Evolution of Flame Shape to a Vortex Pair

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The PSC (Propagation of Surfaces under Curvature) algorithm is adapted to the simulation of a flame propagation in a premixed medium including the effect of volume expansion across the flame front due to exothermicity. The algorithm is further developed to incorporate the flame anchoring scheme. This methodology is successfully applied to numerically simulate the response of an anchored V-flame to two strong free stream vortices, in accord with experimental observations of a passage of Karman vortex street through a flame. The simulation predicts flame cusping when a strong vortex pair interacts with flame front. In other words, this algorithm handles merging and breaking of the flame front and provides an accurate calculation of the flame curvature which is needed for flame propagation computation and estimation of curvature-dependent flame speeds.

Key Words : Flame Propagation, Flame-vortex Interaction, Flame Cusping, Flameholding

Nomenclature

C : Constant
f : Source term of Poisson equation
L : Markstein length scale
m : Volume source strength
n : Normal direction
s : Tangential direction
S : Laminar burning speed
t : Time
U : Velocity
u : x-component of velocity
v : y-component of velocity
(x, y) : Spatial coordinates

Greek

δ : Dirac-delta function
ρ : Density
κ : Curvature
ω : Vorticity
ψ : Stream function
ϕ : Velocity potential of incident flow
Φ : Velocity potential due to volume sources
ϕ : Scalar field
∇ : Gradient operator

Subscripts

b : Burnt
f : Flame front
p : Potential
s : Volume source
u : Unburnt
v : Vortex
∞ : Far upstream

Superscripts

0 : Laminar burning speed of a planar flame

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1. Introduction

The propagation of a premixed flame and its interaction with incoming flowfield is a difficult problem due to the coupling of the exothermic chemical reaction with the hydrodynamic flow field. Some models have relied on statistical methods using a probability density function (Pope, 1976). This pdf is specified empirically or may be obtained from an evolution equation. Alternative descriptions have been based on the

laminar flamelet concept (Peters, 1986), wherein the flowfield consists of a collection of flame elements embedded in a flowfield. The structure of these flamelets (Clavin, 1986) is analyzed separately from the flow so that the complicated chemistry is decoupled from the simulation of the flowfield. In many practical situations, the flame thickness is much smaller than the smallest length scale and the flamelet descriptions are relevant (Williams, 1985).

Here we simplify one step further beyond the flamelet model. A flame sheet model is employed in this study by taking the zero limit for flame thickness and thereby solving only for the flame front geometry (Ashurst, 1987). The flame front acts as an infinitely thin boundary which separates two regions of constant densities and propagates into the fresh mixture at a prescribed curvature-dependent flame speed. The speed may be a function of the local curvature of the flame front, which avoids hydrodynamic instability (Landau, 1944). Fluid elements at the flame front undergo an increase in volume as they burn, creating a jump in the normal component of velocity across the flame front.

The conventional methods of flame front tracking use a set of marker particles equally spaced along the flame interface. Regridding procedure is required when marker particles merge together and so the curvature of the propagating front is large (Hyman, 1984). This regridding procedure resembles diffusion and overshadows the actual effects of curvature. In view of the experimental fact that flames can develop cusps, this method cannot provide an accurate calculation of the flame front curvature which is needed for flame speed evaluation. A new numerical PSC (Propagation of Surfaces under Curvature) algorithm introduced by Osher and Sethian (1988) is adapted to the simulation of flame propagation in a premixed medium including the effect of volume expansion along the flame front due to exothermicity. This algorithm is further developed to incorporate the flame-anchoring scheme and is successfully applied to numerically simulate the response of an anchored V-flame to two strong free stream vortices, in accord with experimental observations of a passage of Karman vortex street through a flame. Simulation results predict flame cusping when a strong vortex pair interacts with a flame front. In other words, this algorithm handles merging and breaking of the flame front naturally and provides an accurate representation of the flame curvature which is needed for the calculation of flame propagation and estimation of curvature-dependent flame speed.

The primary objective of this work is to adapt the PSC algorithm to the simple case of rod-stabilized premixed flame, a configuration which has been investigated extensively both experimentally and theoretically. A numerical study is presented describing the interaction of two strong discrete vortices with the flame front. The capability of the algorithm to predict the flame cusping phenomena which have been observed experimentally is highlighted.

