

Inversion-Based Robust Output Tracking of Differentially Flat Nonlinear Systems

Jin Man Joo, Jin Bae Park, and Yoon Ho Choi

Abstract: In this study, we propose a two degree of freedom robust output tracking control method for a class of nonlinear systems. We consider hyperbolically nonminimum phase single-input single-output uncertain nonlinear systems. We also consider the case that the nominal input-state equation is differentially flat. Nominal stable state trajectory is obtained in the flat output space via the flat output. Nominal feedforward control input is also computed from the nominal state trajectory. Due to the nature of the method, the generated flat output trajectory and control input are noncausal. Robust feedback control is designed to stabilize the system around the nominal trajectory. A numerical example is given to demonstrate that robust output tracking is achieved.

Keywords: differential flatness, flat output, stable inversion, robust output tracking

I. Introduction

Tracking control and regulation are common problems in applications and have attracted considerable attention from control researchers. Several important results have been obtained for output tracking. Asymptotic tracking problem has been solved when the system is minimum phase [5] by many authors and approximate tracking has also been solved when the system is nonminimum phase [2][4]. Recently in [1] a noncausal inversion-based approach to exact nonlinear output tracking control for systems with hyperbolic zero dynamics has been proposed. The method of noncausal inversion tries to find a stable solution for the full state trajectory by steering from the unstable zero dynamics manifold to the stable zero dynamics manifold. The solution is found by repeatedly solving a two point boundary value problem for the linearized zero dynamics driven by the desired trajectory. In this paper we adopt the approach in [1] and further develop the method for a class of nonlinear systems. We consider hyperbolically nonminimum phase input-output systems whose input-state systems are differentially flat.

Differentially flat systems are underdetermined systems of (nonlinear) ordinary differential equations whose solution curves are in smooth one-to-one correspondence with arbitrary curves in a space whose dimension equals the number of equations by which the system is underdetermined [3][9]. The components of the map from the system space to the smaller dimensional space are referred to as the flat outputs. Typically the flat outputs may depend on the original independent and dependent variables in terms of which the ordinary differential equations are written as well as finitely many derivatives of the dependent variables [3]. For single-input systems, single flat output exists and can be written as a function of states only. In addition, feedback linearizability implies differential flatness for single-input systems [11]. However, the flat output has useful geometric

properties for both trajectory generation and feedback stabilization.

For the problem of generation of the stable trajectory, as mentioned above, flat output has an one-to-one correspondence with the states and hence, finding noncausal stable solution of the internal dynamics can be completely formulated in terms of the flat output. This formulation has an computational advantage that the resulting nonlinear ordinary differential equation consists of a single variable with specified inputs and then can be treated more easily. We only need to solve one ordinary differential equation by Picard-like iteration.

For the feedback stabilization problem, the flat output and its derivatives serves as nominal coordinate transformation functions. While it is possible to confirm or exclude the existence of feedback linearizing control laws by testing some properties of the vector fields, unfortunately, there is no general systematic approach to construct analytically the local diffeomorphism, hence the feedback linearizing control. In practice, it is customary to approach the problem by trial and error, using the notion of relative degree, i.e., finding the output function having relative degree equals the dimension of the system. However, the flat output can perfectly serve as the given output function for the input-state linearization problem. With this approach, we linearize the uncertain system around the nominal desired flat output trajectory into linear error dynamics with uncertainty. Based on this linear error dynamics, robust feedback control is designed.

For obvious reasons, the approach of this paper can be regarded as the two degree of freedom design: the generation of stable flat output trajectory and design of robust feedback stabilizing control.

II. Absolute equivalence and differential flatness

In this section we shall introduce the notions of Pfaffian systems, Cartan prolongations and absolute equivalence and provide a definition of differential flatness in terms of absolute equivalence. We assume that all manifolds and mappings are smooth (C^∞) unless explicitly stated otherwise.

