Inverse Dynamic Analysis of Flexible Multibody Systems with Closed-Loops

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The analysis of actuating forces (or torques) and joint reaction forces (or moments) are essential to determine the capacity of actuators, to control the system and to design the components. This paper presents an inverse dynamic analysis algorithm for flexible multibody systems with closed-loops in the relative joint coordinate space. The joint reaction forces are analyzed in Cartesian coordinate space using the inverse velocity transformation technique. The joint coordinates and the deformation modal coordinates are used as the generalized coordinates of a flexible multibody system. The algorithm is verified through the anaysis of a slider-crank mechanism.

Key Words : Flexible Multibody Systems, Inverse Dynamics, Inverse Velocity Transformation, Actuating Forces, Joint Reaction Forces

1. Introduction

The inverse dynamic analysis of multibody systems, which determines actuating forces required to produce a prescribed motion and joint reaction forces, is necessary to determine the capacity of actuators to control the robot manipulators and to design the components of the system.

The kinematic constraints that describe mechanical joints interconnecting each body and driving constraints that describe specified motion trajectories are expressed as nonlinear algebraic equations and these constraints are adjoined to the system equations of motion using the Lagrange's multiplier technique.

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Generalized constraint forces can be obtained from the Lagrange multipliers and constraint Jacobian matrix but the forces are neither actual joint reaction forces nor actual actuating forces when the Cartesian coordinates are used as the system coordinates. Chen and Shabana (1991) derived the procedure determining actual joint reaction forces of mutibody systems with flexible bodies from the constraint force vector in the Cartesian coordinate space. In case that relative joint coordinates are used as system generalized coordinates, actual joint actuating forces for driving constraints are directly obtained from the system equations of motion. So it is efficient to obtain the data for controlling the motion of the system. But in this case, since the kinematic constraints of all joints except cut-joint are not adjoined to the system equations of motion, the joint reaction forces should be computed recursively from the tree-end body along the inward path in the Cartesian coordinate space after analyzing the motion of the system. And studies on this procedure for systems with closed-loops are not yet performed.

In this paper, an inverse dynamic analysis algorithm for flexible multibody systems with closed loops in the relative joint coordinate space is presented.

The constraint acceleration equations expressed in Cartesian coordinates are derived in terms of joint coordinates using the velocity transformation technique (Jerkovsky, 1978). From the system equations of motion combined with the constraint equations, the actual joint actuating forces (torques) or joint reaction forces (moments) corresponding to the constraints (driving or cut-joint constraints) are computed. The reaction forces are analyzed in Cartesian coordinate space using the inverse velocity transformation matrix (Lee, 1997).

A slider crank mechanism with an angle driver is used as an numerical example in order to exemplify the formulation presented.

2. System Equations of Motion

The position coordinate vector of an arbitrary point on the deformable body i can be represented with a vector of Cartesian coordinates r^i, p^i and modal coordinates a^i, that is

$$x^i=[r^{iT}\ p^{iT}\ a^{iT}]^T \tag{1}$$

where r^i is the global position of the origin of the ith body reference frame and $p^i(=[p_0^i,\ p_1^i,\ p_2^i,\ p_3^i]^T)$ is the Euler parameters that represents the orientation of the body reference frame with respect to the inertial reference frame.

The relative joint coordinates q^i of body i are defined as relative rotational angles about joint axes and translational distances between that body and its reference body. Since the reference coordinates of the base body are inertial reference frames, the relative joint coordinates of the base body are Cartesian coordinates of that body.

This paper defines the generalized coordinates q^{i*} of a flexible body i as a collection of relative joint coordinates q^i and modal coordinates a^i of that body, that is

$$q^{i*}=[q^{iT}\ a^{iT}]^T \tag{2}$$

In general, the vector of Cartesian velocities and the time derivatives of modal coordinates $\dot{x}$ can be represented in terms of generalized velocities $\dot{q}^*$ using the velocity transformation matrix S:

$$\dot{x}=S\dot{q}^* \tag{3}$$

The time derivative of Eq. (3) yields the acceleration transformation equation.

$$\ddot{x}=S\ddot{q}^*+\dot{S}\dot{q}^* \tag{4}$$

where ($\ddot{}$) denotes second derivative with respect to time.

Using the Lagrange's equations with the transformation Eqs. (3) and (4), the equations of motion for a flexible multibody system can be written in terms of generalized coordinates as (Lee, 1993)

$$M^*\ddot{q}^*=Q^*-\boldsymbol{\Phi}_{q*}^T\lambda \tag{5}$$

where

$$M^*=S^TMS \tag{6}$$

$$Q^*=S^T[F-M\dot{S}\dot{q}^*-Kx-\dot{M}\dot{x}+\left(\frac{\partial T}{\partial x}\right)^T] \tag{7}$$

in which M and K are system mass and stiffness matrices respectively, F is a force vector in Cartesian coordinate space, T is the kinetic energy of the system, $\boldsymbol{\Phi}_{q*}$ is the Jacobian matrix of the constraint equations($\boldsymbol{\Phi}=0$) and λ is a vector of Lagrange multipliers associated with those constraints.