2. Physical Model

The main assumptions of our model for flame propagation and its response to the accompanying flowfield are:

a) the flowfield is two-dimensional and inviscid,

b) the Mach number is small enough to consider the flowfield as incompressible,

c) the flame acts as an interface which separates two regions of different but constant density and propagates into the unburnt region at a prescribed flame speed Su which depends on the local curvature.

2.1 General background

A common approach to the problem of interaction between the flowfield and the flame consists of separating the fluid dynamic treatment from that of the flame zone. The flame front is treated as a surface of discontinuity separating two fluids of different densities. Landau (1944) found that perturbations along a density interface that propagated at a constant speed would grow rapidly without bounds and that such flames were absolutely unstable to any perturbation. This is the so-called hydrodynamic instability. The

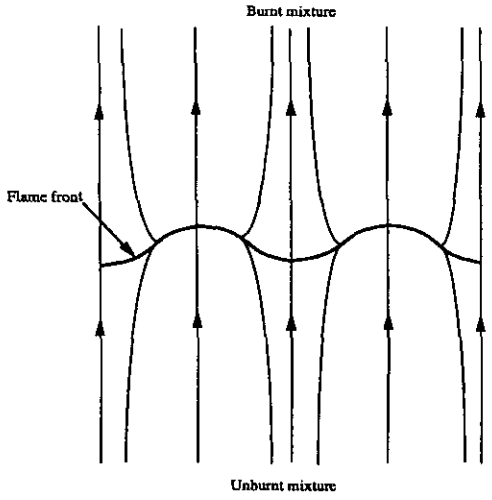

Fig. 1 Hydrodynamic instability

mechanism of this instability can be understood if one considers a plane stationary flame front which is perturbed by a sinusoidal wave, as shown in Fig. 1. The perturbation causes the streamlines to diverge ahead of the convex region of the flame and converge ahead of the concave region. The flow of the reactants decelerates ahead of the crest while it accelerates in front of its trough. If the burning speed is constant, the convex part will protrude more into the reactants and the concave part will keep receding. Thus small perturbations grow and the flame becomes unstable. Heat conduction improves the flame front stability by changing the local value of the flame speed. The concave/convex part of the perturbed flame heats up the reactants more/less than an unperturbed planar flame, thus increasing/decreasing the burning speed.

In this work, we model the stabilizing effect of heat conduction associated with the curvature of the flame. Markstein (1964) showed that if the burning speed varied with the curvature of the flame front such that concave/convex flames were faster/slower than the planar flames, a stabilizing mechanism was realized for short wavelength perturbations, this was verified experimentally. According to this formulation, the laminar burning speed for weak curvature can be given by

$$S_u = S_u^0\left(1-\frac{L}{R_f}\right) = S_u^0(1-L\kappa) \qquad (1)$$

where S_u^0 is the laminar burning flame speed of a planar flame, often denoted in the literature as S_L, κ is the curvature of flame front, $R_f=1/\kappa$ is the radius of curvature of the front and L is the Markstein length scale. An expression for L was deduced by Garcia-Ybarra et al. (1984) using high activation energy asymptotics. L divided by the laminar flame thickness ranges from 2 (rich mixture) to 6 (lean mixture) for ethylene flames. In this simulation, it was assumed that the grid size (=0.02 in non-dimensional units when scaled with respect to the transverse width of the computational domain) corresponded to an integral length scale of 1 mm, and the thickness of the laminar flame was about 0. 5 mm. In the starting, a ratio of L to the laminar flame thickness of 4 was chosen. This leads to a non-dimensional numerical value of $L=0.04$.

2.2 Flowfield description

As was done by Pindera & Talbot (1986), we decompose the velocity field into three components:

$$U = U_s + U_v + U_p \qquad (2)$$

With the individual velocity components in Eq. (2) satisfying the following conditions:

$$\nabla\cdot U_s = m\delta(x-x_f);\ \nabla\times U_s = 0,$$
$$\nabla\times U_v = \omega(x);\qquad \nabla\cdot U_v = 0,$$
$$U_p = \nabla\phi;\qquad \nabla\cdot U_p = 0,$$

where $U=(u, v)$ is the two-dimensional velocity vector, U_s the velocity due to volume expansion across the flame front, U_v the rotational velocity field due to flow vortices, U_p the potential velocity field, m the volume source strength per unit length of the flame, x_f the coordinate describing the position of the flame front, the vorticity field, the velocity potential of incident flow and is the two-dimensional Dirac-delta function.