Definition 1: A Pfaffian system I on a manifold M is a submodule of the module of differential one-forms $\Omega^1(M)$ over the commutative ring of smooth functions $C^\infty(M)$. A set of one-forms $\omega^1, \dots, \omega^n$, generates a Pfaffian system $I =$

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$$\{\omega^1, \dots, \omega^n\} = \{\sum f_k \omega^k \mid f_k \in C^\infty(M)\}.$$

We restrict our attention to finitely generated Pfaffian systems on finite dimensional manifolds. We treat dt as an *independence condition*, i.e., a one-form that is not allowed to vanish on any of the solution curves. For $p \in M$, the *codimension at p* of the system is $\dim M - \dim I(p)$. A system is *trivial* if $I = \{0\}$.

In local coordinates, control systems can be written in the form:

$$I = \{dx_1 - f_1(x, u, t)dt, \dots, dx_n - f_n(x, u, t)dt\}$$

with states $\{x_1, \dots, x_n\}$ and control inputs $\{u_1, \dots, u_p\}$.

Definition 2: Let (I, dt) be a Pfaffian system on a manifold M . The derived systems of I are $I^{(0)} = I$ and, for each $k \geq 0$,

$$I^{(k+1)} = \{\omega \in I^{(k)} \mid d\omega = 0 \pmod{I^{(k)}}\}.$$

If we consider the equivalence between two systems, we usually try to find nonsingular transformations associated with two systems. However, we may consider the equivalence in the sense of a solution curve. Then one can say that two systems are equivalent if they have the same solution curve regardless of the dimension of each system. We introduce the Cartan prolongation and absolute equivalence which give rise to a more general notion of equivalence between systems that live in spaces of possibly different dimensions.

Definition 3: [Cartan Prolongation [10][9]]: Let (I, dt) be a Pfaffian system on a manifold M . Let B be a manifold such that $\pi : B \rightarrow M$ is a fiber bundle. A Pfaffian system $(J, \pi^* dt)$ on B is a Cartan prolongation of the system (I, dt) if the following conditions hold:

1. $\pi^*(I) \subset J$
2. For every integral curve of I , $c : (-\epsilon, \epsilon) \rightarrow M$, there is a unique lifted integral curve of J , $\tilde{c} : (-\epsilon, \epsilon) \rightarrow B$ with $\pi \circ \tilde{c} = c$.

Definition 4: [Absolute Equivalence [10]] Two systems I_1 and I_2 are called absolutely equivalent if they have Cartan prolongations J_1 and J_2 respectively, that are equivalent in the usual sense, i.e., there exists a diffeomorphism ϕ such that $\phi^*(J_2) = J_1$.

Theorem 1: [[10]] A system (I, dt) on M is differentially flat if and only if it is absolutely equivalent to the trivial system $I_t = (\{0\}, dt)$ on a manifold N .

If (t, y_1, \dots, y_p) are local coordinates on N then (y_1, \dots, y_p) are a set of *flat outputs*. Observe that the number of flat outputs is p where $p + 1$ is the codimension of system (I, dt) on M . If the system is a control system then p is also the number of inputs. The absolute equivalence problem has been completely solved by Élie Cartan for codimension 2 systems. All Cartan prolongations are locally equivalent to total prolongations [10]. Starting with any system, taking derived systems enables one to “strip off” prolongations and reach the “core” system, which is not a total prolongation of any system. For differentially flat systems, the core is trivial.

1. Flatness for single input systems

For single input control systems, the corresponding differential system has codimension 2. There are a number of results available in codimension 2 systems which allow us to give a complete characterization of differentially flat single-input control systems [7][9]-[11]. In codimension 2, every Cartan prolon-

gation is a total prolongation around every point of the fibered manifold [10]. This allows the following.

Theorem 2: [[9]] Let I be a time invariant control system:

$$I = \{dx_1 - f_1(x, u)dt, \dots, dx_n - f_n(x, u)dt\},$$

where u is a scalar control, i.e., the system has codimension 2. If I is differentially flat around an equilibrium point, then I is feedback linearizable by static feedback at that equilibrium point.

Theorem 3: [[9]] If a time invariant single input system is differentially flat we can always take the flat output as a function of the states only: $z_f = \mu(x)$.

