

# Fuzzy Estimator for Gain Scheduling and its Application to Magnetic Suspension

Seon-Ho Lee and Jong-Tae Lim

**Abstract:** The external force disturbance is the one of the main causes that deteriorate the performance of the magnetic suspension. Thus, this paper develops a fuzzy estimator for gain scheduling control of magnetic suspension systems suffering from the unknown disturbance. The proposed fuzzy estimator computes the disturbance injected to the plant and the gain scheduled controller generates the corresponding stabilizing control input associated with the estimated disturbance. In the simulation results we confirm the novelty of the proposed control scheme comparing with the other method using a feedback linearization.

**Keywords:** fuzzy estimator, gain scheduling, adaptive law, external force disturbance, magnetic suspension

## I. Introduction

The gain scheduled controllers proved to be a successful design methodology have been developed by many researchers [1]-[3]. Furthermore, in order to improve its regulation performance for fast scheduling parameters, [4] and [5] have proposed extended control laws using the derivative information of the time-varying parameter. Nowadays,  $H_\infty$  control theory has become an effective design methodology in tracking the problem of stability and performance robustness under plant uncertainties. Thus, many researchers have been interested in the fusion technique of  $H_\infty$  synthesis and gain scheduling [6]-[8]. However, since the stabilizing control gains of all the gain scheduled controllers mentioned above are driven by the measured scheduling parameter, these developed controllers become unavailable for the unmeasurable scheduling parameter, which should be estimated and engaged appropriately.

In spite of the high nonlinearity and the unstable dynamics, the magnetic suspension systems have been researched with merits that the absence of contact reduces noise, component wear, vibration, and maintenance costs. It is well known that the external force disturbance and mass uncertainty dominantly deteriorate the quality of system performance. In order to solve such problems, the classical state feedback control with pole-placement has been applied into the linearized model corresponding to a specific operating condition [9]. However, since the operating condition changes according to the disturbance and the uncertainty, the local controller couldn't achieve the satisfactory performance in the global operating points. Nonlinear control schemes have also been reported in the literature such as the gain scheduling [10] and the feedback linearization methodology [11] [12]. However, the force disturbance estimator [10] utilized the exact inverse dynamic of the given plant model using the derivative information of the state vectors. Thus, it was confined to solve the regulation problem only when the exact derivative information of the scheduling parameter is available in the considered plant. Moreover, though the feedback lin-

earization canceled the nonlinear nature in the magnetic suspension, it also had a limitation to tackle the unknown parameter variation and/or disturbance [11] [12]. Most recently, the self-tuning controller was developed for the unknown mass variation and showed improved performance [13]. However, the problem of determining an adequate adaptation rate and combining the cost functions still remained so as to apply to the general nonlinear plants.

The proposed fuzzy-estimated gain scheduled controller in this paper consists of the fuzzy estimator and the gain scheduled controller. The fuzzy estimator tracks the unknown scheduling parameter (external force disturbance) in the closed-loop control system and the stabilizing gains of the gain scheduled controller are appropriately scheduled according to the estimated disturbance. The proposed control scheme has several advantages over the related literature. The developed fuzzy estimator does not require the time derivative of the state vector that induces noise in practice. Moreover, the fuzzy estimator computes the adaptation rate and combine the adaptation laws in the manner of achieving the control objective. Thus, it is applicable to the various nonlinear plants in company with the gain scheduling methodologies.

## II. Gain scheduled controller

The single-axis magnetic suspension in Fig. 1 is modeled by the following nonlinear dynamic equation [9]

$$\begin{aligned} \dot{x} &= f(x, u, f_d) = \begin{bmatrix} x_2 \\ -\frac{C}{4m}(x_3)^2 + g + \frac{f_d}{m} \\ \frac{x_2 x_3}{x_1} - \frac{2R}{C} x_1 x_3 + \frac{2x_1}{C} u \end{bmatrix} \\ y &= h(x) = x_1 \end{aligned} \quad (1)$$

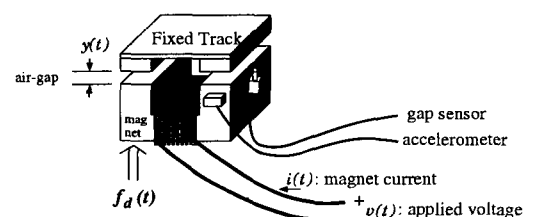


Fig. 1. Magnetic suspension system.

