A Global Optimal Sliding-Mode Control for the Minimum Time Trajectory Tracking with Bounded Inputs

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A new design of the sliding mode control is proposed for the uncertain linear time-varying second order system. The proposed control drives system states to the target point in the minimum time with specified ranges of parametric uncertainties and disturbances, One of the advantages of the proposed control scheme is that the control inputs do not go beyond saturation limits of the actuators. The other advantage is that the minimum arrival time and the acceleration of the second order actuators system can be estimated with given parametric bounds and can be expressed in the closed form; conversely, the designer can select actuators based on the condition of the minimum arrival time to the target point. The superior performance of the proposed control scheme to other sliding mode controllers is validated by computer simulations.

Key Words : Sliding-Mode Control, Bounded Inputs, Minimun Time Trajectory Tracking

1. Introduction

Sliding mode control(SMC) originated from the variable structure control system was proposed and elaborated in the early 1960's in the Soviet Union by Emelyanov (1967) and Ikis (1976). SMC has been extensively studied due to invariance properties and the robustness against uncertain system parameters and disturbances. An extensive survey on the sliding mode control was performed. Fundamental theory, main results, and practical applications of variable structure control was introduced by Hung (1993). Recently, Kim and Lee (2000) devised a SMC with perturbation compensation to reduce the low-frequency tracking errors.

To achieve fast path tracking, an improved sliding mode control employing an optimal sliding surface was proposed by Ashchepkov (1983). The optimal sliding surface was decided by minimizing error performance index for a given initial condition. To improve the tracking behavior of the nonlinear second order dynamical systems, a moving sliding surface was proposed by Choi and Park (1994). The surface is initially designed to pass given initial errors and subsequentially is moved toward a fixed sliding surface.

Despite of the invariance properties and the robustness, Slotine (1991) presented that the conventional SMC had important drawbacks limiting its applicability, such as chattering or large control input requirement. Also, response of conventional SMC is sensitive to system perturbation during the reaching phase. The condition of the robustness of the conventional SMC is based on the assumption of the unlimited control inputs. Conversely, the robustness properties are guaranteed only as long as the control actuators do not saturate. The input limitation is one of important issues need to be considered for controller design in realistic and practical applications. To solve the robustness problem of the conventional SMC under the input torque saturation, Madani-Esfahani et al. (1990) proposed a scheme to estimate the region of the asymptotic stability with bounds on the control inputs but, which is not applicable due to excessive chatter. Lu and Chen

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(1995) devised a global sliding mode control scheme (GSMC), which ensures sliding behaviour throughout an entire response. With given uncertainty bounds, the merit of this control scheme is that the maximum and minimum values of the control action is estimated and the range of allowable reference input is obtained under the input limits.

In this paper, a model of the second time-varying system with uncertain parameters is specified in Sec. 2. To control the system, we propose a global optimal sliding mode control (GOSMC) for tracking to the reference input along the minimum time trajectory in Sec. 3. In Sec. 4, the minimum arrival time expressed in the closed form is derived. Finally, computer simulation results are shown and discussed.

2. Model of the System

We consider the linear time varying second order system with parameter uncertainties and disturbances.

$$\ddot{x}+a_1(t)\dot{x}+a_2(t)x=b(t)(u+d(t)) \quad (1)$$

where we suppose that system parameter $a_1(t)$, $a_2(t)$, and $b(t)$ are difficult to measure and the disturbance is unknown except their upper and lower bounds as follows:

$$\left.\begin{array}{l}\beta_{\min}\le b^{-1}(t)\le\beta_{\max}\\ \alpha_{1\min}\le b^{-1}(t)a_1(t)\le\alpha_{1\max}\\ \alpha_{2\min}\le b^{-1}(t)a_2(t)<\alpha_{2\max}\\ \max\limits_{t}|d(t)|<D\end{array}\right\} \quad (2)$$

Since all the physical systems have input torque limits, the bounds of the input torques are defined as

$$U_{\min}\le u\le U_{\max} \quad (3)$$

In this paper, a global optimal sliding mode control algorithm is devised, which drives the system with the uncertain parameters and disturbances, to the target point in the minimum time with limited inputs.

