Thermo-Elastic Analysis for Chattering Phenomenon of Automotive Disk Brake

Chongdu Cho*†, Sooick Ahn
Department of Mechanical Engineering, Inha University

This study investigates the effects of operating conditions on the chattering of an automotive disk brake by experimental and computational methods. Design factors, which cause chattering in automobiles, have attracted great attentions for long time; but they are not well understood yet. For this study, we construct a brake dynamometer for measuring the disk surface temperature during chattering, and propose an efficient hybrid algorithm (combining FFT-FEA and traditional FEA program) for analyzing the thermo-elastic behavior of three-dimensional brake system. We successfully measure the judder in a brake system via the dynamometer and efficiently simulate the contact pressure variation by the hybrid algorithm. The three-dimensional simulation of thermo-mechanical interactions on the automotive brake, showing the transient thermo-elastic instability phenomenon, is presented for the first time in this academic community. We also find from the experimental study that the disk bulk temperature strongly influences the brake chattering in the automotive disk brakes.

Key Words : Brake Chattering, Judder, Hot Spots, Dynamometer, Fast Fourier Transformation (FFT), Finite Element Analysis (FEA)

1. Introduction

Noise and vibration often radiate from automotive brakes when we operate a brake pedal. We collectively refer to them as brake chattering and the dry contact between the pads and disk is the cause. The brake chattering is classified into two kinds by considering whether it depends on foreign parts connected to brake assembly or not. The (uncoupled) noise originating from the brake assembly has two types: the high frequency chatter noise (higher than 1,000 Hz) is "squeal" while the low frequency noise (lower than 10 Hz) is "judder". And the other noises (boom, grunt, humming, groan, and moan between 10 Hz and 1,000 Hz) are coupled with the foreign parts (Cho, 1997). In order to analyze the noise problem depending on the foreign parts, we must carefully inspect the natural frequencies of every part connecting to the brake assembly. However, the squeal and judder are confined to the brake assembly itself. They have been referred to as traditional brake Noise, Vibration, and Harshness(NVH) problems. Although the brake NVH troubles have been enormously examined, automotive industries did not seriously consider the brake chattering problem in their design until they utilized non-asbestos friction materials for the automotive brake pads.

There exist many experimental and theoretical papers about the squeal noise while only a limited number of papers for the judder problem are found. Although no generally acceptable solution has yet been reported of the problem of brake squeals, a number of theories (Earles and Soar, 1971) prevail (e.g. stick-slip, variable dynamic friction, sprag-slip, and geometric coupling). And still, many diverse reasons for the squeal have

† First Author
* Corresponding Author,
E-mail : cdcho@inha.ac.kr
TEL : +82-32-860-7321; **FAX** : +82-32-868-1716
Department of Mechanical Engineering, Inha University, #253, Yonghyun-dong, Nam-ku, Inchon 402-751, Korea.
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driven recent investigations (Hiromichi *et al.*, 1992; Nishiwaki *et al.*, 1993; Masushima and Kikuchi, 1995; Guan and Jiang, 1998; Kim and Cho, 1999), which propose mainly dynamic modeling and numerical simulation techniques. On the other hand, a completely different mechanism applies to the low frequency noise (judder). Friction work generates heat into the disk and pads while we operate the brake, resulting in thermal energy in those components. The thermal energy leads to the gradual thermal deformation that interferes with the uniform contact between the disk and pads. Even worse than this, periodically distributed hot spots (causing hot roughness) have been frequently observed on the surfaces of disks during their operation (Barber, 1967; Hills and Barber, 1985). The hot roughness clearly induces non-smooth contact that is the origin of low frequency noise in brake systems. There have been some efforts for identifying the factors that affect the judder. It is the first theoretical report that Dow and Burton (1972) investigated a thermo-elastic instability with a two-dimensional brake model. The ideal brake model for their study is a blade (as a frictional material) on a semi-infinite plate (as a brake disk) moving with a constant speed. The model successfully explains the unstable temperature growth from even a small temperature variation on the contact line. Although their model failed to quantitatively predict the phenomenon (their predicted speed for the instability occurrence is higher than the common experience by ten times at least), they enlightened the possibility to analyze the judder. More recently, Lee and Barber (1993, 1994) proposed a practical model that has a finite-thickness plate, instead of a disk, sliding between two half infinite friction materials. They calculated the lowest speed to make the system unstable; but the new model improved the accuracy only moderately, still giving as large as 100% error (so it is still not enough for design applications).