Theorem 4: [[10]] A system (I, dt) of constant codimension 2 is differentially flat if and only if

1. $\dim I^{(i)} = \dim I^{(i-1)} - 1$, for $i = 0, \dots, n = \dim I$. This implies $I^{(n)} = \{0\}$

2. The system $I^{(i)} + \{dt\}$ is integrable for each $i = 0, \dots, n$. These results can only be applied to codimension 2 systems. The characterization of flatness in systems of codimension higher than 2 remains open problem.

III. The stable inversion problem

1. Inversion-Based output tracking scheme

Here we describe how the inversion approach is used to develop output tracking controller. Consider an SISO system described by

$$\begin{aligned} \dot{x}(t) &= \tilde{f}(x(t), \theta) + g(x(t))u(t) \\ y(t) &= h(x(t)) \end{aligned} \quad (1)$$

where $\tilde{f}(\cdot)$ and $g(\cdot)$ are C^∞ vector fields defined on a dense submanifold $X \subset \mathbb{R}^n$, u is a scalar control input, $h(\cdot)$ is a C^∞ function, and y is a scalar output. θ is an unknown parameter vector. It is assumed that \tilde{f} is a smooth vector field for every $\theta \in B \subset \mathbb{R}^l$, where B is a compact set. The nominal parameter vector θ^n is assumed to be known and the perturbation about θ^n are represented as $\theta = \theta^n + \delta\theta$. And assume that the uncertain parameter vector θ appears linearly in (1). Then the vector field \tilde{f} can be written

$$\tilde{f}(x(t), \theta) = f(x(t)) + \Delta f(x(t)).$$

The nominal system can be written as

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)). \end{aligned} \quad (2)$$

Let $y_d(\cdot)$ be the desired output trajectory to be tracked. In the noncausal inversion-based approach we first find a bounded nominal input-state trajectory $\{u_d(\cdot), x_d(\cdot)\}$ that satisfies the system (2) and yields the desired output exactly, i.e.,

$$\begin{aligned} \dot{x}_d(t) &= f(x_d(t)) + g(x_d(t))u_d(t) \\ y_d(t) &= h(x_d(t)), \quad \forall t \in (-\infty, \infty). \end{aligned} \quad (3)$$

Secondly, we use the exact output yielding input trajectory $u_d(\cdot)$ as feedforward and the system is stabilized by using feedback.

2. The internal dynamics

In this subsection, it is shown that finding the inverse input-state trajectory is equivalent to finding bounded solutions to the

system's internal dynamics. Assume that the system (2) has a well-defined relative degree r . The well-defined relative degree assumption enables the system equations to be rewritten, through a coordinate transformation [8]

$$x(t) = T(\xi(t), \eta(t)) \quad (4)$$

in the following form:

$$\begin{aligned} \dot{\xi}_1(t) &= \xi_2(t) \\ &\vdots \\ \dot{\xi}_r(t) &= \alpha(\xi(t), \eta(t)) + \beta(\xi(t), \eta(t))u(t) \\ \dot{\eta}(t) &= s(\xi(t), \eta(t)) \\ y(t) &= \xi_1(t). \end{aligned} \quad (5)$$

Note that the desired $\xi(\cdot)$ is known when the desired output trajectory $y_d(\cdot)$ and its time derivatives are specified. The desired $\xi(\cdot)$ is defined as $\xi_d(\cdot)$. If the output tracking is achieved then the control law for maintaining exact tracking can be written from (5) as

$$u_d(t) = [\beta(\xi_d(t), \eta(t))]^{-1} [\dot{\xi}_d(t) - \alpha(\xi_d(t), \eta(t))] \quad (6)$$

which results in state equations of the form

$$\begin{aligned} y^{(r)}(t) &= \dot{\xi}_d(t) \\ \dot{\eta}(t) &= s(\xi_d(t), \eta(t)). \end{aligned} \quad (7)$$