3. Design of Global Optimal Sliding Mode Controller

3.1 Global sliding mode control

To control the uncertain system defined in Eq. (1), a SMC with estimates of the uncertain parameters, shown in Eq. (4), is applied

$$u=-\hat{\beta}(c\dot{x}-\dot{f})+\hat{\alpha}_1\dot{x}+\hat{\alpha}_2x-\{\Delta\beta\,|\,c\dot{x}-\dot{f}\,|+\Delta\alpha_1|\dot{x}|+\Delta\alpha_2|x|+D\}sgn(s) \quad (4)$$

where

$$\hat{\beta}=\frac{\beta_{\max}+\beta_{\min}}{2},\ \Delta\beta=\frac{\beta_{\max}-\beta_{\min}}{2}$$

$$\hat{\alpha}_1=\frac{\alpha_{1\max}+\alpha_{1\min}}{2},\ \Delta\alpha_1=\frac{\alpha_{1\max}-\alpha_{1\min}}{2}$$

$$\hat{\alpha}_2=\frac{\alpha_{2\max}+\alpha_{2\min}}{2},\ \Delta\alpha_2=\frac{\alpha_{2\max}-\alpha_{2\min}}{2}$$

The proposed control can be applied to higher order systems similarly, but closed-form solution may not be obtained. The sliding mode control suggested by Lu and Chen (1995) is trivial except for the forcing function $f(t)$, where the sliding mode is defined in conjunction with $f(t)$ as

$$s=\dot{e}+ce-f(t) \quad (5)$$

where error state is $e=x-r$ with reference step input $r>0$, where the sign of r represents the direction of input. When $r<0$, similar approach except for the sign of input can be made. The forcing function drives the system states in any state space to the switching plane directly without reaching phase such that the GOSMC is robust during the reaching phase. In this paper, we propose a new forcing function which drives the system states along the minimum time trajectory. In addition to jumping to switching plane, the proposed forcing function makes helps us to calculate the minimum arrival time at the target point in a closed form. In order for the GOSMC to manage the system states to maintain on the sliding surfaces, the conditions on the forcing function $f(t)$ should be satisfied as

$$f(0)=\dot{e}_0+ce_0 \quad (6a)$$

$$f(t)\rightarrow 0 \text{ as } t\rightarrow\infty \quad (6b)$$

$$\dot{f}(t) \text{ should be bounded} \quad (6c)$$

The stability of the GOSMC satisfying the conditions (6) can be shown using Lyapunov function $V=(1/2)s^2>0$. The negative definite of the time derivative of the V except for $s=0$ ensures that the proposed control scheme guarantees asymptotic stability. The proof of the stability of the closed system is simple and was already shown by Luh and Chen (1995).

3.2 Design of the forcing function

In the stability analysis, the asymptotic stability of the closed loop system is guaranteed if the forcing function satisfies the condition Eq. (6). This, eventually, means that $s=0$ is satisfied. In this paper, we propose the following desired trajectory function that not only satisfies the condition in Eq. (6) but also is the minimum arrival time trajectory for pure mass systems. The initial and final conditions of the function are specified as follows

$$\left.\begin{array}{l}\text{for } t=0: x(0)=0,\ \dot{x}(0)=0\\ \text{for } t\geq t_f: x(t_f)=r,\ \dot{x}(t_f)=0\end{array}\right\} \tag{7}$$

where t_f is the final arrival time. The boundary conditions of the desired trajectory are specified as

$$\left.\begin{array}{l}x=\dfrac{a}{2}t^2\\ \dot{x}=at\\ \ddot{x}=a=\dfrac{v}{t_b}\end{array}\right\} \text{ for } 0\leq t<t_b \tag{8a}$$

$$\left.\begin{array}{l}x=r-\dfrac{a}{2}t_f^2+at_ft-\dfrac{a}{2}t^2\\ \dot{x}=a(t_f-t)\\ \ddot{x}=-a\end{array}\right\} \text{ for } t_b\leq t\leq t_f \tag{8b}$$

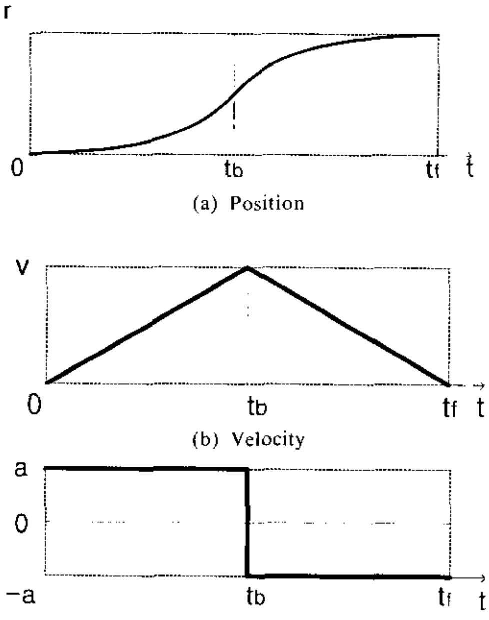
where v is the maximum velocity, a is the acceleration, and t_b is the mid travel time. The profile of the trajectory is shown in Fig. 1.