The inaccuracy may result partly from the over-simplified models. But even their simple models require much mathematical complexity in derivation. In order to overcome the difficulties, we experimentally examine the design factors sensitive to the judder occurrence and utilize computational methods for simulating the phenomenon. In the experiments, the general-purpose brake dynamometer needs to be modified for measuring at sufficiently many test points for observing the judder; and in computation, more efficient techniques must be developed. The traditional Finite Element Analysis(FEA) requires impractically much computation time to simulate three-dimensional brake systems and often fails to converge. This study, therefore, develops a brake dynamometer and examines how the disk temperature affects the judder speed. Also we propose a very efficient computation technique for simulating three-dimensional brake systems.

2. Experiments and Thermo-Elastic Analysis

Lee and Barber (1993) utilized a finite-thickness plate model sliding with a constant speed and pressed two semi-infinite friction solids against each other, analyzing the drag braking. They derived an analytic solution using the superposition of the symmetric and anti-symmetric modes. They reported that the anti-symmetric mode determined the lowest (or critical) speed to make the system unstable and the analysis was verified by experiments on a lathe (Lee and Barber, 1994). Following their analysis of the anti-symmetric mode, the critical disk speed is estimated to be about 77 km/h where the brake system has a cast iron disk and two semi-metallic pads (friction coefficient (μ=0.37). Because the estimated speed is the relative sliding speed between the pads and the disk, the lowest traveling speed of the vehicle to experience the thermo-elastic stability is 210 km/h when the ratio of an effective tire radius to the pad centerline from the axle is 2.72. The predicted speed is higher compared to the road test experiences. This high critical speed may result from neglecting the effects of disk bulk temperature as well as simplifying the disk geometries in the analysis.

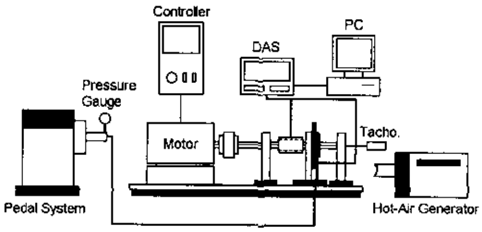


Fig. 1 Schematic of experimental apparatus and data acquisition system

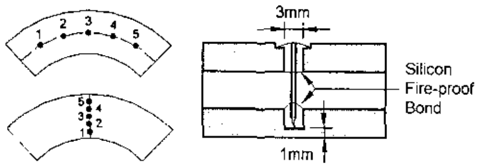


Fig. 2 Thermocouple locations and installation in the brake disc and pads

2.1 Experimental apparatus

We develop a dynamometer without a flywheel for simulating the actual vehicle inertia. Figure 1 shows the experimental apparatus and data acquisition system we used. An automotive disk brake system with a disk and a floating type caliper is assembled on the apparatus that provides power to the brake disk by a 3.7 kW motor. A pedal system is used to actuate the brakes and then the brake oil power lines. The disk brake assembly is vertically installed. The disk and caliper assembly are set in an adiabatic chamber where temperature can be controlled up to 350℃ by a fan heater.

The rotational speed of the disk is monitored by a tachometer and the brake line pressure by a pressure gage on the line. Temperatures are measured by chromel-alumel (K type) thermocouples, all the connections being maintained at the same temperature by an isothermal block for more accurate measurement. Readings of the disk temperatures are transmitted via a slip ring.

Nine thermocouples are installed in the outboard brake pad as shown in Fig. 2 for measuring temperature variations at multiple points on the brake pad during experiments. Note that SAE J2115 recommends the use of one thermocouple in a pad for a conventional brake dynamometer. Typical thermocouple installation in the friction pad is also shown in Fig. 2.

Thermocouple locations in the disk are similar to those in the pad (Fig. 2) and aligned to the sliding centerline. Nine thermocouples are press fitted through holes drilled below the disk surface. Five of these thermocouples are installed for observing the circumferential distribution on the inboard surface of the disk and the rest are prepared for the radial variation.

2.2 Test procedure

All tests are performed at constant rotational speeds with the brake line pressure not exceeding 150 kPa. This mimics a light brake drag application that has been reported to simulate the situation similar to the downhill braking on highway. Also with this light braking condition, the power capacity of the electric motor is not exceeded and the rotational speed of the motor and disk can remain relatively constant up to 2,500 rpm (the motor can rotate up to 3,450 rpm). Three variables (rotational speed, brake pressure, and disk bulk temperature) are varied to investigate their effects on the thermo-elastic instability. Other variables such as the brake pad configuration and disk thickness are not changed. We operate a fan heater that controls the ambient temperature (in an adiabatic chamber). We test the disk at 20,50, 100,150, and 200℃ and 120 kPa braking pressure to determine whether the disk bulk temperature affects the thermo-elastic instability. Revolution speed spectrum is between 700 and 2500 rpm at intervals.