Using the conservation of mass, we derive an expression for the effect of volume expansion on the velocity field, U_s. Let Su and S_u be the relative normal velocities with respect to the flame on unburnt and burnt sides, respectively, and ρ_u and ρ_b be the fluid densities of unburnt and burnt mixtures. Mass continuity yields

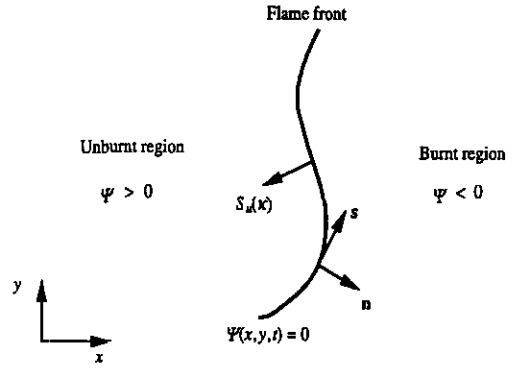

Fig. 2 Flame front configuration

$$\rho_u S_u = \rho_b S_b \tag{3}$$

As reactants are converted into products, the volume per unit length of the flame m is generated during the thermal expansion process.

$$m = (S_b - S_u) = \left(\frac{\rho_u - \rho_b}{\rho_b}\right) S_u \tag{4}$$

2.3 Equation for flame propagation with advection

It is convenient to formulate the flame propagation problem in terms of an equation for a scalar field $\psi(x, y, t)$. Following Osher & Sethian (1988), we define a continuous initial distance function $\psi(x, y, t)$ such that $\psi > 0$ in a unburnt region, $\psi < 0$ in a burnt region and the $\psi = 0$ represents the flame front as depicted in Fig. 2.

In order to find an equation for the evolution of ψ which corresponds to the propagating flame, the motion of some level set $\psi(x, y, t) = C$ is considered. Let $(x(t), y(t))$ be the trajectory of a particle located on this level set, so

$$\psi(x(t), y(t), t) = C \tag{5}$$

The particle speed $\partial x/\partial t$, in the direction $-n$ normal to the flame, is given by the flame speed $S_u(\kappa)$. Thus, with the convention adopted here that n points in the direction of the burnt gas

$$-n \cdot \frac{\partial x}{\partial t} = S_u(\kappa) \tag{6}$$

where the normal vector n is given by $n = -\nabla\psi / |\nabla\psi|$. When the chain rule is applied to Eq. (5),

$$\psi_t + \frac{\partial x}{\partial t} \cdot \nabla\psi = 0 \tag{7}$$

is obtained. A substitution of n to Eq. (6) converts Eq. (7) to

$$\psi_t + S_u(\kappa) |\nabla\psi| = 0 \tag{8}$$

A flame propagates with its own burning speed relative to the accompanying flowfield. The flame front affects the flowfield by volume generation. The flowfield influences the flame location by advection. Hence, there exists a mutual interaction between the flame and flowfield. In this case, the field equation becomes

$$\psi_t + S_u(\kappa) |\nabla\psi| + U \cdot \nabla\psi = 0 \tag{9}$$

The second term represents propagation and the third term denotes advection of the scalar field ψ. The velocity vector U is the convection velocity of the unburnt fluid just upstream of the flame front.

3. Numerical methods

Here we briefly mention the numerical procedure for determining the flame motion. The algorithm uses the method of fractional steps to decompose the flame motion into propagation and advection. First, we propagate the flame as a result of burning. Then we locate the volume sources along the flame due to exothermicity to obtain the irrotational velocity field for the advection part. The rotational velocity field due to flow vortices is also obtained. A flame-anchoring algorithm for flameholder of the V-flame is developed to stabilize the flame against the blowout. Complete details for the flame-anchoring scheme can be found in Rhee (1992).

At each time step, ψ is updated to new values on an Eulerian grid by propagation and advection. A new flame position is obtained by passing scalar field data to a contouring routine and finding a set of flame segments corresponding to the level set $\psi = 0$. The position and length of the flame segment within the cell determines the location and the strength of the volumetric source. This volume source strength becomes the source term in the Poisson equation for the velocity potential computed on an Eulerian grid.

$$\nabla^2\Phi=f \tag{10}$$

where Φ is the velocity potential due to volume sources and f is the source strength. Inflowing vortices generate a source term in the Poisson equation for the vorticity stream function ψ:

$$\nabla^2\psi=-\omega \tag{11}$$

where ω is the vortex strength which is distributed at the cell corners on an Eulerian grid by area weighting. This rotational velocity combined with the volume-source-induced irrotational velocity and the uniform incident velocity specifies the total velocity which moves the scalar field by advection.