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Seon-Ho Lee: Satellite Control Dept., Korea Aerospace Research Institute (shlee@kari.re.kr)

Jong-Tae Lim: Dep. of Electrical Engineering, Korea Advanced Institute of Science and Technology (jtlim@ee.kaist.ac.kr)

with  $C = \mu_o AN^2$  where  $x_1$  is the vertical air-gap,  $x_2$  is the vertical velocity,  $x_3 = i(t)$  is the magnet current,  $u = v(t)$  is the applied voltage,  $f_d$  is the external force disturbance,  $m = 15kg$  is the total mass,  $N = 2000$  is the number of turns of the coil wrapped around the magnet,  $A = 0.0012m^2$  is the pole area,  $\mu_o = 4\pi \times 10^{-7} H/m$  is the permeability of free space,  $R = 8\Omega$  is the coil resistance,  $g = 9.8m/sec^2$  is the gravity constant, and  $r_d$  is the reference air-gap. Defining the disturbance  $f_d$  as the scheduling parameter, the smooth functions  $\mathbf{x}(f_d)$  and  $\mathbf{u}(f_d)$  satisfying

$$0 = f(\mathbf{x}(f_d), \mathbf{u}(f_d), f_d) \text{ and } r_d = h(\mathbf{x}(f_d)) \quad (2)$$

for  $f_d \in \Gamma \subset R$  are computed by

$$\mathbf{x}(f_d) = \begin{bmatrix} r_d \\ 0 \\ \frac{2r_d w(f_d)}{\sqrt{C}} \end{bmatrix} \text{ and } \mathbf{u}(f_d) = \frac{2r_d R w(f_d)}{\sqrt{C}} \quad (3)$$

where  $w(f_d) = \sqrt{f_d + mg}$ . The control objective is to obtain  $\lim_{t \rightarrow \infty} \|r_d - y\| = 0$  while rejecting the disturbance  $f_d$ . Then, for each fixed  $f_d \in \Gamma$ , the corresponding linearized closed loop system with a nonlinear state feedback control law  $u = k(x, f_d)$  is written by

$$\begin{aligned} \dot{x}_\delta &= A(f_d)x_\delta + B(f_d)u_\delta \\ y_\delta &= C(f_d)x_\delta \\ u_\delta &= K(f_d)x_\delta \end{aligned} \quad (4)$$

where the deviation variables are defined by  $x_\delta = x - \mathbf{x}(f_d)$ ,  $u_\delta = u - \mathbf{u}(f_d)$ , and  $y_\delta = y - r_d$ . Moreover, the linearized system coefficients are given by

$$\begin{aligned} A(f_d) &= \frac{\partial f(\mathbf{x}(f_d), \mathbf{u}(f_d), f_d)}{\partial x} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ \frac{2w(f_d)^2}{mr_d} & 0 & -\frac{\sqrt{C}w(f_d)}{mr_d} \\ 0 & \frac{2w(f_d)}{\sqrt{C}} & -\frac{2r_d R}{C} \end{bmatrix} \end{aligned} \quad (5)$$

$$B(f_d) = \frac{\partial f(\mathbf{x}(f_d), \mathbf{u}(f_d), f_d)}{\partial u} = \begin{bmatrix} 0 & 0 & \frac{2r_d}{C} \end{bmatrix} \quad (6)$$

$$C(f_d) = \frac{\partial h(\mathbf{x}(f_d))}{\partial x} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad (7)$$