The brake pads used are of a semi-metallic friction material (Young's modulus, 0.3 GPa; Poisson's ratio, 0.25; thermal expansion coefficient, 57×10^{-6}/℃ ; specific heat, 1000 J/kgK; thermal conductivity, 0.9 W/mK). The disk is of cast iron (Young's modulus, 170 GPa; Poisson's ratio, 0.25; thermal expansion coefficient, 11×10^{-6}/℃ ; specific heat, 434 J/kgK; thermal conductivity, 48 W/mK). Before each of the test series, new friction material is run in at 500 rpm for an hour, with a light braking pressure.

The general test procedure is to bring the brake

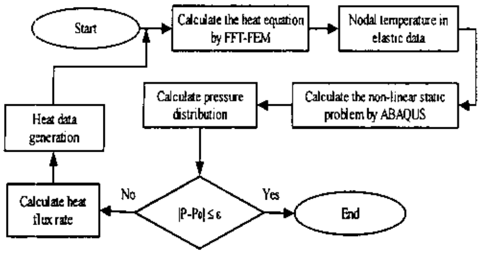


Fig. 3 Thermo-elastic simulation algorithm by using combined FFT-FEM(for heat transfer) and ABAQUS^{TM}(for elastic analysis)

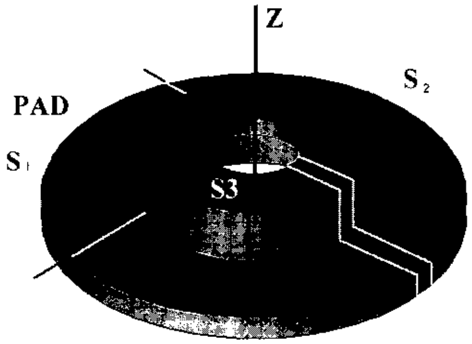


Fig. 4 Three-dimensional automotive disk model. S_1, S_2 on the plot are thermal boundaries for the specified heat flux through contacting pad and the convection, respectively

disk to the desired speed at the preconditioned disk temperature (usually at room temperature). The disk speed drops when the brake is applied and it is readjusted to the desired speed by changing the brake pressure (measured by a pressure gage set in the master cylinder). The disk brake system is dragged for a maximum of 25 minutes until the temperature variation due to the thermo-elastic instability is fully observed. After a long period of dragging, the brake pressure is released and the rotational speed is reduced to zero and the disk brake system is then allowed to cool down to the room temperature before the next run. Data from all the thermocouples are monitored and recorded continuously during the test. The test for the temperature reading follows ASTM (KRISS, 1985) and SAE (SAE Standard, 1993) codes.

2.3 Thermo-elastic analysis

We have known that although a transient numerical simulation of a realistic automotive brake contact is possible by various commercial finite element programs, the mesh refinement in both time and space for 3-D thermo-elastic analysis requires extremely extensive computations. Based on our experience, we simulated the thermo-elastic behavior of a half disk brake model for up to 0.6 sec starting from the contact. It took several days with a commercial program on SUN Sparc20 station. In fact, nobody has successfully reported the transient hot spots due to the thermo-elastic instability on computers. Therefore, in order to analyze the transient heat transfer of a disk brake system under drag braking, we use the fast Fourier transformation (Floquet and Dubourg, 1994) and develop a Visual C++ program that analyzes a 3-D brake system efficiently (Cho and Ahn, 2000). This paper proposes an algorithm (see Fig. 3) that combines the FFT-FEM program (for heat transfer analysis (Floquet and Dubourg, 1994)) with a commercial program (for the elastic analysis). With this algorithm, we can efficiently analyze a transient thermo-elastic behavior of a fully 3-D brake system. We utilize ABAQUS for the elastic analysis part in the algorithm.

Figure 4 shows a model of a solid disk squeezed by two friction materials (not shown in the figure). We specified the following initial and boundary conditions:

- initial temperature over surface $S=S_1+S_2+S_3$
- heat flux corresponding to frictional heating between the pads and disk over S_1
- natural convection boundary during simulation over S_2

We initially apply a uniform pressure, p, of 1.7 MPa to the frictional contact area to determine the heat flux amount. The successive pressure distribution changes according to the thermo-mechanical interaction of each simulation step. In order to determine the heat flux vectors, we assume Coulomb friction on the contact area;

$$q=\mu v p=\mu r\omega p \quad (1)$$

where μ is the friction coefficient, v the sliding speed, r the radius, and ω the angular velocity.

