $\omega_{\tau_c}(0_{\sim})=\tau_c(1_{\sim})=0$, $\omega_{\tau_c}(1_{\sim})=\tau_c(0_{\sim})=0$.

(3) $\omega_\tau(A\cup B)=\tau(A^c\cap B^c)\geq\tau(A^c)\wedge\tau(B^c) = \omega_\tau(A)\wedge\omega_\tau(B)$

and

$$\omega_{\tau_c}(A\cup B)=\tau_c(A^c\cap B^c)\leq\tau_c(A^c)\vee\tau_c(B^c) = \omega_{\tau_c}(A)\vee\omega_{\tau_c}(B).$$

(4) $\omega_\tau(\bigcap A_i)=\tau(\bigcup A_i^c)\geq\bigwedge\tau(A_i^c)=\bigwedge\omega_\tau(A_i)$

and

$$\omega_{\tau_c}(\bigcap A_i)=\tau_c(\bigcup A_i^c)\leq\bigvee\tau_c(A_i^c)=\bigvee\omega_{\tau_c}(A_i).$$

Proposition 3.4 Let $\Omega=(\omega,\omega_c)$ be a mated fuzzy family of closed sets on X and $\Im_{\Omega}=(\tau_\omega,\tau_{\omega_c})$ defined by $\tau_\omega(A)=\omega(A^c)$ and $\tau_{\omega_c}(A)=\omega_c(A^c)$.

Then $\Im_{\Omega}$ is a mated fuzzy topology on X.

Proof. Similar to the above proposition.

Corollary 3.5 Let $\Im=(\tau,\tau_c)$ be a mated fuzzy topology and $\Omega=(\omega,\omega_c)$ a mated fuzzy family of closed sets. Then $\Im_{\Omega_{\Im}}=\Im$ and $\Omega_{\Im_{\Omega}}=\Omega$.

Proposition 3.6 Let (X,τ,τ_c) be a mated fuzzy topological space. Then for each $r\in I$,

$$\tau_r=\{A\in I(X)\mid\tau(A)\geq r\}$$

is an intuitionistic fuzzy topology on X. Moreover $\tau_{r_1}\supseteq\tau_{r_2}$ if $r_1\leq r_2$.

Proof. (1) Since $\tau(0_{\sim})=\tau(1_{\sim})=1\geq r$, we have $0_{\sim}, 1_{\sim}\in\tau_r$.

(2) Let $A, B\in\tau_r$. Then $\tau(A)\geq r$ and $\tau(B)\geq r$. So $\tau(A\cap B)\geq\tau(A)\wedge\tau(B)\geq r$ and hence $A\cap B\in\tau_r$.

(3) Let $A_i\in\tau_r$ for all $i\in\Gamma$. Then $\tau(A_i)\geq r$ for all $i\in\Gamma$. So $\tau(\bigcup A_i)\geq\bigwedge\tau(A_i)\geq r$. Thus $\bigcup A_i\in\tau_r$.

Therefore, τ_r is an intuitionistic fuzzy topology on X. Next, let $r_1\leq r_2$. Take $A\in\tau_{r_2}$. Then $\tau(A)\geq r_2\geq r_1$ and hence $A\in\tau_{r_1}$. Thus $\tau_{r_1}\supseteq\tau_{r_2}$.

Proposition 3.7 Let (X,τ,τ_c) be a mated fuzzy topological space. Then for each $s\in I$,

$$\tau_c{}^s=\{A\in I(X)\mid\tau_c(A)\leq s\}$$

is an intuitionistic fuzzy topology on X. Moreover $\tau_c{}^{s_1}\subseteq\tau_c{}^{s_2}$ if $s_1\leq s_2$.

Proof. (1) Since $\tau_c(0_{\sim})=\tau_c(1_{\sim})=0\leq s$, we have $0_{\sim}, 1_{\sim}\in\tau_c{}^s$.

(2) Let $A, B\in\tau_c{}^s$. Then $\tau_c(A)\leq s$ and $\tau_c(B)\leq s$. So $\tau_c(A\cap B)\leq\tau_c(A)\vee\tau_c(B)\leq s$ and hence $A\cap B\in\tau_c{}^s$.

(3) Let $A_i\in\tau_c{}^s$ for all $i\in\Gamma$. Then $\tau_c(A_i)\leq s$ for all $i\in\Gamma$. So $\tau_c(\bigcup A_i)\leq\bigvee\tau_c(A_i)\leq s$. Thus $\bigcup A_i\in\tau_c{}^s$.