The computational domain is a rectangle with no-flow boundary conditions along $y=0$, 1, and inflow-outflow conditions at $x=0$, 2. The flame propagates in the negative x-direction. The boundary conditions for the Poisson equation should conform to the Neumann compatibility condition. All the volume generated inside the computational domain manifests itself in the uniform increment of the exit boundary velocity at $x=2$.

4. Numerical Examples

The numerical method is applied to the dynamic behavior of a V-shaped premixed flame. Although we are simulating an open flame, numerical work requires a finite computational domain. Hence, the solution domain is truncated to a rectangle of axial (x) length equal to 2 and transverse (y) width equal to 1. Incoming fresh mixture enters the computational domain at $x=0$ with a velocity equal to 1. All velocities and lengths are scaled with the incoming free-stream velocity U_∞ and the transverse width, respectively. The combination of the two gives a time scale. The flameholder is located at $(x, y)=(0.5, 0.5)$. Non-dimensional flame speed S_u^0/U_∞ and density ratio ρ_u/ρ_b are set to 0. 08 and 6. 0, respectively. The Markstein length scale L in the flame speed equation is set to 0.04. The grid size is equal to 0.02 and the time step is 0.004 in order to satisfy the Courant condition.

Figure 3 shows the flame evolution at equal

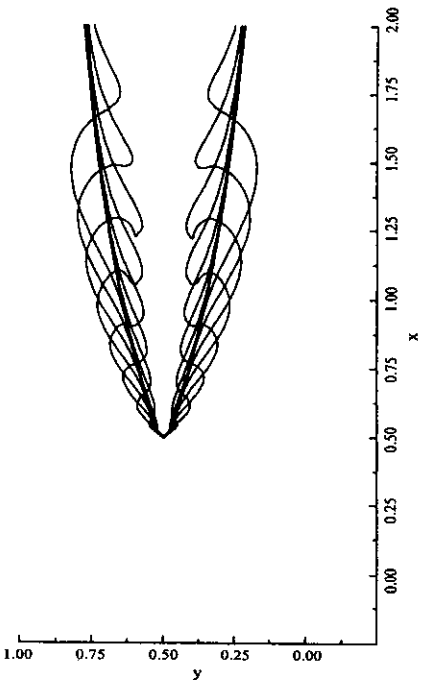

Fig. 3 Evolution of flame motions with passage of two strong flow vortices

successive time intervals under the influence of two strong discrete flow vortices of non-dimensional circulation ± 0.2 initially located near the flameholder. This non-dimensional value of circulation with the vortex core radius of 0. 02 yields a maximum azimuthal velocity-flame speed ratio, U_{max}/S_u^0, of 20. This corresponds to the severely wrinkled flame regime. The senses of rotation of flow vortices are indicated in Fig. 4 (a). The locations of these vortices with time along with flame excursions are shown in Figs. 4 (a)~(d). The two line vortex pair moves outward from the original release positions by advection of the divergent reactant flowfield, noting that the spacing between the vortex pair on the reactant side increases in (a), (b) and (c). As these vortices approach and then pass through the flame front as shown in (c) and (d), cusps develop at the front and eventually smooth out as they traverse the product region behind the flame. The more flame cusps develop, the greater the flame speeds up due to the increase of the curvature of the flame front, which leads to the smoothing-out of the flame front curvature.