Transient thermal analysis via FFT-FEM formulation

Automotive disk brakes have two parts: one is the rotating disk and the other the fixed frictional materials (pads). The rotating disk has an axi-symmetric geometry, but the pads retain three-dimensional shape. As a result, the loading condition invalidates the axi-symmetric simulation of the thermo-elastically coupled problem in a disk. The basic idea of FFT-FEM transforms the three-dimensional formulation into a plane problem through the Fourier transformation with respect to the space variable, θ. The finite element analysis is then performed on the transformed plane, and the discrete results are superposed to recover a three-dimensional solution by the inverse FFT technique.

We have the heat conduction equation with respect to the cylindrical coordinate system(r, θ, z) fixed on the rotating disk:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = \frac{1}{D}\left(\frac{\partial T}{\partial t}+\omega\frac{\partial T}{\partial \theta}\right) \tag{2}$$

with boundary conditions:

- $T(r, \theta, z, t)=T_c(r, \theta, z, t)$ on S
- $k\vec{\nabla} T\cdot\vec{n}=q_c(r, \theta, z, t)$ on S_1
- $-k\vec{\nabla} T\cdot\vec{n}=h(T-T_a)$ on S_2

And the initial condition is $T(r, \theta, z, 0)=T_i(r, \theta, z)$. In Eq. (2), D is the diffusivity, h the convection transfer coefficient, and k the heat conductivity. After transforming Eq. (2) and the specified conditions with respect to θ, we end up with Eq. (3) by defining $\alpha=2\pi f$ and $\beta=2\pi f\omega$:

$$\frac{\partial^2 \overline{T}}{\partial r^2}+\frac{1}{r}\frac{\partial \overline{T}}{\partial r}+\frac{\partial^2 \overline{T}}{\partial z^2}-\left(\frac{\alpha^2}{r^2}+j\frac{\beta}{D}\right)\overline{T} = \frac{1}{D}\frac{\partial \overline{T}}{\partial t} \text{ on } \Omega \tag{3}$$

and

- $\overline{T}(r, z, t)=\overline{T}_c(r, z, t)$ on Γ
- $k\vec{\nabla}\overline{T}\cdot\vec{n}=\overline{q}_c(r, z, t)$ on Γ_1
- $-k\vec{\nabla}\overline{T}\cdot\vec{n}=h\left(\overline{T}-\overline{T}_a\right)$ on Γ_2

and the transformed initial condition is $\overline{T}(r, z, 0)=\overline{T}_i(r, z)$. The space variable, θ, is replaced by discrete frequencies, f. The three-dimensional body (disk), V, and its surface, S, are transformed to a closed area, Ω, and its boundary, Γ, respectively. After pre-multiplying by a weighting function, W, on each side of Eq. (3), integration by parts over domain Ω gives a finite element weak formulation:

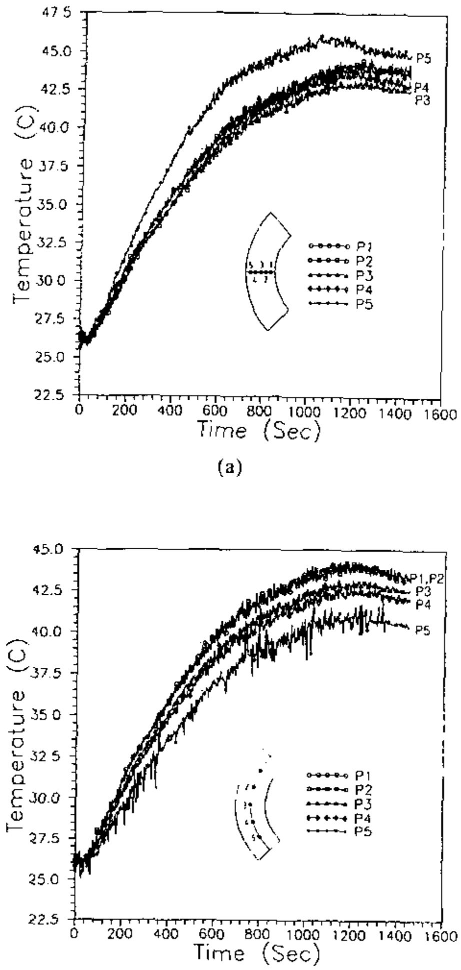
$$-\int_\Omega \vec{\nabla}\overline{T}\cdot\vec{\nabla} W d\Omega+\int_\Omega \frac{1}{r}\frac{\partial \overline{T}}{\partial r} W d\Omega -\alpha^2\int_\Omega \frac{\overline{T}}{r^2} W d\Omega - j\beta\int_\Omega \frac{\overline{T}}{D} W d\Omega+\int_{\Gamma_1}\frac{\overline{q}_c}{k} W d\Gamma -\int_{\Gamma_2}\frac{h}{k}\left(\overline{T}-\overline{T}_a\right) W d\Gamma=\int_\Omega \frac{1}{D}\frac{\partial \overline{T}}{\partial t} W d\Omega \tag{4}$$