Therefore, $\tau_c{}^s$ is an intuitionistic fuzzy topology on X. Next, let $s_1\leq s_2$. Take $A\in\tau_c{}^{s_1}$. Then $\tau_c(A)\leq s_1\leq s_2$ and hence $A\in\tau_c{}^{s_2}$. Thus $\tau_c{}^{s_1}\subseteq\tau_c{}^{s_2}$.

From the above two propositions we have the following result.

Corollary 3.8 Let $\Im=(\tau,\tau_c)$ be a mated fuzzy topology on X. Then for each $r,s\in I$,

$\Im_r^s=\{A\in I(X)\mid\tau(A)\geq r \text{ and } \tau_c(A)\leq s\}$

is an intuitionistic fuzzy topology on X and $\Im_r^s=\tau_r\cap\tau_c{}^s$. Moreover $\Im_{r_2}^{s_1}\subseteq\Im_{r_1}^{s_2}$ if $r_1\leq r_2$ and $s_1\leq s_2$.

4. Fuzzy (r,s)-interiors and fuzzy (r,s)-closures

Let $I\oplus I=\{(r,s)\mid r,s\in I \text{ and } r+s\leq 1\}$.

Definition 4.1 Let A be an intuitionistic fuzzy set of a mated fuzzy topological space (X, τ, τ_c) and $(r, s) \in I \oplus I$. Then A is called;

(1) a fuzzy (r, s)-open set if $\tau(A) \geq r$ and $\tau_c(A) \leq s$,

(2) a fuzzy (r, s)-closed set if $\tau(A^c) \geq r$ and $\tau_c(A^c) \leq s$.

Definition 4.2 Let (X, τ, τ_c) be a mated fuzzy topological space. For each $(r, s) \in I \oplus I$ and for each $A \in I(X)$, the fuzzy (r, s)-interior is defined by

$$\mathrm{Int}(A, r, s) = \bigcup \{B \in I(X) \mid A \supseteq B \text{ and } B \text{ is a fuzzy } (r, s)\text{-open set}\}$$

and the fuzzy (r, s)-closure is defined by

$$\mathrm{Cl}(A, r, s) = \bigcap \{B \in I(X) \mid A \subseteq B \text{ and } B \text{ is a fuzzy } (r, s)\text{-closed set }\}.$$

The operators $\mathrm{Int} : I(X) \times I \oplus I \to I(X)$ and $\mathrm{Cl} : I(X) \times I \oplus I \to I(X)$ are called the fuzzy interior operator and the fuzzy closure operator in (X, τ, τ_c), respectively.

Obviously, $\mathrm{Int}(A, r, s)$ is the greatest fuzzy (r, s)-open set which is contained in A and $\mathrm{Cl}(A, r, s)$ is the smallest fuzzy (r, s)-closed set which contains A. Also, $\mathrm{Int}(A, r, s) = A$ for any fuzzy (r, s)-fuzzy open set A and $\mathrm{Cl}(A, r, s) = A$ for any fuzzy (r, s)-closed set A. Moreover, we have the following results.

Proposition 4.3 Let (X, τ, τ_c) be a mated fuzzy topological space and let $\mathrm{Int} : I(X) \times I \oplus I \to I(X)$ the fuzzy interior operator in (X, τ, τ_c). Then for any $A, B \in I(X)$ and $(r, s) \in I \oplus I$,

(1) $\mathrm{Int}(0_\sim, r, s) = 0_\sim$, $\mathrm{Int}(1_\sim, r, s) = 1_\sim$.

(2) $\mathrm{Int}(A, r, s) \subseteq A$.

(3) $\mathrm{Int}(A, r_1, s_1) \supseteq \mathrm{Int}(A, r_2, s_2)$ if $r_1 \leq r_2$ and $s_1 \geq s_2$.

(4) $\mathrm{Int}(A \cap B, r, s) = \mathrm{Int}(A, r, s) \cap \mathrm{Int}(B, r, s)$.

(5) $\mathrm{Int}(\mathrm{Int}(A, r, s), r, s) = \mathrm{Int}(A, r, s)$.

Proof. (1), (2) and (5) are obvious.