$$K(f_d) = \frac{\partial k(\mathbf{x}(f_d), f_d)}{\partial x} \quad (8)$$

In order to achieve the desired pole-placement in the linearized closed-loop system,  $K(f_d)$  is determined so that the eigenvalues of  $A(f_d) + B(f_d)K(f_d)$  have specified values with negative real parts for each  $f_d \in \Gamma$  with the controllable pair of  $A(f_d)$  and  $B(f_d)$  [1]. Then,

$$k(x, f_d) = \mathbf{u}(f_d) + K(f_d)(x - \mathbf{x}(f_d)) \quad (9)$$

is obtained with  $K(f_d) = \left[ \frac{-\sqrt{C}\lambda(6w(f_d)^2 + \lambda^2 mr_d)}{2r_d w(f_d)}, \frac{3\sqrt{C}\lambda^2 m}{2w(f_d)}, \frac{3C\lambda}{2r_d} + R \right]$  where  $\lambda = -40$  is the triple eigenvalues of the linearized closed loop system. The unknown disturbance  $f_d$  in (9) is substituted with the estimated value  $\hat{f}_d$  in the closed-loop control system as shown in Fig. 2.

In order to compare the simulation result of the proposed control scheme in this paper, we introduce the feedback linearizing controller from [11]. It is easily shown that  $\{g, ad_f g, ad_f^2 g\}$  is

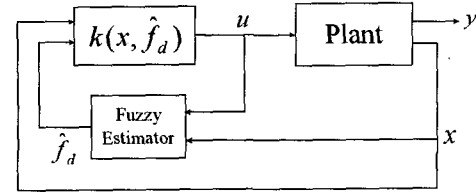


Fig. 2. Overall control system of fuzzy-estimated gain scheduling.

linear independent and the distribution  $\text{span}\{g, ad_f g\}$  is involutive where

$$g = \begin{bmatrix} 0 \\ 0 \\ \frac{2x_1}{C} \end{bmatrix} \quad (10)$$

$$ad_f g = \begin{bmatrix} 0 \\ \frac{x_3}{mx_1} \\ \frac{4Rx_1^2}{C^2} \end{bmatrix} \quad (11)$$

$$ad_f^2 g = \begin{bmatrix} \frac{-x_3}{mx_1} \\ 0 \\ \frac{4R}{C^2}x_1x_2 - \frac{x_3^2}{mx_1^2} + \frac{8Rx_1^3}{C^3} \end{bmatrix} \quad (12)$$

Moreover, for an input-state feedback linearization, a diffeomorphism is computed by

$$z = T(x) = \begin{bmatrix} x_1 - r_d \\ x_2 \\ -\frac{Cx_3^2}{4mx_1^2} + g \end{bmatrix} \quad (13)$$

Then, the resulting feedback linearizing controller is obtained by

$$u = m \frac{x_1}{x_3} (k_1 z_1 + k_2 z_2 + k_3 z_3 + \frac{Rx_3^2}{mx_1}) \quad (14)$$

where  $k_1 = 343000$ ,  $k_2 = 14700$ , and  $k_3 = 210$ .

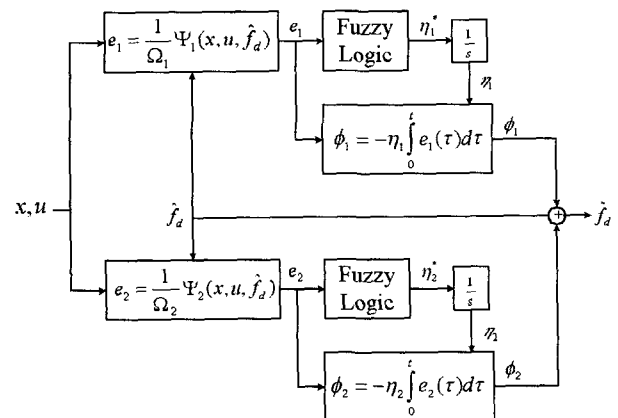


Fig. 3. Structure of fuzzy estimator.