and the initial condition is $\overline{T}(r, z, 0)=\overline{T}_i(r, z)$. Decomposition of Eq. (4) into the real and imaginary parts gives two real equations with two field variables $\overline{T}_R$, $\overline{T}_I$. We programmed Eq. (4) and adopted the Crank-Nicolson algorithm for the time integration.

3. Results and Discussion

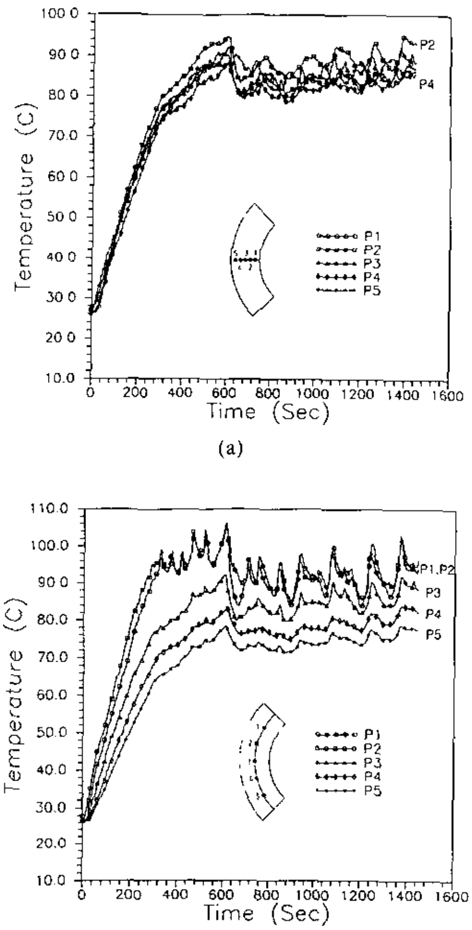
3.1 Experimental observations

When the brakes run in a stable manner, frictionally-induced temperature rise occurs steadily with time. It must be very informative to understand how the temperature profile prevails over the pad surface during the brake operation. Figure 5 shows a typical example when the system is in a stable condition. The temperature initially rises faster at the outer radius (Pad 5 position in Fig. 5(a)) and slower at the inner radius (Pad 1 position); this is due to the higher sliding speed at the outer radius of the friction pad. As the temperature rises, the friction pad expands thermally, and its shape changes according to the contact pressure variation. Due to the easier cooling, the temperature of the pad should be lower along the outer and inner edges than inside the contact region. As shown in Fig. 5, we measure the lowest temperature at the center (Pad 3 position) inside the contact surface of the pad. Thus the overall temperature profile along the radius becomes "M" shaped. The temperatures near leading edges (Pad



(b)

Fig. 5 Measured temperature distribution in the thermo-elastically stable case (p=64.6 kPa, ω=780 rpm) : (a) in radial direction and (b) in circumferential direction



(b)

Fig. 6 Measured temperature distribution in the thermo-elastically unstable case (p=85.2 kPa, ω=1840 rpm) : (a) in radial direction and (b) in circumferential direction

1 and 2 positions in Fig. 5(b)) are higher than near trailing edges (Pad 4 and 5 positions in Fig. 5(b)).

On the other hand, Fig. 6 shows a typical case of thermo-elastically unstable states. Temperature variations appear as oscillations with relatively long wavelengths, superposed on the stable bulk temperature distribution; this observation is also found in Lee and Barber (1994). These long wavelengths are attributed to slow movement of contact spots on the pads when the judder occurs. The hot spots on the disk and pads move at very low speeds. In dry friction between dissimilar bodies, the hot spots are generally observed to move faster on the softer materials (i.e. the pads in disk brake systems). We observe from Figs. 6(a) and (b) that there exists a transition instant at approximately 600 sec. It is observed from Fig. 6 (a) that the measured temperatures in the radial direction are almost the same prior to the instant; but after the transition, the inner temperature (Pad 2 position) fluctuates with 180° phase difference from the outer temperature (Pad 5 position). Also Fig. 6(b) shows that the temperature readings near leading edges and the rest are different before the transition; but after the instant, all measured temperature values vary in phase