(3) Let $r_1 \leq r_2$ and $s_1 \geq s_2$. Then every fuzzy (r_2, s_2)-open set is also fuzzy (r_1, s_1)-open. Hence we have

$$\begin{aligned} &\mathrm{Int}(A, r_1, s_1) \\ &= \bigcup \{B \in I(X) \mid A \supseteq B,\ \tau(B) \geq r_1,\ \tau_c(B) \leq s_1\} \\ &\supseteq \bigcup \{B \in I(X) \mid A \supseteq B,\ \tau(B) \geq r_2,\ \tau_c(B) \leq s_2\} \\ &= \mathrm{Int}(A, r_2, s_2). \end{aligned}$$

(4) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, we have $\mathrm{Int}(A \cap B, r, s) \subseteq \mathrm{Int}(A, r, s)$ and $\mathrm{Int}(A \cap B, r, s) \subseteq \mathrm{Int}(B, r, s)$.
Thus $\mathrm{Int}(A \cap B, r, s) \subseteq \mathrm{Int}(A, r, s) \cap \mathrm{Int}(B, r, s)$.

Conversely, it is clear that
$A \cap B \supseteq \mathrm{Int}(A, r, s) \cap \mathrm{Int}(B, r, s)$.

Also, $\tau(\mathrm{Int}(A, r, s) \cap \mathrm{Int}(B, r, s))$
$\geq \tau(\mathrm{Int}(A, r, s)) \wedge \tau(\mathrm{Int}(B, r, s))$
$\geq r \wedge r = r$
and
$\tau_c(\mathrm{Int}(A, r, s) \cap \mathrm{Int}(B, r, s))$
$\leq \tau_c(\mathrm{Int}(A, r, s)) \vee \tau_c(\mathrm{Int}(B, r, s))$
$\leq s \vee s = s$.
By the definition of fuzzy (r, s)-interior,
$\mathrm{Int}(A \cap B, r, s) \supseteq \mathrm{Int}(A, r, s) \cap \mathrm{Int}(B, r, s)$.
Hence $\mathrm{Int}(A \cap B, r, s) = \mathrm{Int}(A, r, s) \cap \mathrm{Int}(B, r, s)$.

Proposition 4.4 Let $\mathrm{Int} : I(X) \times I \oplus I \to I(X)$ be a map satisfying (1) - (5) of Proposition 4.3. Let $\tau : I(X) \to I$ and $\tau_c : I(X) \to I$ be maps defined by
$\tau(A) = \bigvee \{ r \in I \mid \mathrm{Int}(A, r, s) = A \}$ and
$\tau_c(A) = \bigwedge \{ s \in I \mid \mathrm{Int}(A, r, s) = A \}$.
Then $\mathfrak{T} = (\tau, \tau_c)$ is a mated fuzzy topology on X.

Proof. (i) Since $(r, s) \in I \oplus I$, we have $r + s \leq 1$. Hence $s \leq 1 - r$. Thus

$$\begin{aligned} 1 - \tau(A) &= 1 - \bigvee \{r \in I \mid \mathrm{Int}(A, r, s) = A\} \\ &= \bigwedge \{1 - r \in I \mid \mathrm{Int}(A, r, s) = A\} \\ &\geq \bigwedge \{s \in I \mid \mathrm{Int}(A, r, s) = A\} \\ &= \tau_c(A). \end{aligned}$$

So $\tau(A) + \tau_c(A) \leq 1$ for each $A \in I(X)$.

(ii) Since $\mathrm{Int}(0_\sim, 1, 0) = 0_\sim$ and $\mathrm{Int}(1_\sim, 1, 0) = 1_\sim$, we have $\tau(0_\sim) = \tau(1_\sim) = 1$, $\tau_c(0_\sim) = \tau_c(1_\sim) = 0$.

(iii) Suppose that $\tau(A_1 \cap A_2) < \tau(A_1) \wedge \tau(A_2)$. Then there is a $t \in I$ such that

$$\tau(A_1 \cap A_2) < t < \tau(A_1) \wedge \tau(A_2).$$

Since $t < \tau(A_i) = \bigvee \{r \in I \mid \mathrm{Int}(A_i, r, s) = A_i\}$ for each $i = 1, 2$, there are $(r_1, s_1), (r_2, s_2) \in I \oplus I$ such that $t < r_i \leq \tau(A_i)$ and $\mathrm{Int}(A_i, r_i, s_i) = A_i$ for each $i = 1, 2$. Let $r = r_1 \wedge r_2$ and $s = s_1 \vee s_2$. Since $(r_1, s_1), (r_2, s_2) \in I \oplus I$,

$$\begin{aligned} 1 - r &= 1 - (r_1 \wedge r_2) \\ &= (1 - r_1) \vee (1 - r_2) \geq s_1 \vee s_2 = s. \end{aligned}$$