This is the inverse system and (7) is referred to as the internal dynamics. Solving the internal dynamics is a key to finding the inverse input and state trajectories. If a bounded solution, $\eta_d(\cdot)$, to the internal dynamics (7) can be found, then the feedforward input can be found through (6) as

$$u_d(t) = [\beta(\xi_d(t), \eta_d(t))]^{-1} [\dot{\xi}_d(t) - \alpha(\xi_d(t), \eta_d(t))] \quad (8)$$

and the reference state trajectory can be found as

$$x_d(t) = T(\xi_d(t), \eta_d(t)). \quad (9)$$

IV. Robust tracking controller design

1. Computation of the inverse

Standard inversion scheme that integrate (forward in time) the internal dynamics (7) lead to unbounded solutions if the origin of the internal dynamics is unstable (nonminimum phase systems). Noncausal inversion leads to bounded but noncausal solution of the internal dynamics. In this subsection we describe how noncausal inversion can be simplified if the system is differentially flat. We first briefly describe the approach in [1]. Consider the system (7) which describes the internal dynamics:

$$\dot{\eta}(t) = s(\xi_d(t), \eta(t)). \quad (10)$$

If we linearize (10), we can rewrite the equation in the following form.

$$\dot{\eta}(t) = A\eta(t) + \phi(\eta(t), \xi_d(t)), \quad (11)$$

where A represents the (hyperbolic) linearization of the dynamics, and $\phi(\cdot, \cdot)$ is the residual error in the linearization. We assume that $\phi(0, 0) = 0$ and $\phi(\cdot, \cdot)$ satisfies the Lipschitz condition

$$\|\phi(x, u) - \phi(y, v)\| \leq K_1\|x - y\| + K_2\|u - v\|. \quad (12)$$

Devasia-Chen-Paden [1] solve for the "steady state" response of the system (11) on $(-\infty, \infty)$, given $\xi_d(t) : -\infty < t < \infty$. We assume that the variables η have been partitioned into η_1 and η_2 such that

$$A = (A_1, A_2)$$

with the eigenvalues of A_1 in \mathbb{C}_- (complex open left half plane) and those of A_2 in \mathbb{C}_+ (complex open right half plane). Then we can rewrite (11) as:

$$\begin{aligned} \dot{\eta}_1(t) &= A_1\eta_1(t) + \phi_1(\eta(t), \xi_d(t)) \\ \dot{\eta}_2(t) &= A_2\eta_2(t) + \phi_2(\eta(t), \xi_d(t)). \end{aligned}$$

Define the state transition function of A on $(-\infty, \infty)$ as

$$\Phi(t) = (e^{A_1 t} \mathbf{1}(t), -e^{A_2 t} \mathbf{1}(-t)), \quad (13)$$

where $\mathbf{1}(t)$ represents the unit step function. By using the variation of constants formula [6], we have

$$\eta_1(t) = \int_{-\infty}^t e^{A_1(t-\tau)} \phi_1(\eta(\tau), \xi_d(\tau)) d\tau \quad (14)$$

$$\eta_2(t) = \int_{\infty}^t e^{A_2(t-\tau)} \phi_2(\eta(\tau), \xi_d(\tau)) d\tau. \quad (15)$$

It may be verified that a bounded solution to (11) on $(-\infty, \infty)$ must satisfy the integral equation

$$\eta(t) = \int_{-\infty}^{\infty} \Phi(t-\tau) \phi(\eta(\tau), \xi_d(\tau)) d\tau. \quad (16)$$

Denoting by \mathcal{N} the integral operator given by

$$\mathcal{N}(\eta(\cdot))(\sqcup) = \int_{-\infty}^{\infty} \Phi(\sqcup - \tau) \phi(\eta(\tau), \xi_d(\tau)) d\tau.$$

If the L_1 norm of Φ is M , then

$$\|\mathcal{N}(\S) - \mathcal{N}(\dagger)\|_{\infty} \leq MK_{\infty} \|\S - \dagger\|_{\infty}.$$

The same estimate holds for the L_1 norm of $\mathcal{N}(\S) - \mathcal{N}(\dagger)$ as well. Thus, when $MK_1 < 1$, the map \mathcal{N} is a contraction map, and the solution to (16) exists, is unique, and may be found by the Picard-Lindelöf iteration scheme

$$\eta_{n+1}(\cdot) = \mathcal{N}(\eta_n(\cdot)). \quad (17)$$

The fixed point of the map \mathcal{N} is the so-called "steady state response" of the system (11). For detailed assumptions and proofs, refer to [1].