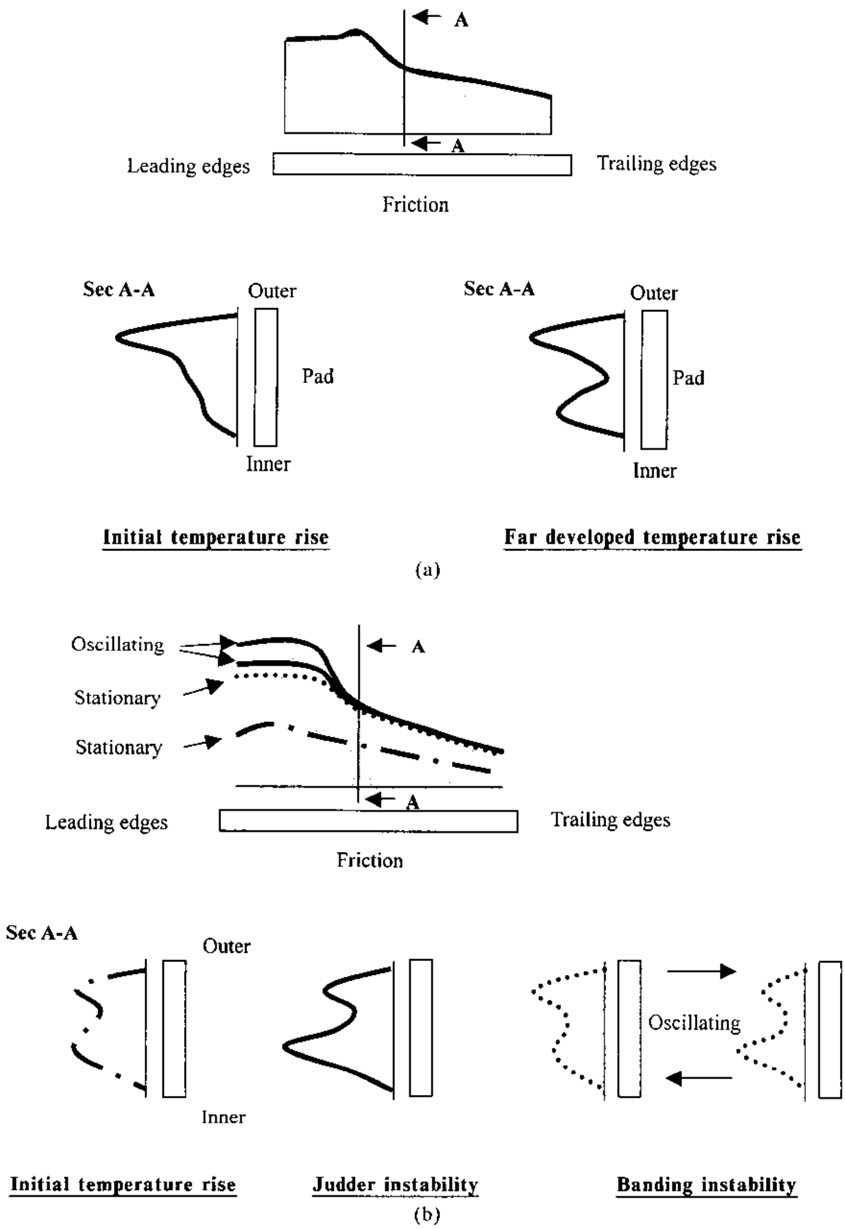


Fig. 7 Schematics of temperature profiles measured on friction pad: (a) stable distribution and (b) unstable distribution

with each other. These mean that the temperature distribution before the transition shows slow variation in the circumferential direction on the friction pad but a constant profile in the radial direction; but after the transition the distribution changes radially, it remains stationary circumferentially with respect to the friction pad. The instability after the transition is named "banding" in the brake industries.

Figure 7 is drawn to show the typical behaviors of transient temperature profiles observed on the pad during the whole drag braking process. Figure 7(a) describes the temperature profiles both at the early stage of the brake engaged and at the fairly late stage but no later than instability occurrence. In the plot, the radially measured pro

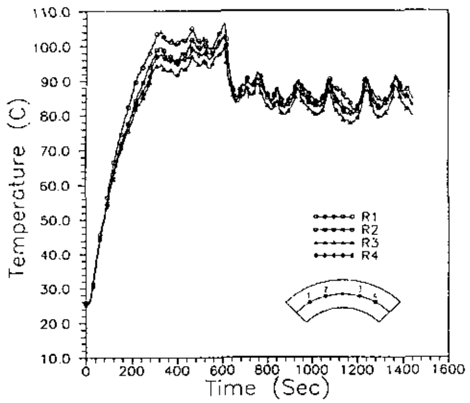


Fig. 8 Temperature fluctuations at thermocouple positions $R1$, $R2$, $R3$, $R4$ of the brake disk (p=85.2 kPa, ω=1840 rpm)