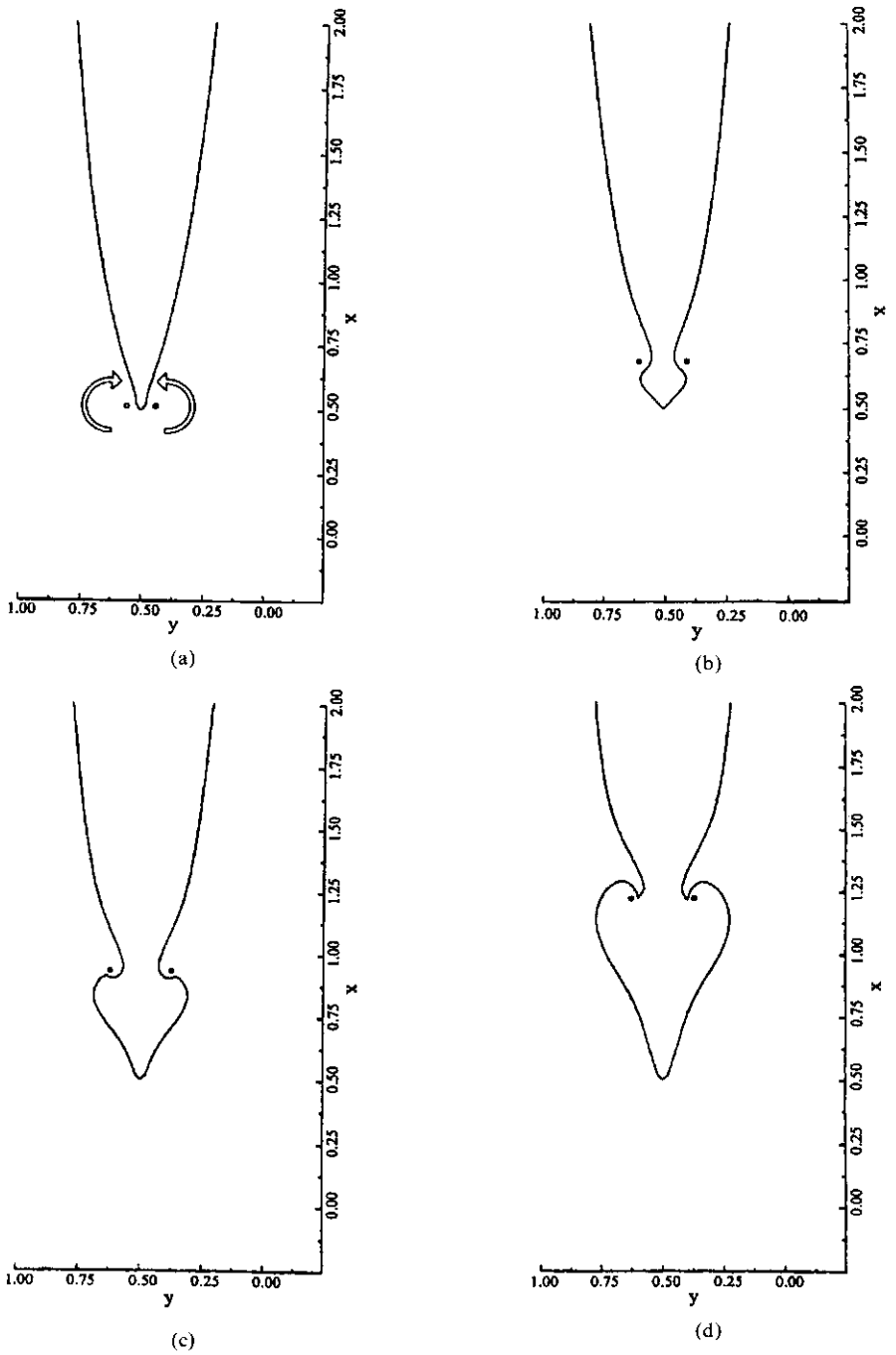

Fig. 4 Flame shape in relation to the locations of vortices in equal time intervals

5. Conclusions

V-flame motion in a Karman vortex street has been investigated experimentally by Namer et al. (1984) and Herzberg et al. (1984). Roberts and Driscoll (1991) studied the response of a flame sheet to a laminar vortex ring. The computed flame response to two strong vortices compares well with the experimental observations of

Herzberg et al. (1984) in that the flame cusping phenomenon which is observed experimentally is well reproduced. The cusping observed in the experiments was not as pronounced as in this numerical simulation. This may be partly due to the truncation of the computational domain especially in the transverse (y) direction to 1. The boundary conditions are set to no-flow conditions at the lateral boundaries ($y=0$ and 1) and uniform velocity increment due to exothermicity at the exit ($x=2$), whereas volume generated during the combustion process is supposed to freely expand in the experimental conditions. The resultant outward deflection and acceleration of the divegent reactant flow overestimates the flame behavior to a certain extent, compared to experimental observations. At the same time, the flame configuration produced in the cusping process clearly depends on the ratio of the maximum vortex-induced velocity to the freestream velocity at the moment of interacting with the flame. According to Blake (1986), this ratio is in the vicinity of 0. 3 for a Karman vortex street at a Reynolds number of 240, whereas this value was about 1. 6 for the numerical simulation. This rough estimate suggests that the cusping observed in the experiment might be expected to be less severe than that found in the numerical simulation, although it is clear that they share the same general structure.

Acknowledgement

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