The forcing function is specified as

$$\begin{aligned}f(t)&=\dot{x}+c(x-r)\\ &=at+c\left(\frac{a}{2}t^2-r\right) \quad \text{for } 0\leq t<t_b \end{aligned} \tag{9a}$$

$$f(t)=a\{(t_f-t)+c(t_ft-\frac{1}{2}t_f^2-\frac{1}{2}t^2)\} \quad \text{for } b_b\leq t\leq t_f \tag{9b}$$

where $f(t)=0$ for $t>t_f$.



(c) Acceleration

Fig. 1 The minimum time trajectory

4. Estimation of the Minimum Time Trajectory

The proposed control yields the asymptotic stability of the uncertain second order system. However, the input torques of the system are bounded as in Eq. (3). Within this bounded range, we design a control scheme to estimate the minimum arrival time to the reference inputs. To do this, we divide the proposed controller into two parts according to its magnitude depending on the sign of s as follows:

$$\begin{aligned}u_h=&\hat{\beta}(-c\dot{x}+\dot{f})+\hat{\alpha}_1\dot{x}+\hat{\alpha}_2x+\{\Delta\beta\,|-c\dot{x}\\ &+\dot{f}\,|+\Delta\alpha_1|\dot{x}|+\Delta\alpha_2|x|+D\} \text{ for } s<0\end{aligned} \tag{10a}$$

$$\begin{aligned}u_l=&\hat{\beta}(-c\dot{x}+\dot{f})+\hat{\alpha}_1\dot{x}+\hat{\alpha}_2x-\{\Delta\beta\,|-c\dot{x}\\ &+\dot{f}\,|+\Delta\alpha_1|\dot{x}|+\Delta\alpha_2|x|+D\} \text{ for } s>0.\end{aligned} \tag{10b}$$

Since the closed-loop system is asymptotically stable, $s=\dot{s}=0$ becomes satisfied for $t\geq 0$. Using this, we have the following equation by exploiting Eq. (5)

$$\ddot{x}=-c\dot{x}+\dot{f}. \tag{11}$$

Rewriting Eq. (10) using Eq. (11) yields

$$u_h=\hat{\beta}\ddot{x}+\hat{\alpha}_1\dot{x}+\hat{\alpha}_2 x+\{\Delta\beta\,|\ddot{x}|+\Delta\alpha_1|\dot{x}|+\Delta\alpha_2|x|+D\} \quad (12a)$$

$$u_l=\hat{\beta}\ddot{x}+\hat{\alpha}_1\dot{x}+\hat{\alpha}_2 x-\{\Delta\beta\,|\ddot{x}|+\Delta\alpha_1|\dot{x}|+\Delta\alpha_2|x|+D\} \quad (12b)$$

Equation (12) can be rearranged according to the input profile and trajectory tracking time. Equation (12a) is expressed as:

$$\left.\begin{aligned} u_h&=\beta_{\max}\ddot{x}+\alpha_{1\max}\dot{x}+\alpha_{2\max}x+D \text{ for } 0\le t<t_b \\ u_h&=\beta_{\min}\ddot{x}+\alpha_{1\max}\dot{x}+\alpha_{2\max}x+D \text{ for } t_b\le t\le t_f \end{aligned}\right\}. \quad (13a)$$

For the time division $0<t<t_b$, we have $|\ddot{x}|=\ddot{x}=a$, and for $t_b\le t\le t_f$, also, we have $|\ddot{x}|=-\ddot{x}=-a$. With the same procedure, Eq. (13b) is expressed as

$$\left.\begin{aligned} u_l&=\beta_{\max}\ddot{x}+\alpha_{1\min}\dot{x}+\alpha_{2\min}x-D \text{ for } 0\le \tau<t_b \\ u_l&=\beta_{\max}\ddot{x}+\alpha_{1\min}\dot{x}+\alpha_{2\min}x-D \text{ for } t_b\le \tau\le t_f \end{aligned}\right\} \quad (13b)$$