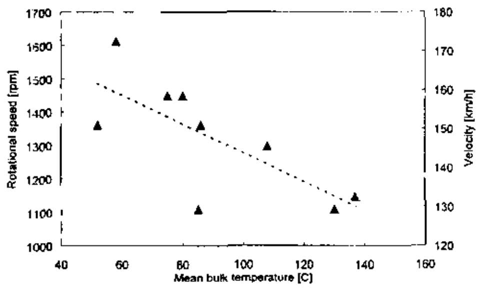


Fig. 9 Triangles shows instability occurrence conditions during experiments. Dotted line was drawn for reference by curve fitting

file clearly explains the appreciable effect of thermo-mechanical interaction during engaging the brake. Once the thermo-mechanical interaction develops, the profiles grow monotonically during this period. On the other hand, Fig. 7(b) depicts the observations during instability process. During the judder instability, we observe the angular profile oscillation between two temperature distribution patterns while the radial profile remains constant (see the solid lines in the plot). Also the banding instability case is drawn by dotted lines in the plot. In this case, the radial patterns oscillate between two types, but the angular direction keeps a constant temperature distribution. These are the reverse of the judder instability observation. In a brief account, Fig. 7 represents schematically the temperature profiles for the stable and unstable cases. In the

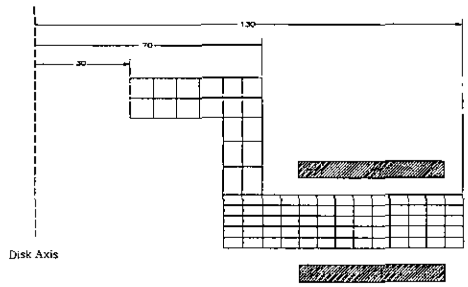


Fig. 10 A disk model used for analyzing heat transfer in a disk brake assembly with FFT-FEM program

judder and banding instabilities, the relative temperature profile in the circumferential direction of the disk does not change. Those temperature profiles do not move with respect to the disk (see Fig. 8). Thus those could cause permanent damage on the disk in a severe situation.

We also test thermo-elastic instabilities under various initial disk bulk temperatures in the following. We heat the test chamber with a hot air generator to control the initial disk bulk temperature; the initial bulk temperatures are 20, 50, 100, 150, 200℃. We conduct tests at 120 kPa brake pressure for disk speeds between 700 and 2500 rpm at intervals. Figure 9 shows the cases in which the instability occurs. There is an inverse relationship between the sliding speed and the disk bulk temperature: the critical sliding speed causing the instability decreases linearly with increasing disk temperature. This test suggests that the disk bulk temperature must be considered as an important factor to predict the judder occurrence in a disk brake system. In case of the initial disk temperature over 200℃, the thermo-elastic instabilities do not occur in our experiments; but the disk temperature rapidly decreases when we operate the foot brake pedal. We assume that it is due to the brake fade phenomenon. Brake fade is termed to describe the decrease in braking effectiveness as the temperature of the brake increases. Generally, the disk brake fade phenomenon could occur at 250℃ (Lee and Barber, 1994; Cho, 1997). Therefore, the disk temperature affects the judder speed only before the fade occurs. In summary, at temper-