Combining the second time derivatives of the constraint equations($\boldsymbol{\Phi}_{q*}\ddot{q}^*=\lambda$) with Eq. (5) yields the following matrix form:

$$\begin{bmatrix} M^* & \boldsymbol{\Phi}_{q*}^T \\ \boldsymbol{\Phi}_{q*} & 0 \end{bmatrix}\begin{bmatrix} \ddot{q}^* \\ \lambda \end{bmatrix}=\begin{bmatrix} Q^* \\ \gamma \end{bmatrix} \tag{8}$$

where

$$\gamma=-\dot{\boldsymbol{\Phi}}_{q*}\dot{q}^*-\dot{\boldsymbol{\Phi}}_t \tag{9}$$

in which $\boldsymbol{\Phi}_t\equiv[\partial\boldsymbol{\Phi}/\partial t]$.

After the generalized coordinates, velocities are determined, the motion of the system can be described in the Cartesian coordinate space using the position and velocity transformation equations.

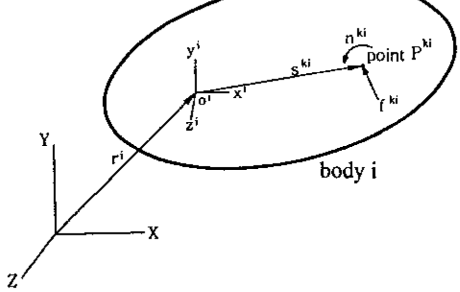

Fig. 1 External forces acting on body i

3. Cartesian and Generalized Forces

The actual force f^{ki} and moment n^{ki} acting on an arbitrary point P^{ki} of body i shown in Fig. 1 can be expressed as a force vector F^i in the Cartesian coordinate space

$$F^i=\begin{bmatrix} f^{ki} \\ 2E^{iT}\tilde{s}^{ki}f^{ki}+2E^{iT}n^{ki} \\ (A^i\phi_t^{ki})^T f^{ki}+(A^i\phi_r^{ki})^T n^{ki} \end{bmatrix} \tag{10}$$

where s^{ki} is the position vector of point P^{ki} in the body reference frame, A^i is the transformation matrix from the ith body reference frame to the inertial frame, $\tilde{s}^{ki}$ is a (3×3) skew-symmetric matrix associated with the vector s^{ki} (Nikravesh, 1988). ϕ_t^{ki} and ϕ_r^{ki} are modal matrices whose columns are composed of translational or rotational displacement of point P^{ki} respectively, and E^i is a (3×4) matrix defined with the elements of Euler parameters p^i as

$$E^i=\begin{bmatrix} -p_1 & p_0 & -p_3 & p_2 \\ -p_2 & p_3 & p_0 & -p_1 \\ -p_3 & -p_2 & p_1 & p_0 \end{bmatrix}^i \tag{11}$$

Cartesian force vector for the system with N_b bodies is

$$F=[F^{1T}\ F^{2T}\ \cdots\ F^{NbT}]^T \tag{12}$$

Using the velocity transformation matrix S, the generalized force vector Q can be obtained as

$$Q=S^TF \tag{13}$$

Meanwhile, the generalized forces can be transformed into forces in Cartesian coordinate space. The virtual work done on the system by the generalized forces is written as

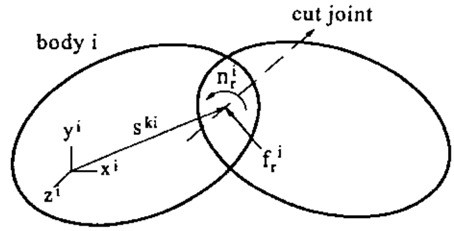

Fig. 2 A body with kinematic constraint

$$\delta W=Q^T\delta q^* \tag{14}$$

Using the chain rule of differentiation, the generalized virtual displacements δq^* are related to the Cartesian virtual displacements δx as

$$\delta q^*=\frac{\partial q^*}{\partial x}\delta x=\frac{\partial \dot{q}^*}{\partial \dot{x}}\delta x=R\delta x \tag{15}$$

where the identity $\partial q^*/\partial x=\partial\dot{q}/\partial\dot{x}$ is used and R is the inverse velocity transformation matrix (Lee, 1997) which can be obtained systematically according to the system reference matrix.