To estimate the maximum of the input, we snbstitute the Eq. (8) with $\gamma=1/4at_f^2$ into Eq. (13a), which yields

$$\begin{aligned} u_h&=aW+D \quad \text{for} \quad 0\le t<t_b \\ &aX+D \quad \text{for} \quad t_b\le t\le t_f \end{aligned} \quad (14)$$

where

$$W=\beta_{\max}+\alpha_{1\max}t+\frac{1}{2}\alpha_{2\max}t^2$$

$$X=-\beta_{\min}+\alpha_{1\max}(t_f-t)-\alpha_{2\max}\left(\frac{1}{4}t_f^2-t_f t+\frac{1}{2}t^2\right).$$

With the same approach, substituting the functions of the minimum time trajectory into Eq. (12b) and arranging it yields

$$\left.\begin{aligned} u_l&=aY-D \text{ for } 0\le t<t_b \\ &=aZ-D \text{ for } t_b\le t\le t_f \end{aligned}\right\} \quad (15)$$

where

$$Y=(\beta_{\min}+\alpha_{1\min}t+\alpha_{2\min}t^2)$$

$$Z=-\beta_{\max}+\alpha_{1\min}(t_f-t)-\alpha_{2\min}\left(\frac{1}{4}t^2{}_f-t_f t+\frac{1}{2}t^2\right)$$

The maximum of u_h in Eq. (14) and the minimum of u_l in Eq. (15) exists at a certain time in the tracking time range. Using the maximum and minimum values of the input we can estimate the maximum value of the acceleration of the uncertain system with the bounded input. We express the maximum and minimum values of the Eqs. (14) and (15) as

$$\max_{t\ge 0}\ u(t)_h=aT_{\max}+D, \quad (16)$$

$$\min_{t\ge 0}\ u(t)_l=aT_{\min}-D. \quad (17)$$

The input torques are bounded as specified in Eq. (3). The physical bounds always exist on all the systems. Hence, for the realistic and practical application, the applied input values should be within the physical bounds. The maximum and minimum input values are bounded as

$$U_{\min}\le \min u(t)\le \max\ u(t)\le U_{\max} \quad (18)$$

Employing the specified torque bounds, we can range the permissible acceleration of the system as follows:

$$\frac{U_{\min}+D}{T_{\min}}\le a\le\frac{U_{\max}-D}{T_{\max}} \quad (19)$$

To estimate the minimum time of the trajectory tracking, we need to obtain the maximum value of the u_h in Eq. (16) and the minimum value of the u_l in Eq. (17). To do this, we should differentiate u_h and u_l with respect to time. The trajectory functions are bounded and closed except the acceleration profile at the mid time t_b for $t\in[0, t_f]$. At this time, we can not differentiate but can get the limit value. In addition to this, we can get several points where the time derivative of the control inputs in Eqs. (14) and (15) become zero. The maximum or minimum values are obtained out of these candidates. Differentiating Eqs. (14) and (15) with respect to time within the differentiable range yields

$$\left.\begin{aligned} \frac{du_h}{dt}&=\alpha_{1\max}a+\alpha_{2\max}at && \text{for } 0\le t<t_b \\ \frac{du_h}{dt}&=-\alpha_{1\max}a+\alpha_{2\max}a(t_f-t) && \text{for } t_b\le t\le t_f \end{aligned}\right\} \quad (20)$$

In Eq. (20), for each time division, we get the solution of $\frac{du_h}{dt}=0$ for $t\ge 0$ as

$$t_{hd}=t_f-\frac{\alpha_{1\max}}{\alpha_{2\max}} \text{ for } t_b\le t\le t_f \quad (21)$$

Substituting the solution in Eq. (21) into Eq. (16)

yields

$$u_h|_{t=t_{hd}}=aT_{\max}|_{t=t_{hd}}+D \text{ for } t_b\leq t\leq t_f. \quad (22a)$$