Hence $(r, s) \in I \oplus I$.
Since $r \leq r_i$ and $s \geq s_i$ for each $i = 1, 2$, we have $\mathrm{Int}(A_i, r, s) \supseteq \mathrm{Int}(A_i, r_i, s_i) = A_i$. Hence $\mathrm{Int}(A_i, r, s) = A_i$ for each $i = 1, 2$. By (4),
$\mathrm{Int}(A_1 \cap A_2, r, s) = \mathrm{Int}(A_1, r, s) \cap \mathrm{Int}(A_2, r, s)$
$= A_1 \cap A_2$.
Thus $t > \tau(A_1 \cap A_2)$
$= \bigvee \{r^* \in I \mid \mathrm{Int}(A_1 \cap A_2, r^*, s^*) = A_1 \cap A_2\}$
$\geq r = r_1 \wedge r_2 > t$. It is a contradiction. Hence $\tau(A_1 \cap A_2) \geq \tau(A_1) \wedge \tau(A_2)$. Next, suppose that $\tau_c(A_1 \cap A_2) > \tau_c(A_1) \vee \tau_c(A_2)$. Then there is a $t \in I$ such that $\tau_c(A_1 \cap A_2) > t > \tau_c(A_1) \vee \tau_c(A_2)$. Since $t > \tau_c(A_i) = \bigwedge \{s \in I \mid \mathrm{Int}(A_i, r, s) = A_i\}$ for each $i = 1, 2$, there are

$(r_1, s_1), (r_2, s_2) \in I \oplus I$ such that $t > s_i \geq \tau_c(A_i)$ and $\mathrm{Int}(A_i, r_i, s_i) = A_i$ for each $i = 1, 2$. Let $r = r_1 \wedge r_2$ and $s = s_1 \vee s_2$. Then $(r, s) \in I \oplus I$ and $\mathrm{Int}(A_i, r, s) = A_i$ for each $i = 1, 2$. By (4),
$\mathrm{Int}(A_1 \cap A_2, r, s) = \mathrm{Int}(A_1, r, s) \cap \mathrm{Int}(A_2, r, s) = A_1 \cap A_2$.
Thus $t < \tau_c(A_1 \cap A_2)$
$= \wedge\{s^* \in I \mid \mathrm{Int}(A_1 \cap A_2, r^*, s^*) = A_1 \cap A_2\}$
$\leq s = s_1 \wedge s_2 < t$. It is a contradiction. Hence $\tau_c(A_1 \cap A_2) \leq \tau_c(A_1) \vee \tau_c(A_2)$.

(iv) First, suppose that $\tau(\bigcup A_i) < \wedge \tau(A_i)$. Then there is a $t \in I$ such that $\tau(\bigcup A_i) < t < \wedge \tau(A_i)$. Since $t < \tau(A_i) = \vee\{r \in I \mid \mathrm{Int}(A_i, r, s) = A_i\}$ for each i, there are $(r_i, s_i) \in I \oplus I$ such that $t < r_i \leq \tau(A_i)$ and $\mathrm{Int}(A_i, r_i, s_i) = A_i$ for each i. Let $r = \wedge r_i$ and $s = \vee s_i$. Since $(r_i, s_i) \in I \oplus I$ for each i,