### III. Fuzzy estimator

Since  $\hat{f}_d$  drives the stabilizing gain of the controller  $k(x, \hat{f}_d)$ , it should be appropriately estimated in the way of stabilizing the closed loop system. Since both  $x_{\delta_3}$  and  $u_\delta$  are the functions of  $f_d$ , we define  $\psi \in R^2$  such that

$$\psi(x, u, f_d) = \begin{bmatrix} x_{\delta_3} \\ u_\delta \end{bmatrix} = \begin{bmatrix} x_3 - \frac{2r_d w(f_d)}{\sqrt{C}} \\ u - \frac{2r_d R w(f_d)}{\sqrt{C}} \end{bmatrix} \quad (15)$$

Here, we note that  $\psi(\cdot)$  is a vector composed of the elements of  $x_\delta$  and  $u_\delta$  whose derivatives with respect to  $f_d$  are nonzero. In order to estimate  $f_d$ , we define cost functions

$$e_i = \frac{1}{\Omega_i} \psi_i(x, u, \hat{f}_d) \quad (16)$$

for  $i \in \{1, 2\}$  with  $\Omega_i = \frac{\partial \psi_i(x, u, 0)}{\partial \hat{f}_d}$  where  $e_i$  is engaged to the input of the fuzzy rule-table and  $\hat{f}_d$  is the estimate of  $f_d$ . Furthermore, based on (16) we propose an adaptation law of  $\phi_i$  such that

$$\dot{\phi}_i = -\eta_i \int_0^t e_i d\tau \quad (17)$$

for  $i \in \{1, 2\}$  where  $\phi_i$  is the estimated portion of  $f_d$  contributed from  $\psi_i$ , and  $\eta_i$  is an adaptation rate computed by the fuzzy logic in Fig. 3.

Before we proceed further, we consider the case when  $\eta_2$  is fixed to zero for simplicity, i.e.,  $\phi_2 = 0$  in order to give an intuition of the adaptation law. For the notational convenience, we let

$$\xi = -\frac{\Omega_1}{\eta_1} \phi_1 = \Omega_1 \int_0^t e_1 d\tau \quad (18)$$

Then, from (18) we note that if  $\phi_1 = f_d$  is achieved,  $\xi$  converges to  $\xi^{eq} = -\frac{\Omega_1 f_d}{\eta_1}$ . Furthermore, using the relation of  $\hat{f}_d = \phi_1 = -\frac{\eta_1}{\Omega_1} \xi$  Taylor series expansion of  $\xi$  about  $\hat{f}_d = 0$  in (18) gives

$$\begin{aligned} \dot{\xi} &= \psi_1(x, u, 0) + \frac{\partial \psi_1(x, u, 0)}{\partial \hat{f}_d} \hat{f}_d + o(\hat{f}_d) \\ &= \psi_1(x, u, 0) - \eta_1 \xi + o(\hat{f}_d) \end{aligned} \quad (19)$$

where  $o(\hat{f}_d) = \psi_1(x, u, \hat{f}_d) - \psi_1(x, u, 0) - \frac{\partial \psi_1(x, u, 0)}{\partial \hat{f}_d} \hat{f}_d$ .

In order to investigate the convergence of the adaptation law of the proposed scheme intuitively, we assume that for a sufficiently large  $\eta_1$ ,  $\xi$  has faster dynamics than  $x_\delta$ . Then, we evaluate the time derivative of  $V = \frac{1}{2}(\xi - \xi^{eq})^2$  and obtain

$$\begin{aligned} \dot{V} &= (\xi - \xi^{eq}) \dot{\xi} \\ &= (\xi - \xi^{eq})(\psi_1(x, u, 0) - \eta_1 \xi + o(\hat{f}_d)) \end{aligned} \quad (20)$$