At initial, final, and mid points of tracking time, the values of the control inputs are calculated as:

$$u_h|_{t=0}=a\beta_{\max}+D \quad (22b)$$

$$u_h|_{t=t_{b-0}} =a(\beta_{\max}+\alpha_{1\max}t_b+\frac{1}{2}\alpha_{2\max}t_b^2)+D \quad (22c)$$

$$u_h|_{t=t_{b+0}} =a(-\beta_{\min}+\alpha_{1\max}t_b+\frac{1}{2}\alpha_{2\max}t_b^2)+D \quad (22d)$$

$$u_h|_{t=t_f}=a(-\beta_{\min}+\frac{1}{2}\alpha_{2\max}t_f^2)+D. \quad (22e)$$

Since the input values of the Eqs. (22b) and (22d) are obviously smaller than that of the Eq. (22c), the maximum input value can be written as

$$\max u(t)=\max\{u_h|_{t=t_{hd}}, u_h|_{t=t_{b-0}}, u_h|_{t=t_f}\} \text{ for } 0\leq t<t_b \quad (23)$$

In the same way as shown in obtaining the maximum value, the minimum value candidates of u_l can be obtained. By referring to the trajectory profile, we can get a minimum candidate t_{ld} by differentiating u_l as

$$t_{ld}=t_f-\frac{\alpha_{1\min}}{\alpha_{2\min}} \text{ for } t_b\leq t\leq t_f. \quad (24)$$

Substituting t_{ld} in Eq. (24) into Eq. (17) yields

$$u_l|_{t=t_{ld}}=aT_{\min}|_{t=t_{ld}}-D \text{ for } t_b\leq t\leq t_f. \quad (25a)$$

We can get other minimum candidates at the piece-wise continuous mid and final points of the tracking time, which can be obtained as

$$u_l|_{t=t_{b-0}}=a(\beta_{\min}+\alpha_{1\min}t_b+\alpha_{2\min}t_b^2)-D \quad (25b)$$

$$u_l|_{t=t_{b+0}} =a(-\beta_{\max}+\alpha_{1\min}t_b+\frac{1}{2}\alpha_{2\min}t_b^2)-D \quad (25c)$$

$$u_l|_{t=t_f}=a(-\beta_{\max}+\frac{1}{4}\alpha_{2\min}t_f^2)-D. \quad (25d)$$

Since the value of Eq. (25b) is bigger than that of Eq. (25c), the minimum value of the control input is selected among the candidates in Eqs. (25a), (25c), and (25d) as

$$\min u(t)=\min\{u_l|_{t=t_{ld}}, u_l|_{t=t_{b+0}}, u_l|_{t=t_f}\} \text{for } t_b\leq t\leq t_f. \quad (26)$$

In this paper, the goals of the proposed control scheme are to achieve tracking of the desired trajectory and to estimate the arrival time at the reference input of the second order time-varying system with unknown but bounded parameters and disturbances. The proposed scheme is more realistic and applicable than the scheme to find the range of allowable reference input proposed by Lu and Chen. One of the eminent merits of the proposed control scheme is that we can get the closed-form solution of the arrival time t_f which we can calculate easily without numerical approaches.

In the electric motor system, where the stiffness coefficient is not considered such that $a_2=0$, the minimum arrival time is expressed in a closed form. To derive the closed-form solution, we use the Eqs. (21) through (26). Since α_2 is zero, there does not exist a solution in Eq. (21). By evaluating the values of Eqs. (22c) and (22d), we can tell that the value of Eq. (22c) is larger than that of Eq. (22d). Hence, the maximum value becomes

$$\max u(t)=u_h|_{t=t_{b-0}} \text{ for } 0\leq t. \quad (27)$$

Also, in finding the minimum value, since $\alpha_2=0$, we know that there does not exist a solution in Eq. (24). By evaluating the last two Eqs. (25c), and (25d), we can tell that $u_l|_{t=t_f}$ is the minimum, which is expressed as

$$\min u(t)=u_l|_{t=t_f} \text{ for } 0\leq t \quad (28)$$

From the minimum and maximum values, we can decide the minimum arrival time within the input value limit. By substituting $a=4r/r_f^2$ into Eqs . (22c) and (25d), and arranging them within the bounded region in Eq. (3), we obtain the following equations as

$$\frac{4r}{t_f^2}(\beta_{\max}+\alpha_{1\max}t_b)+D\leq U_{\max} \quad (29)$$

$$U_{\min}\leq-\frac{4r}{t_f^2}\beta_{\max}-D. \quad (30)$$

Rearranging Eq. (29) yields second order inequality as:

$$(U_{\max}-D)t_f^2-2r\alpha_{1\max}t_f-4r\beta_{\max}\geq 0. \quad (31)$$

By solving the Eq. (31), the minimum time candidate to arrive at the desired final position r can be obtained in a closed form as

$$t_{hmin}=\frac{r\alpha_{1\max}+\sqrt{r^2\alpha_{1\max}^2+4r\beta_{\max}(U_{\max}-D)}}{U_{\max}-D}. \quad (32)$$