From Eqs. (14) and (15), the following equation is obtained:

$$\delta W=Q^TR\delta x=(R^TQ)^T\delta x=F_{eq}^T\delta x \tag{16}$$

where F_{eq} is defined as the equivalent Cartesian force for the generalized force Q.

$$F_{eq}=R^TQ \tag{17}$$

4. Actuating and Joint Reaction Forces

In the system equations of motion (5) derived in the generalized coordinate space, the second term of right side $\Phi_{q*}^T\lambda$ denotes the joint actuating forces to drive the system for given driving conditions or to satisfy the kinematic constraints generated when cutting a joint of a closed-loop system.

The Cartesian force vector corresponding to those driving constraints and kinematic constraints expressed in the Cartesian coordinates is $\Phi_x^T\lambda$ in which λ is the vector obtained from Eq. (8). If the driving constraints are expressed in the joint coordinates($\Phi(q^*,\ t)=0$), the equivalent Cartesian force vector of the joint actuating force $\Phi_{q*}^T\lambda$ becomes $R^T\Phi_{q*}^T\lambda$ using the inverse velocity transformation matrix R.

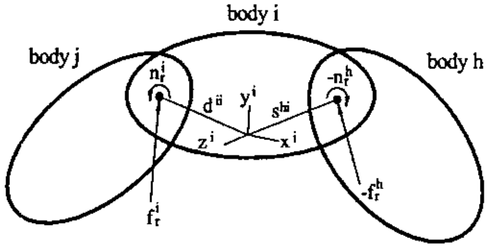

Fig. 3 Joint reaction forces acting on body i

Figure 2 shows a cut-joint in a closed-loop system. The joint reaction force f_r^i and reaction moment n_r^i of body i at the cut-joint can be calculated from the following equation:

$$\Phi_x^T\lambda=\begin{bmatrix} f_r^i \\ 2E^{iT}\tilde{s}^{hi}f_r^i+2E^{iT}n_r^i \\ (A^i\phi_t^{hi})^T f_r^i+(A^i\phi_r^{hi})^T n_r^i \end{bmatrix} \quad (18)$$

In order to analyze the joint reaction forces acting on each joint along the inward path of the system, consider the following adjacent three bodies connected by joints as shown in Fig. 3. Body i and h are bodies on inward and outward path of body i respectively.

In the Cartesian coordinate space, the equation of motion of body i is written as

$$M^i\ddot{x}^i=F_r^i+F_a^i+F_e^i-K^ix^i-\dot{M}^i\dot{x}^i+\left(\frac{\partial T^i}{\partial x^i}\right)^T \quad (19)$$

where M^i is the mass matrix, T^i is the kinetic energy, F_a^i is the Cartesian force vector of the actuating forces for driving and kinematic constraints. F_e^i is the Cartesian force vector of the external forces acting on body i. F_r^i is the Cartesian force vector of the joint reaction forces acting on joints between body h and j. F_r^i can be written as

$$F_r^i=\begin{bmatrix} (1-\delta_{1i})f_r^i-(1-\delta_{ni})f_r^h \\ (1-\delta_{1i})[2E^{iT}(-\tilde{d}^{ii}f_r^i)+2E^{iT}n_r^i] \\ -(1-\delta_{ni})[2E^{iT}\tilde{s}^{hi}f_r^h+2E^{iT}n_r^h] \\ (1-\delta_{1i})[(A^i\phi_t^{ii})^Tf_r^i+(A^i\phi_r^{ii})^Tn_r^i] \\ -(1-\delta_{ni})[A^i\phi_t^{hi})^Tf_r^h+(A^i\phi_r^{hi})^Tn_r^h] \end{bmatrix} \quad (20)$$

where f_r^i and n_r^i are the joint reaction force and reaction moment acting on the joint of body i with its reference body. The subscripts 1 and n of δ_{1i} denote the base body and tree-end body number respectively and δ_{ij} is Kronecker delta.

After analyzing the absolute motion of a system, joint reaction forces acting on each joint are determined successively from the tree-end body to the base body along the inward path. The joint reaction force f_r^i and n_r^i acting on body i can be calculated from Eqs. (19) and (20).

Table 1 Inertia properties and dimensions of a slider-crank mechanism

Body	Length (cm)	Mass (g)	Moment of inertia ($g\cdot cm^2$)		
			I_{xx}	I_{yy}	I_{zz}
Crank	15.2	37.8	1.9	732.4	732.4
Connecting rod	30.5	75.6	3.8	5853.3	5853.3
Slider	2.0	62.6	41.8	41.8	41.8

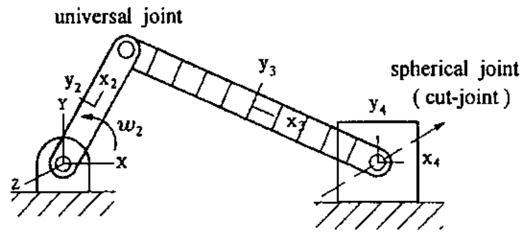

Fig. 4 A silder crank mechanism with an angle driver

5. Numerical Example

A slider-crank mechanism shown in Fig. 4 consists of a rigid crank, a flexible connecting rod with uniform circular cross section and a rigid sliding block. Inertia property of each component in the undeformed state is presented in Table 1. Young's modulus and diameter of the connecting rod are 2.0×10^{12} dyne/cm^2 and 0.64 cm respectively. To illustrate the effect of the elastic deformation of connecting rod, it is divided into ten beam elements and the first two simply supported beam bending modes are used for flexible body analysis.