Since  $\psi_1(x, u, 0) - \eta_1 \xi^{eq} + o(\hat{f}_d) = 0$  at equilibrium point  $\xi = \xi^{eq}$ , by choosing a sufficiently large value of  $\eta_1$  it can be derived that

$$\psi_1(x, u, 0) - \eta_1 \xi + o(\hat{f}_d) \begin{cases} > 0 & \text{if } \xi < \xi^{eq} \\ = 0 & \text{if } \xi = \xi^{eq} \\ < 0 & \text{if } \xi > \xi^{eq} \end{cases} \quad (21)$$

Then,  $\dot{V} < 0$  is guaranteed regardless of  $\xi$  and we conclude that  $\lim_{t \rightarrow \infty} \xi(t) = \xi^{eq}$  and  $\lim_{t \rightarrow \infty} \phi_1 = f_d$  in (18). The above explanation can also be made for the case when  $\eta_1$  is fixed to zero similarly. However, it is noted that there are two remaining problems in the developed adaptation scheme above. First, although  $\phi_i$ ,  $i \in \{1, 2\}$  are computed for the two adaptation laws, there is no rule to combine the adaptation laws. Second, although the adaptation rate  $\eta_i$  are required to be sufficiently large, it is a plant-dependent problem requiring much trial and error procedure in real practice. Thus, in order to overcome these problems we introduce a fuzzy logic to combine the adaptation laws and to determine the adaptation rates simultaneously.

Table 1. Fuzzy rule-table for  $i \in \{1, 2\}$ .

$e_i \setminus \dot{e}_i$	NB	NS	ZE	PS	PB
NB	NB	NS	ZE	PS	PB
NS	NS	NS	ZE	PS	PS
ZE	ZE	ZE	ZE	ZE	ZE
PS	PS	PS	ZE	NS	NS
PB	PB	PS	ZE	NS	NB

As shown in Fig. 3 the fuzzy logic computes the increment of adaptation rate  $\eta_i^*$  by using both  $e_i$  and  $\dot{e}_i (= \frac{de_i}{dt})$  as inputs and  $\tilde{\eta}_i$  as a output of the rule-table in Table 1 for  $i \in \{1, 2\}$ . The associated membership functions  $M_e^i(e_i)$ ,  $M_{\dot{e}_i}^i(\dot{e}_i)$ , and  $M_{\tilde{\eta}_i}^i(\tilde{\eta}_i)$  are constructed as shown in Fig. 4 where  $e_i^{max}$ ,  $de_i^{max}$ , and  $\tilde{\eta}_i^{max}$  are positive constants. The rule-table is constructed and is expandable by the following concept. In the case of  $e_i \dot{e}_i < 0 (> 0)$ ,  $\tilde{\eta}_i$  is selected by a positive (negative) fuzzy value, which means that  $\eta_i$  satisfying  $e_i \dot{e}_i < 0 (> 0)$  is a winner (loser) and gains a corresponding positive (negative) increment  $\eta_i^*$ . Moreover,  $|\tilde{\eta}_i|$  becomes decreased as  $|e_i \dot{e}_i|$  decreases regardless of the sign of  $e_i \dot{e}_i$ , which means that as  $\eta_i$  becomes a winner (loser) more frequently, the amount of the increment  $\eta_i^*$  becomes smaller (larger). We note that the number of winners is not fixed to one but varying as time goes on. Thus, in this manner, the combined output membership function  $\mu_i$  is obtained by reasoning process. In defuzzification, the center of gravity method is adopted such that  $\eta_i^* = \frac{\int \mu_i(\tilde{\eta}_i) \tilde{\eta}_i d\tilde{\eta}_i}{\int \mu_i(\tilde{\eta}_i) d\tilde{\eta}_i}$  where  $\eta_i^*$  is the defuzzified value of the increment and  $\mu_i(\tilde{\eta}_i)$  is the value of the combined output membership function corresponding to  $\tilde{\eta}_i$ . Then, the adaptation rate  $\eta_i$  is updated by the increment of  $\eta_i^*$ , i.e.,  $\dot{\eta}_i = \eta_i^*$  and the estimated disturbance is determined by  $\hat{f}_d = \sum_{i=1}^2 \phi_i$ .