In the same way, another minimum time candidate can be obtained by solving the inequality (30) as follows:

$$t_{lmin}=\sqrt{\frac{4r\beta_{\max}}{-U_{\min}-D}}. \quad (33)$$

The minimum arrival time is

$$t_{\min}=\max\{t_{hmin},\ t_{lmin}\} \quad (34)$$

As shown in Eqs. (33) and (34), the minimum arrival time is expressed in closed form clearly. Therefore, we can easily calculate the minimum arrival time at the desired final condition. Conversely, we can easily design or select appropriate motors to drive mechanical systems according to control specifications

4. Computer Simulation

4.1 Model of the motor system

In the computer simulation, we apply the proposed GOSMC to the BLDC motor system, and show that the performance of the proposed control scheme is more realistic and superior to that of the GSMC and SMC. We apply the proposed GOSMC to the motor system with unknown but bounded parameters and disturbances, and show that the estimated minimum arrival time becomes quite near to the simulation result. We describe the well known motor dynamics as

$$\ddot{\theta}+a_1\dot{\theta}=b(u+d) \quad (35)$$

where θ is the position angle, $a_1=B/J$ is composed of the damping coefficient B and the moment of inertia J, $b=K_tK_c/J$ is composed of the torque coefficient of motor K_t and PWM inverter currents K_c. The disturbance d is the Coulomb friction. The only parameter K_t is given in the catalogue, and other parameter should be measured or be estimated. In this reason, the parameters are uncertain, but their upper bounds can be specified in general.

The values of the parameters for a 700 W permanent-magnet synchronous BLDC motor are taken from those of Lu and Chen's reference to compare controller performance. The bounds on the uncertain parameter, disturbances, and control torques are specified as

$$\beta_{\min}=2.8743\times10^{-4}\leq b^{-1}\leq 4.3114\times10^{-3}=\beta_{\max}$$
$$\alpha_{1\min}=1.6679\times10^{-3}\leq b^{-1}a_1\leq 3.7528\times10^{-3}=\alpha_{1\max}$$
$$|d|<0.1=D(\mathrm{V})$$
$$-5\leq \max u\leq 5 \quad (\mathrm{V})$$

4.2 Results of the computer simulation and comparison

In the simulation, we set the desired final position $r=15$(rad), and set system parameters a_1, b, and d to 8, 2.5×10^3, and 0.08 sgn(θ), respectively. With given parameter bounds, we can calculate the maximum acceleration, the maximum input torques, and the minimum arrival time using Eqs. (27), (28), and (34), which are

$$t_{\min}=0.2415(\mathrm{s}),$$
$$a=4r/t_f^2=1028.4(\mathrm{m/s^2}),$$
$$\max u=u_h|_{t=0.1205}=5(\mathrm{V})$$

where $t_{\min}=t_f$ is obtained using the closed-form equation. We compare the calculated values with simulation results.

In the simulation, we compared three different controllers: GOSMC, GSMC, and the conventional SMC. We analyze the performance of the proposed controller by evaluating the simulation results. Especially, we compares the controllers in two aspects: the desired trajectory tracking capability and the obeyance of control torque limits.

According to the simulation results, it took 0. 242 (seconds) for GOSMC to arrive at the reference input as shown in Fig. 4(a), but took 0. 4 (seconds) for GSMC in Fig. 3(a), and even took more than 0.5 (seconds) for SMC controller in Fig. 2(a). The arrival time shown in GOSMC is the fastest out of the three controllers. The reason is that GOSMC utilizes the control torques fully within the torque limits but not the other controllers as shown in Figs. (b). The SMC shows