$$1 - r = 1 - (\wedge r_i) = \vee (1 - r_i) \geq \vee s_i = s.$$

Hence $(r, s) \in I \oplus I$. And since $r \leq r_i$ and $s \geq s_i$ for each i, $\mathrm{Int}(A_i, r, s) \supseteq \mathrm{Int}(A_i, r_i, s_i) = A_i$.
Hence $\mathrm{Int}(\bigcup A_i, r, s) \supseteq \mathrm{Int}(A_i, r, s) \supseteq A_i$ for each i. So $\mathrm{Int}(\bigcup A_i, r, s) \supseteq \bigcup A_i$ and hence
$\mathrm{Int}(\bigcup A_i, r, s) = \bigcup A_i$.
Thus $t > \tau(\bigcup A_i)$
$= \vee\{r^* \in I \mid \mathrm{Int}(\bigcup A_i, r^*, s^*) = \bigcup A_i\}$
$\geq r = \wedge r_i \geq t$.
It is a contradiction. Hence $\tau(\bigcup A_i) \geq \wedge \tau(A_i)$. Next, suppose that $\tau_c(\bigcup A_i) > \vee \tau_c(A_i)$. Then there is a $t \in I$ such that $\tau_c(\bigcup A_i) > t > \vee \tau_c(A_i)$.
Since $t > \tau_c(A_i) = \wedge\{s \in I \mid \mathrm{Int}(A_i, r, s) = A_i\}$ for each i, there are $(r_i, s_i) \in I \oplus I$ such that $t > s_i \geq \tau_c(A_i)$ and $\mathrm{Int}(A_i, r_i, s_i) = A_i$ for each i. Let $r = \wedge r_i$ and $s = \vee s_i$. Then $(r, s) \in I \oplus I$ and $\mathrm{Int}(\bigcup A_i, r, s) = \bigcup A_i$. Thus
$t < \tau_c(\bigcup A_i)$
$= \wedge\{s^* \in I \mid \mathrm{Int}(\bigcup A_i, r^*, s^*) = \bigcup A_i\}$
$\leq s = \vee s_i \leq t$.
It is a contradiction. Hence $\tau_c(\bigcup A_i) \leq \vee \tau_c(A_i)$. Therefore $\mathfrak{T} = (\tau, \tau_c)$ is a mated fuzzy topology on X.

Proposition 4.5 Let (X, τ, τ_c) be a mated fuzzy topological space and let $\mathrm{Cl} : I(X) \times I \oplus I \rightarrow I(X)$ the fuzzy closure operator in (X, τ, τ_c). Then for any $A, B \in I(X)$ and $(r, s) \in I \oplus I$,

(1) $\mathrm{Cl}(0_\sim, r, s) = 0_\sim$, $\mathrm{Cl}(1_\sim, r, s) = 1_\sim$.
(2) $\mathrm{Cl}(A, r, s) \supseteq A$.
(3) $\mathrm{Cl}(A, r_1, s_1) \subseteq \mathrm{Cl}(A, r_2, s_2)$ if $r_1 \leq r_2$ and $s_1 \geq s_2$.
(4) $\mathrm{Cl}(A \cup B, r, s) = \mathrm{Cl}(A, r, s) \cup \mathrm{Cl}(B, r, s)$.
(5) $\mathrm{Cl}(\mathrm{Cl}(A, r, s), r, s) = \mathrm{Cl}(A, r, s)$.

Proof. Similar to Proposition 4.3.

Proposition 4.6 Let $\mathrm{Cl} : I(X) \times I \oplus I \rightarrow I(X)$ be a map satisfying (1) - (5) of Proposition 4.5. Let $\omega : I(X) \rightarrow I$ and $\omega_c : I(X) \rightarrow I$ be maps defined by

$\omega(A) = \vee\{r \in I \mid \mathrm{Cl}(A, r, s) = A\}$
and $\omega_c(A) = \wedge\{s \in I \mid \mathrm{Cl}(A, r, s) = A\}$.

Then $\Omega = (\omega, \omega_c)$ is a mated fuzzy family of closed sets on X.

Proof. Similar to Proposition 4.4

For a family $\{A_i\}_{i \in \Gamma}$ of intuitionistic fuzzy sets of a mated fuzzy topological space X and $(r, s) \in I \oplus I$, $\bigcup \mathrm{Cl}(A_i, r, s) \subseteq \mathrm{Cl}(\bigcup A_i, r, s)$ and the equality holds when Γ is a finite set. Similarly,
$\bigcap \mathrm{Int}(A_i, r, s) \supseteq \mathrm{Int}(\bigcap A_i, r, s)$ and
$\bigcap \mathrm{Int}(A_i, r, s) = \mathrm{Int}(\bigcap A_i, r, s)$ for a finite set Γ.

Proposition 4.7 For an intuitionistic fuzzy set A of a mated fuzzy topological space (X, τ, τ_c) and $(r, s) \in I \oplus I$, we have

(1) $\mathrm{Int}(A, r, s)^c = \mathrm{Cl}(A^c, r, s)$.
(2) $\mathrm{Cl}(A, r, s)^c = \mathrm{Int}(A^c, r, s)$.

Proof. (1) $\mathrm{Int}(A, r, s)^c$
$= (\bigcup\{B \in I(X) \mid A \supseteq B,\ \tau(B) \geq r,\ \tau_c(B) \leq s\})^c$
$= \bigcap\{B^c \in I(X) \mid A^c \subseteq B^c, \omega_\tau(B^c) \geq r, \omega_{\tau_c}(B^c) \leq s\}$
$= \mathrm{Cl}(A^c, r, s)$.
Similarly, we can show (2).

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