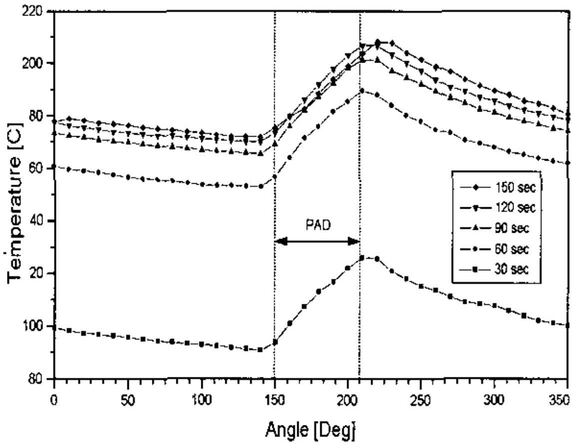
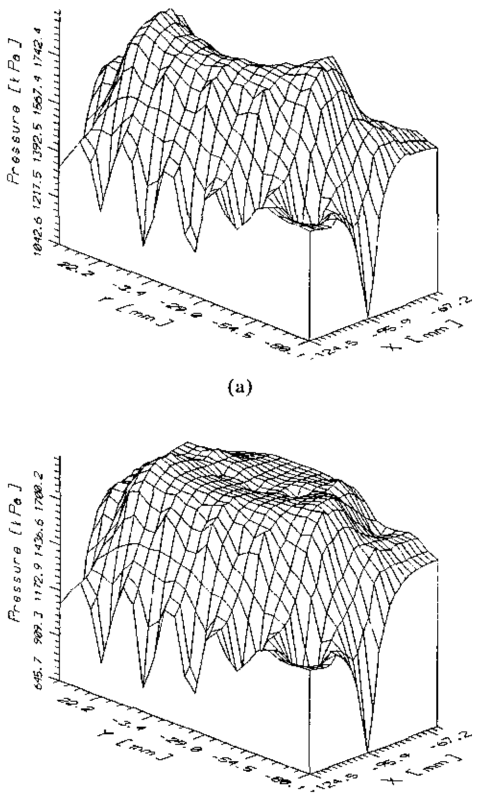


Fig. 11 Temperature distributions along sliding centerline in automotive disk



(a)

(b)

Fig. 12 Contact pressure developed on a pad at (a) 30 seconds, and (b) 120 seconds

atures below the fade, the sliding speed on the occurrence of judder linearly deceases with increasing disk volume temperature. Once the brake fade occurs, the brake system becomes thermo-elastically stable but the friction force drop will deteriorate the brake system safety.

3.2 Numerical simulation

We constitute a finite element disk model (see Fig. 10) for running FFT-FEA and ABAQUS programs. Mechanical properties (provided by the manufacturer) are listed in Section 2.2. The friction coefficient on the contact surface is assumed as 0.37 (Cho, 1997). We simulate drag braking at a constant speed of 1900 rpm.

We apply 1.7 MPa contact pressure on the frictional area and the disk is initially at 20℃. Figure 11 shows the temperature distribution of the disk at four instants (30 sec, 60 sec, 90 sec, and 120 sec) along the sliding centerline. As shown in Fig. 11, we predict that the temperature rises faster under the pad contact area (150~210°) where the disk rotates counterclockwise. The effects of heat input on the pads and the natural convection into atmosphere are clearly noticed in Fig. 11 as the gradient near the leading edge is steeper than that on the trailing edge.

Figure 12 shows the contact pressure variations at two instants (30 and 120 sec.) due to the thermal deformation of the disk. Note that the contact pressure is initially uniform (1.7 MPa) over the pads. It is found from the simulation that the assumption of uniformly distributed pressure throughout the analysis of three-dimensional brake systems leads to false conclusions. As the

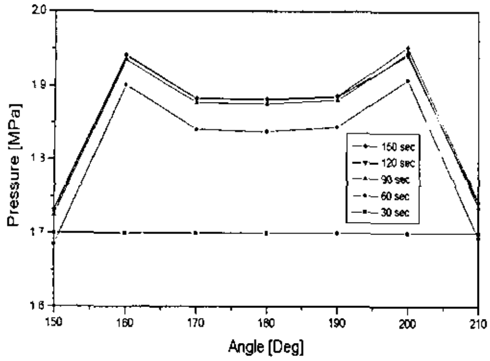


Fig. 13 Contact pressure distributions along the pad sliding centerline at various instants

disk and pads expand thermally, the distribution of the contact pressure is convex outwards and then approximately plateaus over the smaller area where the profile shows a weak "M" in the circumferential direction on the pad. Figure 13 summarizes the contact pressure profiles along the pad sliding centerline.

4. Conclusions

This paper proposes an effective three-dimensional thermo-elastic analysis technique for an automotive disk brake system, and conducts experiments on a brake dynamometer for examining factors that affect the judder occurrence. We draw following conclusions:

(1) We have identified thermo-elastic instabilities in an automotive disk brake by constructing a brake dynamometer that allows *multi-position temperature measurements*. The experiments have shown hot spots and banding during chattering where the temperature profile does not change with respect to the disk.

(2) It has been experimentally found that the disk bulk temperature severely affects the brake judder speed with inversely linear relationship; that is, the critical speed decreases linearly with increasing disk bulk temperature.

(3) We have proposed an effective hybrid algorithm for numerically analyzing the transient thermo-elastic problem for automotive disk brakes. The algorithm combines FFT-FEA (for thermal analysis) and non-linear static stress analysis programs. The algorithm considerably saves the computation time.

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