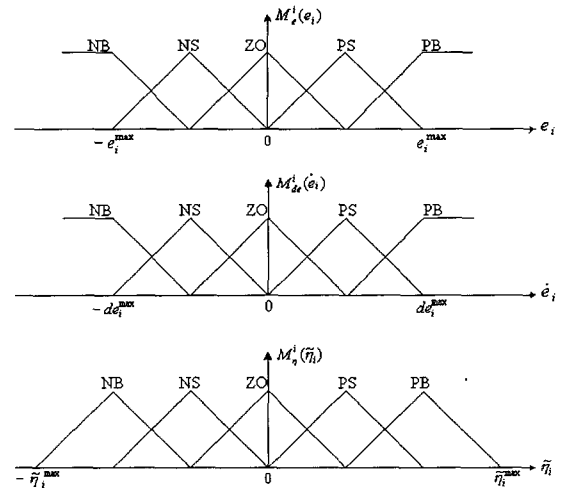


Fig. 4. Membership functions for  $i \in \{1, 2\}$ .

#### IV. Simulation results

The simulation of the magnetic suspension system is executed using the proposed control scheme (9) with the fuzzy estimator, while comparing with the feedback linearization method (14). Both control schemes are investigated for the cases of the constant and the sinusoidally varying external force disturbance. For the construction of the fuzzy estimator, its associated

Table 2. Coefficients of membership functions.

$i$	$e_{max}$	$de_{max}$	$\tilde{\eta}_{max}$
1	0.02	0.002	50
2	0.001	0.0002	50

coefficients are chosen as Table 2. Fig. 5-6 shows that the output responses for the constant disturbance case. We note that the adaptation rates with initial values of zeros converge to a certain constant value in the steady state. Moreover, the fuzzy estimator shows good estimation of the unknown disturbance while the controlled output achieves the control objective (i.e., zero steady-state error), whereas the feedback linearizing controller does not. Here, we note that the steady-state error by the linearizing controller is computed as  $\frac{k_3 f_d}{m k_1}$  from the result of [11]. In Fig. 7-8, the similar enhanced performance is observed for the sinusoidally varying disturbance, while achieving more reduced ultimate output error than the feedback linearization.

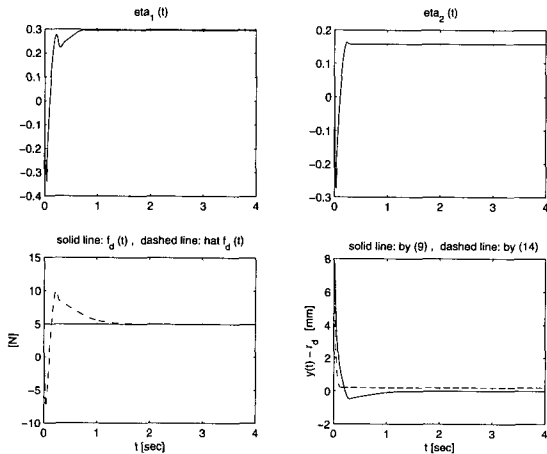


Fig. 5. Output response of the proposed control scheme for the constant disturbance:  $f_d(t) = 5$ .

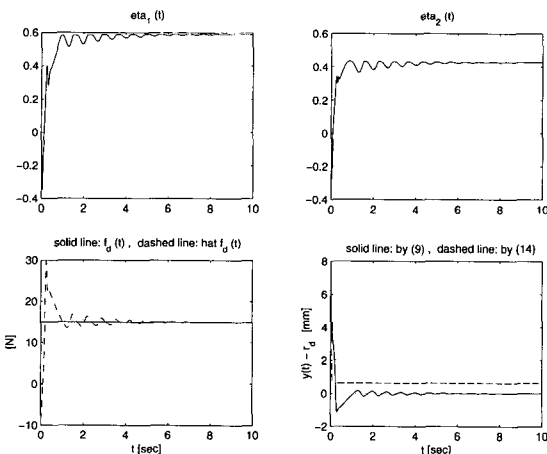


Fig. 6. Output response of the proposed control scheme for the constant disturbance:  $f_d(t) = 15$ .