Now we describe how the above procedure can be simplified by utilizing the flat output. Note that the flat output can sum up the whole dynamic behavior of the system. In other words, the flat output has a one-to-one correspondence with the system state. Therefore, if we compute bounded desired flat output trajectory from the desired output profile directly, we can compute the full state trajectory and the feedforward control input by algebraic mapping. By Theorem 3, we can obtain the flat output of the nominal system:

$$z_f(t) = \mu(x(t)). \quad (18)$$

And from the definition of the flat output, the system states can be written as a function of the flat output and its derivatives, i.e., there exists a surjective submersion $\psi(\cdot)$ such that,

$$x(t) = \psi(z_f(t), \dot{z}_f(t), \dots, z_f^{(m)}(t)), \quad (19)$$

for some integer m . The upper bound for the integer m exists, for details refer to [12]. Consider only the purely unstable partition of the internal dynamics, i.e., $\eta_2(\cdot)$:

$$\dot{\eta}_2(t) = A_2\eta_2(t) + \phi_2(\eta(t), \xi_d(t)). \quad (20)$$

By the relationship (4) and (19), equation (20) can be rewritten as the following form:

$$\dot{\zeta}(t) = \gamma(\zeta(t), \xi_d(t)) \quad (21)$$

where $\zeta(t) = [z_f(t), \dot{z}_f(t), \dots, z_f^{(m)}(t)]^T$. And it is a set of ordinary differential equations consist of a single variable $z_f(t)$ with $\xi_d(\cdot)$ input. Then, by the equation (15), we can compute the bounded solution of the flat output rather than compute all the bounded solution of the internal dynamics. Once the solution $\zeta(t)$ is found, we can easily compute the nominal stable but noncausal state trajectory $x_d(t)$ by (19). The nominal feedforward control input is calculated by (8) and it is also noncausal. This approach has a computational advantage, since we only need to consider the unstable partition of the internal dynamics. We solve a set of differential equations in the single variable backward in time.

2. Feedback control design

The next task is to design feedback control to stabilize the uncertain system (1) around the nominal state trajectory. However, we shall linearize the uncertain system around the nominal flat output trajectory rather than around the output variable. Hence, the tracking error dynamics is obtained from the error between nominal flat output trajectory and real flat output trajectory. We shall now illustrate how to construct coordinate transformation functions and design robust stabilizing feedback control utilizing the flat output. From here, we drop the argument t for notational ease. We assume that the uncertain vector field $\Delta f(x)$ satisfies matching condition, i.e.: The vector field $\Delta f(x)$ satisfies

$$\Delta f(x) \in \{d\mu(x), dL_f\mu(x), \dots, dL_f^{n-2}\mu(x)\}. \quad (22)$$

Define a new state as

$$z_1 \equiv z_f = \mu(x). \quad (23)$$

Based on this coordinate, we can define coordinate transformation functions as

$$\begin{aligned} z_1 &= z_f \\ z_2 &= L_f\mu(x) \\ z_3 &= L_f^2\mu(x) \\ &\vdots \\ z_n &= L_f^{n-1}\mu(x). \end{aligned} \quad (24)$$

In the new coordinates, we have

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_n &= L_f^n\mu(x) + L_gL_f^{n-1}\mu(x)u_f + L_{\Delta f}L_f^{n-1}\mu(x). \end{aligned} \quad (25)$$

Here, we abuse a notation u_f to discriminate feedback control input u_f from the feedforward control input u_d . Next, we define error $e_i = z_i - z_{fd}^{(i-1)}$. Here, z_{fd} represents the desired flat output trajectory. Then the error dynamics can be represented:

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ &\vdots \\ \dot{e}_n &= L_f^n\mu(x) + L_gL_f^{n-1}\mu(x)u_f + L_{\Delta f}L_f^{n-1}\mu(x) - z_{fd}^{(n)}. \end{aligned} \quad (26)$$