To change this mechanism into an open-loop system, a spherical joint between the connecting rod and slider is cut. Simulation is carried out for

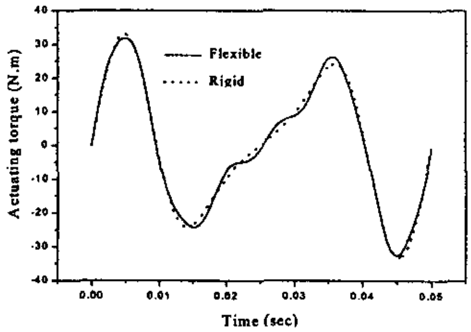

Fig. 5 Actuating torque for joint driving

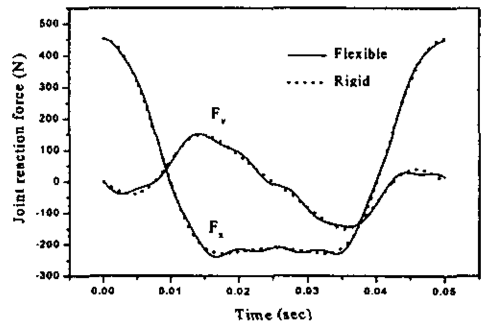

Fig. 6 Joint reaction forces at cut-joint

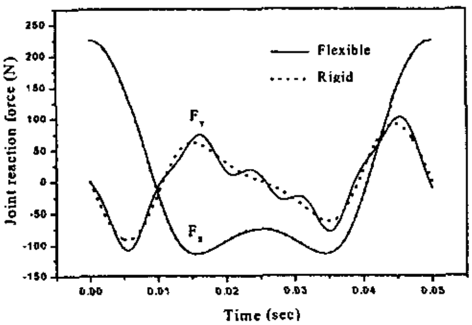

Fig. 7 Joint reaction forces at universal joint

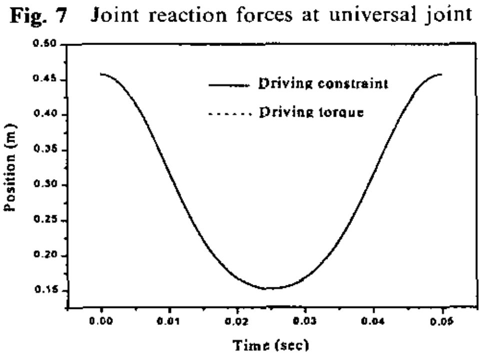

Fig. 8 Position of the slider in X-direction

0.05 sec with a constant crank angular velocity $\omega_2 = 125.7$ rad/sec.

Figure 5 shows the actuating torque for the given crank driving constraint. Figure 6 and 7 show the joint reaction forces acting on the cut-joint (spherical joint) and reaction forces on the universal joint. The results are quite similar to those obtained by DADS (CADSI, 1997). For more verification of the algorithm proposed, forward dynamic analysis is carried out with the actuating torque obtained from inverse dynamics (Fig. 5) imposing on the crank instead of imposing a driving constraint. The motion trajectoriy of the slider in the X-direction is compared with that of the case imposing a driving constraint in Fig. 8 and the two results agree quite well. This is an indication of the validity of the inverse dynamic analysis algorithm which calculates the joint reaction forces and actuating forces of flexible multibody systems with closed-loops.

6. Conclusions

An inverse dynamic algorithm that calculates actuating forces and joint reaction forces of flexible multibody systems with closed-loops in the joint coordinate space has been presented.

The relative joint coordinates and the deformation modal coordinates are used as the generalized coordinates of a flexible multibody system.

The joint reaction forces are analyzed in Cartesian coordinate space. The reaction force acting on the cut-joint is calculated with the Lagrange multipliers obtained from the system equations of motion derived in the joint coordinate space. After calculating the joint reaction force at the cut-joint, the reaction forces acting on the other joints are determined successively from the tree-end body to the base body along the inward path. To transform the joint actuating force for joint driving constraint expressed in the relative joint coordinates into the equivalent Cartesian force vector, the inverse velocity transformation matrix is used.

The algorithm proposed is verified through the analysis of a slider-crank mechanism with a crank angle driver.

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