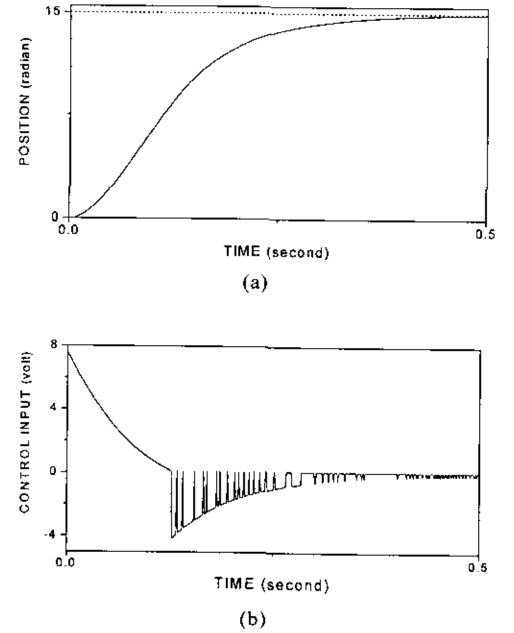


Fig. 2 SMC:(a) Reference trajectory tracking (b) Control input

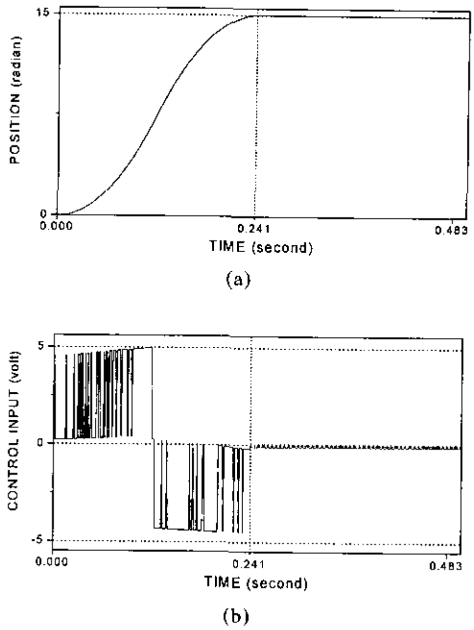


Fig. 4 GOSMC: (a) reference input tracking (b) control input

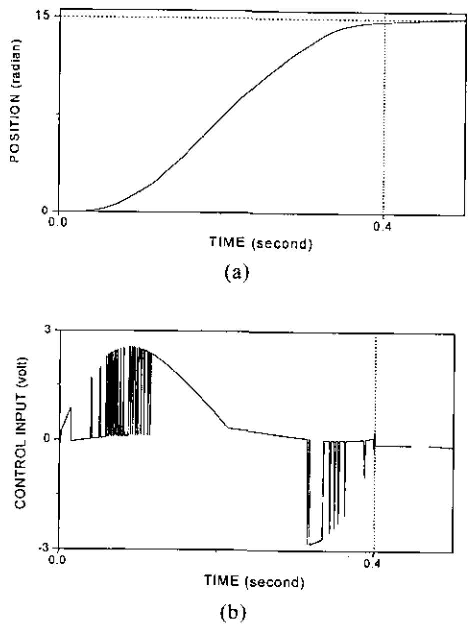


Fig. 3 GSMC: (a) reference input tracking (b) control input

poor response despite of applying excessive control torques as shown in Fig. 2(a). GOSMC does not show any overshoot or chattering over transient mode despite of the fastest dynamic tracking. GOSMC does not trespass against the control torque limits as shown in Fig. 4(b). Though GSMC does not trespass against the control torque limits, it does not fully exploit the control torques.

One of the eminent advantages of GOSMC is to estimate the arrival time at the reference input. The estimated arrival time based on the closed-form equation become quite near to simulation results. Hence, we don't have to simulate the closed-loop system by using a numerical algorithm such as the Runge-Kutta method. The estimation scheme of the arrival time in conjunction with the maximum acceleration estimation would be very helpful in selecting and designing motor systems.

5. Conclusion

A global optimal sliding mode control (GOSMC) was proposed to control the second order system with uncertain but bounded parameters and disturbances within limited control input. The proposed controller drives the system states along "the minimum time trajectory" within the control input limit. If the desired final and the bounds of the uncertain parameters and disturbances are specified, the arrival time and the acceleration are expressed in closed-form equations. The proposed controller was applied to the BLDC motor with uncertain parameters. Simulation results of the proposed controller are quite similar to the closed-form equation results, and showed the best performance compared with other SMCs. The closed-form equation can be utilized in selecting the actuators for the mechanical system without a computer simulation.

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