For the system (26), one can calculate the robust stabilizing feedback control by solving the following equation in terms of u_f :

$$L_f^n\mu(x) + L_gL_f^{n-1}\mu(x)u_f - z_{fd}^{(n)} = \sum_{i=1}^n k_i e_i + v \quad (27)$$

where k_i 's are constant coefficients such that the associated polynomial $s^n - k_n s^{n-1} - \dots - k_1$ is Hurwitz and v is a control variable to compensate the uncertainty $L_{\Delta f}L_f^{n-1}\mu(x)$. By the control input in (26), we can write error dynamics as:

$$\dot{e} = (A + BK)e + Bv + BL_{\Delta f}L_f^{n-1}\mu(x) \quad (28)$$

where the pair (A, B) is in Brunovsky canonical form and $K = [k_1, \dots, k_n]$ is a feedback gain vector. Define a bounding function for the uncertainty in (28)

$$\rho(x) \geq \|L_{\Delta f}L_f^{n-1}\mu(x)\|. \quad (29)$$

If we design v such that

$$v = \begin{cases} -\frac{\rho(x)}{\|B\|}, & e^T P B \geq 0 \\ \frac{\rho(x)}{\|B\|}, & e^T P B < 0 \end{cases} \quad (30)$$

where P is the unique symmetric positive definite solution to the Lyapunov equation

$$P(A + BK) + (A + BK)^T P = -Q \quad (31)$$

with Q a given symmetric positive definite matrix. Then the error dynamics of (28) is exponentially stable with the Lyapunov function

$$V(e) = e^T P e.$$

The overall control design completes if we let

$$u = u_d + u_f. \quad (32)$$

Figure 1 represents the overall two degree of freedom design approach.

V. An illustrative example

To demonstrate the approach, consider the following SISO uncertain nonlinear system

$$\begin{aligned} \dot{x}_1 &= -\theta x_1^2 + 2x_2 + u \\ \dot{x}_2 &= x_1 + x_2 \\ y &= x_1 \end{aligned} \quad (33)$$

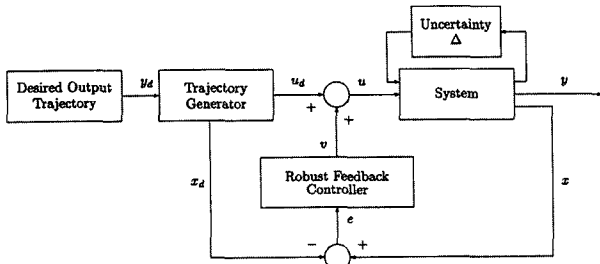


Fig. 1. Block diagram of the two degree of freedom design.

where θ is an uncertain parameter. We can see that the uncertainty satisfies Assumption. To check whether the nominal input-state system is flat, we shall calculate the derived systems. The Pfaffian system of the nominal input-state system of (33) is as follows:

$$I = \{\omega_1, \omega_2\}$$

$$= \{dx_1 + \theta^n x_1^2 dt - 2x_2 dt - u dt, dx_2 - x_1 dt - x_2 dt\}.$$

From the definition of the derived systems, we have to check all the span of one-forms in I whether the condition holds. Let us choose ω_2 , then the exterior differentiation of ω_2 is given:

$$d\omega_2 = -dx_1 \wedge dt - dx_2 \wedge dt.$$

We can easily find out that $d\omega_2 = 0 \pmod I$. Then we obtain the first derived system as follows:

$$I^{(1)} = \{dx_2 - x_1 dt - x_2 dt\}.$$

Taking derived systems only involves exterior differentiation and linear algebra. Finally, the second derived system is the following:

$$I^{(2)} = \{0\}.$$

Calculations also show that the system $I^{(i)} + \{dt\}$ is integrable for each $i = 0, 1, 2$. By Theorem 4 the nominal input-state system is differentially flat. The first derived system $I^{(1)}$ indicates that the flat output is given as $z_f = x_2$. We let desired output $y_d = \sin(t)$. And it can be easily seen that the internal dynamics of the system can be written

$$\dot{z}_f = z_f + y_d. \tag{34}$$

We can find noncausal stable solution z_{fd} of (34) by (15). Then nominal feedforward control input is calculated as

$$u_d = \dot{y}_d + \theta^n y_d^2 - 2z_{fd}.$$

And following the procedure explained in the previous section, we can design a robust stabilizing feedback control. Figure 2 represents the desired flat output trajectory and nominal feedforward control input. Noncausal nature of the method is clearly illustrated. We truncate the negative-time part to -5 seconds. Figure 3 depicts robust output tracking performance and overall control input. In Figure 3, we considered 30% uncertainty level of the parameters. Finally, Figure 4 shows output tracking error.