VI. Conclusions

In this paper, we develop a fuzzy estimator for the gain scheduling control of the magnetic suspension experiencing the external force disturbance. The fuzzy estimator computes the

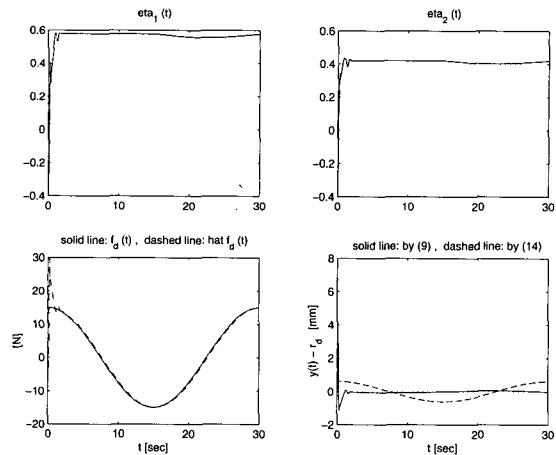


Fig. 7. Output response of the proposed control scheme for the sinusoidal disturbance:  $f_d(t) = 15 \sin(2\pi/30t)$ .

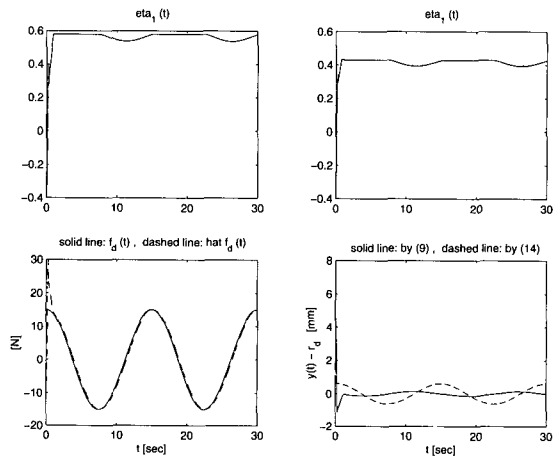


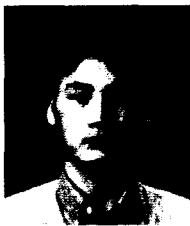
Fig. 8. Output response of the proposed control scheme for the sinusoidal disturbance:  $f_d(t) = 15 \sin(2\pi/15t)$ .

unknown disturbance injected to the plant and the gain scheduled controller is driven according to the estimated disturbance. The fuzzy logic combines the adaptation laws and determines their rates adequately at the same time. Furthermore, since the fuzzy estimator is constructed based on the general gain scheduling idea, it is applicable to the other existing gain scheduled controllers.

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Seon-Ho Lee

He received the B.S.E.E degree from Kyungpook National University, Korea in 1994, the M.S.E.E. degree from KAIST in 1996, and the Ph.D. degree in Electrical Engineering from KAIST in 2000. He is currently a senior researcher at KARI. His research interests are in the

areas of system and control theory, communications networks, and satellite control systems.



Jong-Tae Lim

He received the B.S.E.E degree from Yonsei University, Korea in 1975, the M.S.E.E. degree from the Illinois Institute of Technology, Chicago in 1983, and the Ph.D. degree in Computer, Information and Control Engineering from the University of Michigan, Ann Arbor in 1986. From 1975 to 1981, he worked as an engineer in Korea Electric Company. From 1987 to 1988, he was a research fellow at the University of Michigan. He is currently a professor in the Dep. of Electrical Engineering at KAIST. His research interests are in the areas of system and control theory, communications networks, and manufacturing systems. He is a member of IEEE, KIEE, KITE, and ICASE.