VI. Conclusions

In this study, we have presented a two degree of freedom robust output tracking control of a class of uncertain nonlinear

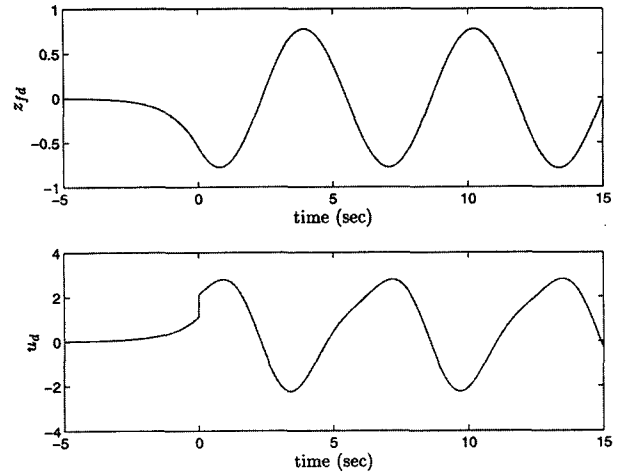


Fig. 2. Desired flat output and feedforward control input.

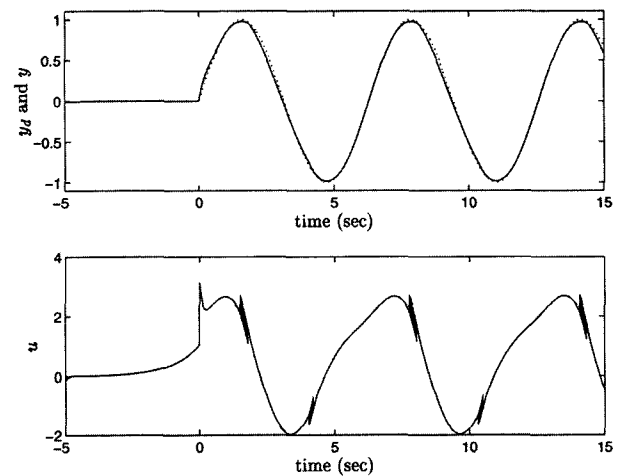


Fig. 3. Actual output (solid) and overall control input.

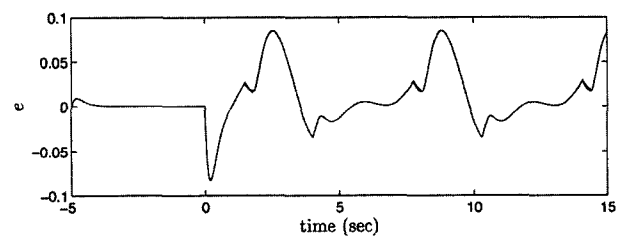


Fig. 4. Tracking error.

systems. Differentially flatness of the input-state system has been shown to be a useful property for the design of robust tracking controller. The flat output of the nominal input-state system has not only been utilized for the generation of bounded solution of the hyperbolically unstable internal dynamics but also for the design of robust feedback stabilizing control. Noncausal inversion problem was formulated in terms of the flat output and its derivatives by considering unstable partition of the internal dynamics. Nominal feedforward control input followed the noncausal solution of the internal dynamics. For robust stabilization problem, the flat output and its derivatives have shown to serve as the nominal coordinate transformation functions and we have designed robust feedback control based on the result-

ing linear error dynamics. Differential flatness has shown to be a useful geometrical property for the two degree of freedom design approach.

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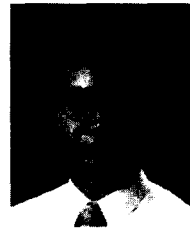
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